Международный научный Симпозиум
к 110-летию со дня рождения Саймона Кузнеца

“НАУЧНОЕ НАСЛЕДИЕ САЙМОНА КУЗНЕЦА И ПЕРСПЕКТИВЫ РАЗВИТИЯ ГЛОБАЛЬНОЙ И НАЦИОНАЛЬНЫХ ЭКОНОМИК В XXI ВЕКЕ”

СБОРНИК НАУЧНЫХ ТРУДОВ

том 2

МИРСК

Киев, 25-27 мая 2011 г.

Comments on the Occasion of the Award of the Simon Kuznets Medal by Robert W. Fogel

I am honored to receive the Simon Kuznets medal from the Simon Kuznets International Institute for Development and Self-Organization. I regret that I am unable to join you at the event in Kiev, but I send my best wishes for a successful event and am pleased that I am able to participate in this limited way.

I congratulate the Institute for promoting innovative and cooperative methods of economic development, something that was at the heart of Kuznets' life work.

To many of those who studied under Kuznets, his demonstrations and discussions of the art of measurement were the most valuable aspect of their training. By the art of measurement I mean not merely statistical and econometric theory, which are more important but quite adequately conveyed in papers and books. A far more difficult question in practice is how to apply statistical methods and economic models to the incomplete and biased data with which economists normally work and still produce reliable estimates of key economic variables and parameters. That question cannot be answered by a simple rule because economic data are so variable in quality and because the circumstances under which a given set of defects in the data are tolerable depends on the issues that are being addressed, on the statistical and analytical procedures that are being employed, and on the sensitivity of the results to systematic errors in the data, to the choice of behavioral models, and to the choice of statistical procedures.

Good judgment on these issues is developed with experience, and Kuznets tried to convey his rich experience on these matters in the same way that doctors use rounds to teach medical students the art of diagnosing illnesses. Kuznets conducted his "rounds" in three different ways: first, in his lectures on economic growth where he discussed problems of measurement and gave numerous examples of good and bad attempts to measure key economic variables and relationships; second, in his seminar on the application of quantitative methods to the analysis of time series, which was largely a laboratory course in which students applied various procedures to typical bodies of economic data, and collectively discussed the problems and interpreted the outcomes; third, in his supervision of dissertations, during which Kuznets varied his approach to the degree of independence desired by the student, while always serving as a sympathetic, thorough, and penetrating critic.

SIMON KUZNETS: CAUTIOUS EMPIRICIST OF THE EASTERN EUROPEAN JEWISH DIASPORA

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The construction of hypotheses is a creative act of inspiration, intuition, invention; its essence is the vision of something new in familiar material. The process must be discussed in psychological, not logical, categories; studied in autobiographies and biographies, not treatises on scientific method; and promoted by maxim and example, not syllogism or theorem.

Milton Friedman, "The Methodology of Positive Economics"

The announcement, in September 1971, that Simon Smith Kuznets (April 30, 1901 - July 9, 1985) was to receive the third Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel surprised no one[1] in the economics community. Kuznets built the system of national income accounting that allows accurate measurement of national product. Over the course of his more than half a century of service to the profession, Kuznets laid much of the foundation of modern development economics by providing the first comprehensive analysis of international growth data from developing countries. His research also made substantive contributions to the study of economic development, emphasizing the links between inequality and economic growth and highlighting important distinctions, not understood at the time, between today's underdeveloped countries and the state of today's rich countries before industrialization. He also pioneered, jointly with Milton Friedman, the foundational concepts of human capital and lifetime income.

Yet there is another side of Simon Kuznets less familiar to his colleagues, which this book highlights. Despite being one of the most distinguished American economists, Kuznets was actually born to a family of well-off Jewish bankers and furriers in Pinsk (formerly in Russia, now in Belarus) and grew up in what is now Ukraine before immigrating via Poland to the United States. Astonishingly, given that his impact on the methodology of economics rivals that of the much-acclaimed economists Kenneth Arrow and Paul Samuelson, there has been hardly any scholarship on Kuznets's life and thought. The few that have studied him see his background as little more than a preamble to his scholarly work.[2] Yet, as I argue below, Kuznets's identity and past, and his attempt to understand them quantitatively through the empirical study of the Eastern European Jewish Diaspora, were central to his

[1] A possible exception was Wassily Leontief, who upon hearing that a Russian economist was to be announced to have won the Nobel Prize prepared to make a statement.

[2] See, for example, Fogel, Robert W. 1987. Some Notes on the Scientific Methods of Simon Kuznets; Fogel, Robert W. 2000. Simon S. Kuznets: April 30, 1901-July 9, 1985; Kapuria-Foreman and Perlman 1995; Abramovitz, Moses. 1986; Simon Kuznets, 1901-1985. *Journal of Economic History* 46 (1):241--246.

understanding of economic development. The standard neglect of Kuznets's background, and of him entirely, is not altogether surprising, however, given that Kuznets labored assiduously to maintain a wall of separation between the two facets of his life. The same cautious empirical methodology that has made Kuznets a challenging subject for historians of economics also hid the personal motivation behind the studies to which he applied it. The secular cosmopolitan life he built for his family obscured his Eastern European[3] Jewish ancestry. A universalistic commitment to empirical rigor and appropriate subjects of economic inquiry protected from the economics community his abiding fascination with his past.

My window on Kuznets is therefore his writing about and relation to the history and economics of the Jews. These works are collected for the first time in these volumes. Some of them have been previously published, two of them even in their complete form and in English. Many of the most interesting works were unpublished, published only in Hebrew or scattered so broadly as to obscure the corpus they represent. Once assembled, even the fairly superficial inspection effected by this introduction demonstrates their close connection to the innovative ideas he brought to early development economics.

In "Economic Structure and the Life of the Jews", Kuznets builds a model of the path of Jewish inequality closely resembling that in his celebrated Presidential Address to the American Economic Association, published in 1955. Beyond the similarity in the formal approach of these two pieces, his substantive claims about the inverted-U shape of income inequality among Jews parallel his broader "Kuznets curve" hypothesis about economic development and income inequality. Thus, Kuznets's path-breaking work, perhaps the first to take seriously the relationship between development and inequality, seems inextricable from his coincident work on the economic history of the Jews. In fact, it seems likely that the severe inequality among Jews that Kuznets documents quantitatively in later work[4] and saw throughout his life, along with its connection to the economic history of the Jews,
played a key role in motivating his focus on distribution.

The influence of Kuznets's past extends to his emphasis, late in his career, on the role of culture, institutions and context in economic development. His views, now fairly widely accepted, were initially highly controversial coming against the backdrop of the linear, materialistic and universalistic theories of development prevalent at the time, such as those of Paul Rosenstein-Rodan, Arthur Lewis, Raul Prebisch and W. W. Rostow. The turn away from purely measurable economic factors and toward these "softer" considerations begins with, and may well have been driven by, his early study of Jewish economic history, as well as the course of his own multicultural life.

[3] To avoid Russian chauvinism, I use the term "Eastern European" to broadly refer to the entirety of the Russian imperialist-Jewish pale. However, it should be noted that Kuznets in his work, along with many others at the time, did not respect such contemporary distinctions and typically refers to what I call Eastern European Jewry as simply Russian Jewry.

[4] Kuznets, Simon S. 1972. Economic Growth of U.S. Jewry. *Papers of Simon Smith Kuznets, 1923-1985 (inclusive),1950-1980 (bulk), Correspondence and other papers relating to Jewish studies, ca.1959-1977 , Box 1, in folder \em Economic Structure of U.S. Jewry. Call Number: HUGFP88.25.*

Population, and the promises and threats it posed for development, was one of the last themes Kuznets took up in the late 60's and 70's. As a firm, if always balanced, opponent of neo-Malthusian hysteria about population, Kuznets clearly echoes his earlier arguments about the contributions (especially Jewish) immigrants made to the American economy. I would suggest that Kuznets saw in the "population bombers" repeats of the anti-immigrant hysteria that helped halt the wave of Eastern European Jewish immigration that had carried him to America. In his work on the "Israel's Economic Development", which appears in English translation for the first time in second volume of these works, Kuznets sees that nation's ideological embrace of immigrants as the lifeblood of that nation's exceptionally rapid economic growth.

A final connection between Kuznets's economics and his background is the most speculative, but perhaps most exciting as well. In the 1940's Kuznets wrote one of his last major works of pure data assembly on income flows jointly with Milton Friedman, *Income from Independent Professional Practice*. This work made an important step beyond data collection, wading deep into controversy that almost sunk the book's publication by arguing that medical licensure acted to raise doctor's wages by limiting competition. The book also pioneered the methodology of human capital accounting.

The former is striking given Kuznets's interest in the role of Jewish employment restrictions in spurring emigration and his singular unpublished[5] writing on "The Doctrine of Usury in the Middle Ages". Human capital, on the other hand, clearly plays a prominent role in Kuznets's work beginning with his study of Jewish educational patterns and his concurrent work on income inequality. While his work with Friedman is sufficiently rote and technically empirical that it is difficult to decipher with any certainty either the motivations that led to the study or conclusions drawn from it, it again seems unlikely that here, its thematic association with the struggles of Eastern European Jews is an accident.

In fact, this opacity of Kuznets's substantive views on economics as well as their motivation are the rule, not the exception, in his work in all fields, as I discuss in the penultimate section of this introduction. Ever the consummate student of his advisor, Wesley Clair Mitchell, Kuznets was the ultimate cautious empiricist, offering caveat upon caveat throughout his career for even the modest hypotheses he dared to venture. This careful positivist attempt to separate facts from conjecture was but one manifestation of a broader set of dualities in his life and work. Never did he reveal in his work the motivation leading him to it and almost never did he show the broader conclusions that might be drawn from it. In fact, whenever motivation was too apparent, as in his work on Jewish economic history, he did his best to conceal his work from his economics colleagues. Despite his status as a first generation Eastern European immigrant and his passionate identification with the state of Israel, he made every effort to raise his children as any other secular, mainstream, native-born American. Thus, Kuznets poses something of an enigma: motivated and inspired by understanding his past, he assiduously labored for universalism, both methodologically through empiricism and culturally through Americanism.

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[5] I believe I am the first to discover this writing in the course of my research for this paper. I owe a tremendous debt to Stephanie Lo, co-editor of this volume, for transcribing it in a legible form that made it possible for me to review it in detail. So other scholars may have the same benefit, this article is available at http://www.glenweyl.com, given that it is not directly relevant to this volume.

Yet while Kuznets's story may superficially seem paradoxical, precisely what makes it so interesting, and of at least some broader significance, is how it parallels the broader story of Jews of Eastern European descent in American economics. Jews rose more in economics than in any other academic discipline during the twentieth century, soaring from total exclusion to dominance of the field. As Derek Penslar[6] argues, while (especially Eastern European) Jews were well integrated into the natural sciences, they had been long excluded from the mainstream of European political and social affairs. The political events of the late 19th and early 20th centuries (emancipation, immigration and anti-Semitism) gave American Jews a socio-political voice and motivation for the first time. This process paralleled, and often intertwined with, the transformation of economics into a quantitative science. I conclude with the speculation that this unique intersection of technical skill, reinforced by traditional separation from Gentile social affairs, and fresh political motivation, which Kuznets typified, may have ideally suited the Eastern European Jewish Diaspora to transform contemporary economics. Obviously, this is a mere conjecture, drawn largely from a single anecdote, but it potentially offers an important avenue for future research. **The Life of a Scholar** Little is known[7] about the history of the Kuznets family. The name, which means "blacksmith" in Russian, is thought to have been adopted only a few generations before the family's migration to the United States and designed to conceal[8] the family's Jewish background in a culture where few Jews were in fact blacksmiths.[9] Despite their name, Kuznets's father was a banker.[10] Pinsk, where Kuznets spent his childhood and attended primary school, was immortalized in Chaim Weizmann's autobiography as a hotbed of Zionist youth activism.[11] At the age of nine or ten, Kuznets's family moved to Rovno in the Ukraine[12] to live with his mother's family, who were well-off furriers.[13] There he was raised in a combination of Russian from his mother and aunt and Yiddish from his grandparents.[14]

While his primary scholastic interests were secular, rather than Talmudic, Kuznets received training in Judaism and Jewish history.[15] After the Jewish expulsion

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[6] Penslar, Derek J. 2001. *Shylock's Children: Econonomics and Jewish Identity in Modern Europe*: Berkeley and Los Angeles: University of California Press; pp. 56-7.

[7] In several places, which I flag, secondary sources disagree on the sequence, and sometimes substances, of events. I have done my best to reconcile the sources, privileging those whose authors are more confident of their facts or closer to the actual events, such as family members.

[8] In fact, Simon was the only member of the family who maintained his name upon arriving in the United States; the rest of the family adopted the anglicized "Smith" (Britannica, Encyclopedia. *Kuznets, Simon* 2007).

[9] Kuznets, Paul. *Personal Interview: May 3* 2007.

[10] Stein, Judith. 2010. Personal Communication, February 10, 2010.

[11] Weizmann, Chaim. 1949. *Trial and Error: The Autobiography of Chaim Weizmann*: New York: Harper; pp. 16-28.

[12] Hauptman, Ruth Kuznets Pearson. *Personal Communication: February 6* 2010.

[13] Stein, Judith. 2010. Personal Communication, February 10, 2010.

[14] Hauptman, Ruth Kuznets Pearson. *Personal Communication: February 6* 2010.

[15] Kapuria-Foreman and Perlman (1995. An Economic Historian's Economist: Remembering Simon Kuznets. *The Economic Journal* 105 (433):1524--1547) and Fogel (2000. Simon S. Kuznets: April 30, 1901-July 9, 1985) disagree about whether Kuznets attended primary

from Ukraine during the Great War, Kuznets moved to Kharkov for his secondary education at the *gymnasium* and university.[16] His education spanned from Kharkov High School #2, from October 1916 to May 1917, to the Commercial Institute of Kharkov, from 1918 to July 1921.[17] In Kharkov, Kuznets was exposed to the Bundist school of Jewish, anti-zionist Marxism,[18] though his interest in and reaction to these influences are far from clear and do not clearly manifest in his later work..

Around the time of his move to Kharkov, his father and older brother left for the United States through Turkey, while he stayed behind with his mother and younger brother.[19] Because his mother was an invalid,[20] the remaining brothers were hesitant to follow their father. However, Kharkov University shut down with the onset of Civil War in Russia following the revolution of October 1917 and Kuznets briefly took up a position as a section head at the bureau of labor statistics in the Ukraine. In 1921 the family was, with many other Jews, deported back to Poland. Simon was briefly arrested for a reason that is not clear from available accounts, persuading the rest of family to join their father in the United States.[21] His mother, who for years had been suffering from symptoms resembling multiple sclerosis, died on the way to the West in Warsaw and the family eventually left through Dantzig. [22]

Kuznets arrived in New York in 1922 and his life[23] as known to the economics community began. Within two years he had received his B.A. and M.A. and after two further years of research he was awarded a Ph.D. in 1926 under the supervision of Wesley Clair Mitchell.[24] Mitchell, the founder of the National Bureau of Economic Research, was undoubtedly the greatest intellectual influence on Kuznets's career. In fact, he was the only economist Kuznets explicitly thanked in his Nobel Prize

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school in Kharkov or Pinsk. I privilege the Kapuria- Foreman and Perlman (1995) account as the authors cite a personal interview. Indeed, Stein (2009. Personal Correspondence with Vladimir M. Moskovkin) points to a memoir that Kuznets's niece wrote to deduce that the family moved from Pinsk to Kharkov when Kuznets was 14 years old.

[16] Fogel, Robert W. 2000. Simon S. Kuznets: April 30, 1901-July 9, 1985; p. 1.

[17] Stein, Judith. 2009. Personal Correspondence with Vladimir M. Moskovkin.

[18] Kapuria-Foreman, Vibha, and Mark Perlman. 1995. An Economic Historian's Economist: Remembering Simon Kuznets. *The Economic Journal* 105 (433):1524--1547.

[19] How and through where his brother and father left for the United States are not exactly clear, but this was the best I was able to piece together from various secondary accounts. See Britannica, Encyclopedia. *Kuznets, Simon* 2007 and Kapuria-Foreman, Vibha, and Mark Perlman. 1995. An Economic Historian's Economist: Remembering Simon Kuznets. *The Economic Journal* 105 (433):1524--1547

[20] Kuznets, Paul. *Personal Interview: May 3* 2007.

[21] Hauptman, Ruth Kuznets Pearson. *Personal Communication: February 6* 2010.

[22] Kuznets, Paul. *Personal Interview: May 3* 2007.
Hauptman, Ruth Kuznets Pearson. *Personal Communication: February 6* 2010.

[23] I do not provide a comprehensive biography of Kuznets's career, as its relevance to the contents of these volumes is limited. Instead, I aim here to provide an outline with emphasis on the aspects of his life most relevant to the connection between his thinking and his Eastern European Jewish heritage. For a more complete intellectual biography, see Kapuria-Foreman and Perlman (1995. An Economic Historian's Economist: Remembering Simon Kuznets. *The Economic Journal* 105 (433):1524--1547).

[24] Fogel, Robert W. 2000. Simon S. Kuznets: April 30, 1901-July 9, 1985.

autobiography, saying that he "owe(d Mitchell) a great intellectual debt".[25] In collaboration with and under the guidance of Mitchell, Kuznets began his early career by investigating empirical regularities in macroeconomic data in a series of books. First, his *Cyclical Fluctuations* investigated cyclical variation in retail commerce.[26] In *Secular Movements in Production and Prices,* Kuznets discovered for the first time the so-called "long" or "Kuznets" cycle, a low-frequency (fifteen to twenty year), low-amplitude fluctuation in economic activity previously unknown to researchers.[27] Finally, Kuznets completed the trilogy by considering extremely high-frequency seasonal movements in manufacturing output in *Seasonal Variations in Industry and Trade*.[28]

While working on his trilogy, Kuznets met and then married his wife, Russian-Canadian Jewish Edith Handler, in 1929.[29] They lived and had two children, Paul and Judith, in the dominantly Gentile Upper West Side.[30] Reinforcing this spatial divide from his past, Kuznets raised his children in a strictly secular, American manner, never attending synagogue and providing them no education in Russian language or culture. Nonetheless, Kuznets maintained a firm personal interest in Russian affairs, as a strong opponent of the Soviet Union, and was seen by his colleagues as something of an amateur expert on the Soviet economy. He also was an avid consumer of emerging Soviet literature, particularly dissident literature, perhaps building on the education in Russian literature his mother and aunt instilled in him.[31] Despite this private interest in Russia, his encounters with Soviet economists left him with the impression that they were more political apparatchiks than social scientists and he engaged in little scholarly dialogue with Russian academics. Furthermore, none of his interest in Russian culture and affairs filtered into his relationship to his wife or children. In addition to the strict line he drew between his past and the family life he was creating, Kuznets divided his personal and professional lives equally stringently, almost never discussing work at home or with friends outside the field. He had many such friends; though they were mostly academics, they were drawn from a variety of fields:
psychology, philosophy, sociology, public affairs, religion and art.[32]

The process of studying data on economic aggregates seems to have persuaded Kuznets that the available information was insufficient to supply the rigor and broad scope economists demanded. Kuznets therefore set out during the 1930's to build a system of comprehensive accounting for productive activity at the national level. His basic insight and approach, familiar to any student who has taken an introductory macroeconomics class, was to measure a nation's productive output by the income it generated. Kuznets set out to comprehensively measure income from all sources within

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[25] Kuznets, Simon S. *Autobiography* 1971.

[26] Kuznets, Simon S. 1926. *Cyclical Fluctuations*: New York: National Bureau of Economic Research.

[27] Kuznets, Simon S. 1930. *Secular Movements in Production and Prices: Their Nature and Their Bearing Upon Cyclical Fluctuations*: Boston: Houghton Miflin.

[28] Kuznets, Simon S. 1933. *Seasonal Variations in Industry and Trade*: New York: National Bureau of Economic Research.

[29] Ibid., p. 1.

[30] Stein, Judith. 2010. Personal Communication, February 10, 2010.

[31] Hauptman, Ruth Kuznets Pearson. *Personal Communication: February 6* 2010.

[32] Kuznets, Paul. *Personal Interview: May 3* 2007.

the United States; the framework he developed was eventually applied across the world and forms the basis of modern methods of measuring national product.[33] After rapid success in this ambitious project, Kuznets moved on to measure other, more detailed forms of income. In collaboration with Milton Friedman,[34] he began the work discussed extensively in the *Work with Milton Friedman* section below. During World War II, Kuznets applied his talent for aggregate accounting and statistical analysis to explore the limits of American productive capacity. His analysis helped impose discipline on a political process that demanded far more in service of the war effort than the U.S. economy was capable of turning out.[35]

After the war, Kuznets and his family moved from New York to Philadelphia, where since the early 1930's Kuznets had been commuting to teach at the University of Pennsylvania. When it came time to find a house in Philadelphia, Kuznets reversed course and placed the family in an overwhelmingly Jewish suburb north of the city. The war's end brought other changes. As news of the Holocaust horrors spread throughout the United States, Kuznets, like other American Jews, was deeply shaken. He greeted the founding of the state of Israel with enthusiasm. Almost immediately, Kuznets began to make nearly annual trips to the holy land, meeting with and assisting the nation's nascent economic policy elite and eventually becoming a primary force behind the founding of the Maurice Falk Institute for Economic Research in Israel, which remains a primary locus for economic research in the Jewish state.[36]

The end of the Second World War also brought a shift in Kuznets's attention to what he described as "a wider view, using national income estimates and their components to compare the performance of countries in different parts of the world on an international scale."[37]

This interest led him to write a series of ten articles under the titles "Quantitative Aspects of the Economic Growth of Nations," published in *Economic Development and Cultural Change* between 1956 and 1967. This set of articles formed the basis for Kuznets's most famous book , *Modern Economic Growth*, published in 1966. Yet, the most cited article of Kuznets's whole career, which emerged from his work on economic growth, was not actually a developed piece of research; rather, it was a hypothesis about the relationship between economic growth and income inequality that he debuted in his address[38] to the American Economic Association as President in 1955.

As his interest shifted from income to development, Kuznets twice changed universities. He left Pennsylvania in 1954 for six years in Baltimore at Johns Hopkins before spending the last decade of his career at Harvard University.

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33 Kapuria-Foreman, Vibha, and Mark Perlman. 1995. An Economic Historian's Economist: Remembering Simon Kuznets. *The Economic Journal* 105 (433):1524-1547; pp. 1529-33.

34 Friedman, Milton, and Simon S. Kuznets. 1945. *Income from Independent Professional Practice*: New York: National Bureau of Economic Research.

35 Kapuria-Foreman, Vibha, and Mark Perlman. 1995. An Economic Historian's Economist:

36 Kuznets, Paul. *Personal Interview: May 3* 2007.

37 Kuznets in statements transcribed by Fogel (1987, 34).

His last major work focused on the relationship between population, demographics and economic development. The connections between this work and his immigrant past are perhaps obvious and were first discussed by Kapuria-Foreman and Perlman.

After winning the Nobel Prize in 1971, Kuznets retired from Harvard and his career entered a new phase. He was in constant demand to lecture around the world and under no pressure to produce cutting edge research; the mathematicization of economic theory and the increasing availability of empirical data eroded the importance of Kuznets's comparative advantages in the field. While he continued to write, he began to explore various areas of economics that had previously been shut out by his drive to address quantitatively the crucial questions of economic development. First, he began, after a long career of sole authorship, to collaborate more closely and more often with his colleagues. Second, he further developed his interest in Jewish history (discussed extensively below), which had lain dormant since his influential "Economic Structure and Life of the Jews" was published in 1960. Finally, he increasing wrote broader articles, addressed more to methodology, survey and interpretation than to original empirical analysis.[39]

As he entered the final stage of his life, he also increasingly took advantage of the nearly unlimited opportunities he had to travel. The frequency of his trips to Israel increased, especially with the Falk Institute he helped found flourishing. Despite all this, he remained extraordinarily productive until health intervened: from 1980-1982 he published twelve articles. Then, after three years of struggling with Parkinson's disease, Simon Kuznets died on July 8, 1985.

The Development of Development Economics

"Development Economics", the branch of the discipline concerned with poor nations, is a young sub-field, even in a comparatively young discipline. As late as the early 1930's, most citizens of the developed world, even economists, did not understand that much of the world's population lived in relative poverty, essentially outside the system of industrial capitalism. Despite pervasive rhetoric about the "barbarism" or "lack of civilization" of colonized regions, Bardhan[40] argues that it was not until the development (by Kuznets) of national income accounting that it became possible to quantify the vast differences in material wellbeing between the developed and developing worlds.

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38 Kuznets, Simon S. 1955. Economic Growth and Income Inequality. *The American Economic Review* 45 (1):1-28. Remembering Simon Kuznets. *The Economic Journal* 105 (433):1524-1547; pp. 1534-5.

39 Kuznets, Edith, Robert W. Fogel, Marilyn Coopersmith, and Kathleen McCauley. 1989. Bibliography of Simon Kuznets. *Economic Development, the Family and Income Distribution,* 439-460.

40 Bardhan, Pranab. 1993. Economics of Development and the Development of Economics. *Journal of Economic Perspectives* 7 (2):129-142; p. 130.

Following Colin Clark's[41] seminal publication of systematic quantitative evidence of the "economic underdevelopment" in many parts of the world, there were a number of prominent "big theories" of development. Paul Rosenstein-Rodan[42] argued that industrialization is only profitable when undertaken simultaneously by many industries and thus requires a "big push" to succeed. Kurt Mandelbaum[43] attempted, with little success, to apply demand side Keynesian theory to explain underdevelopment. Raúl Prebisch[44] pointed to colonial legacy trade patterns that victimized developing nations, while W. Arthur Lewis[45] emphasized the misallocation of labor supply to the rural, rather than industrial, sector. Robert Solow[46] proposed an influential mathematical theory of economic growth in which poor nations were poor because of a lack of capital and technology. Perhaps most infamously, W. W. Rostow[47] argued that developing nations simply needed to position their economies as currently developed nations had been when they developed to begin a "take-off" to sustained economic growth through a series of "linear stages".

All these theories had at least two important broad features in common, which Kuznets called into question. First, all focused overwhelmingly on the aggregate problem of industrialization and growth, rather than on the effects of policies on, or through, their within-country distributions. Second, all viewed currently developing countries as following roughly the same growth trajectory (sharing the same production function, in Solow's terms) today as developed countries had followed in the past. While they disagreed about the causes of development, all believed in a universal recipe that had worked in the past for currently wealthy nations and would work in the future for currently underdeveloped nations. The following section discusses how insights Kuznets drew from his understanding and study of the Eastern European Jewish Diaspora led him to challenge the first of these views, while the section after it discusses the second.

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41 Clark, Colin. 1939. *The Conditions of Economic Progress*: London: MacMillan.

42 Rosenstein-Rodan, Paul N. 1943. Problems of Industrialisation of Eastern and Southeastern Europe. *Economic Journal* 53 (210-211):202-211.

43 Mandelbaum, Kurt. 1945. *Industrialisation of Backward Areas*: Oxford: Oxford University Press.

44 Economic Commission for Latin America, United Nations, and Raúl Prebisch. 1950. *The Economic Development of Latin America and Its Principal Problems*: United Nations Department of Economic Affairs.

45 Lewis, W. Arthur. 1954. Economic Development with Unlimited Supply of Labour. *The Manchester School* 22 (2):139-91.

46 Solow, Robert M. 1956. A Contribution to the Theory of Economic Growth. *Quarterly Journal of Economics* 70 (1):65-94.

47 Rostow, Walt W. 1960. *The Stages of Economic Growth: A Non-communist Manifesto*: Cambridge, UK: Cambridge University Press.

Jewish Inequality and the Kuznets Curve

Economic inequality has proved a severe and persistent feature of the economic life of Jews, especially those of Eastern European descent, for at least the last century and a half. As Kuznets argues on page ??? of his seminal 1975 article "Immigration of Russian Jews to the United States: Background and Structure," which is reproduced in our second volume, the combination of legal discrimination and urbanity likely combined to create enormous inequality within the Jewish community between a wealthy commercial and financial elite and the dislocated and discriminated-against masses. In fact, extreme inequality due to professional insecurity among European Jews was bemoaned as early as 1793 by prominent Jewish enlightenment (*maskilim*) intellectual David Friedländer in his classic *Akten-stücke, die Reform jüdischen Kolonien in den Preussischen Staaten betreffend*[48] and has long been seen as the source of the paradox in anti-Semitism that Jews have been viewed both as exploitative economic overlords and detestable paupers.

Kuznets argues that this inequality may have played an important role in the emigration of Eastern European Jews in two ways. First, inequality within the Jewish community may have reinforced prejudices within the non-Jewish population both in creating resentment of Jewish wealth and disdain for Jewish poverty, a theme that Penslar also picks up. Second, the dislocation and low economic position of much of the Jewish population, particularly when contrasted to the wealthy community elite, may have created a strong desire among some for selective migration to countries such as the United States with broader opportunity. While not discounting the role of Jewish persecution in Eastern Europe in spurring emigration, he argues that much of the differential Jewish migration may be attributable to greater Jewish urbanity and therefore greater exposure to dislocation and inequality associated with early stages of industrialization. While quantifying the extent of these differential rates of wealth disparity is nearly impossible given the lack of data, Kuznets documents in the 1972 manuscript "Economic Growth of U.S. Jewry," which appears in print for the first time on page ??? of this volume, that this trend has persisted, if not steepened, after Jewish immigration to the United States. He shows that while Jewish median income is only 10-20% higher than that of urban American Gentiles, mean income is almost twice that of the reference group, which suggests far greater Jewish inequality. Dramatic inequalities between impoverished newly arrived immigrants and wealthy established American Jewry, documented by Kuznets in his 1960 "Economic Structure and the Life of the Jews,"[49] were followed, after acculturation, by the wide cleavages of income between and within the professions (almost universally well- educated) Jews chose. Inequality among Jews is made all the more potent by the relative cultural segregation of Jews, which led to close contact among Jews of different classes. These inequalities were not merely an engaging subject for academic study in Kuznets's life, but of pressing personal relevance.

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48 Penslar, Derek J. 2001. *Shylock's Children: Econonomics and Jewish Identity in Modern Europe*: Berkeley and Los-Angeles: University of California Press.

49 Kuznets, Simon S. 1960. Economic Structure and Life of the Jews. *The Jews: their History, Culture and Religion*, 1597-1666; pp. 1621-3.

From the inequality between wealthy Jewish professional and lower middle class academic friends[50] to that surrounding him in his life in New York,[51] inequality among Jews appeared at all stages of his life. One can only speculate that the view down from the wealthy heights of his youth in Pinsk and Kharkov[52] fit the rough patterns described in his academic work.

Thus, it should not be surprising that income inequality became a central theme of Kuznets's understanding of both the economic structure of Jews and the development of economies. The latter theme is perhaps the most widely known of Kuznets's contributions to economics. In his 1954 Presidential Address to the American Economic Association, Kuznets argued that the evolution of income inequality and its relationship to economic growth should be central to the study of economic development. He also laid out a hypothesis about the nature of this relationship, which remains influential to this day, despite having been recently falsified even in the countries Kuznets studied with the advent of richer data.[53] His basic theory was that income inequality should first rise and then fall as a country developed economically. His reasoning ran roughly as follows: an industrializing country may be seen as being divided, à la Lewis,[54] into two broad sectors, one urban and industrial; the other rural, communal, and agricultural. Economic development involves the transfer of population from the second sector into the first. Given the greater inequality of outcomes and uncertainty in urban life, at least the initial stages of this move were sure to exacerbate the divide between rich and poor, even as they spurred the nation's overall economic development. Furthermore, the increasing wealth of the urban sector relative to the rural sector and the accumulation of savings by this capitalist sector exacerbate inequality.

However, countervailing forces emerge as the process of development proceeds. First, the continued thrust of industrialization eventually erases differences of income between urban and rural sectors, as increased mobility and labor market efficiency demand the equalization of wages for comparable work. Second, the increasing availability of education, social welfare, and other government services demanded by urban masses eventually spread economic opportunity widely, holding down early entrepreneurial profits through competition and expanding the range of people to whom the most attractive economic opportunities are available. Finally, the process of development is largely one of capital accumulation and with such accumulation comes decreasing returns to capital; in fact, in most standard economic models, the share of national income accruing to capital is constant as capital accumulates. Workers, who now have more machines to use, see the returns to human capital rise. Given increasing mass education, human capital is more equitably spread than physical capital. Therefore, wages rise and economic inequality eventually declines.

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50 This was described to me by his son Paul Kuznets in 2007 in a personal interview.

51 Fogel, Robert W. 2000. Simon S. Kuznets: April 30, 1901-July 9, 1985.

52 Kuznets, Paul. *Personal Interview: May 3* 2007.

53 Atkinson, Tony, Thomas Piketty, and Emmanuel Saez. 2009. Top Incomes in the Long Run of History.

54 Lewis, W. Arthur. 1954. Economic Development with Unlimited Supply of Labour. *The Manchester School* 22 (2):139-91.

Much less well-known are Kuznets's closely related theories of inequality among Jewish Eastern European migrants. In an early working draft of "Economic Structure," edited and published for the first time in this volume, Kuznets lays out what might be termed the "immigrant Kuznets Curve" hypothesis on pages ???-???. He argues that inequality within an immigrant population should first increase and then fall as that community develops economically within its destination country. His reasoning is that immigrants are likely to rise economically as they become accustomed to the economic conditions and culture of a country. So long as a steady stream of migration continues, inequality will arise between the wealthier migrants who have spent longer in the country and the poorer new arrivals. However, if migration tapers or ceases, inequality will abate as all members of the arrived group equilibrate to their natural income in the new country. Note that this reasoning largely parallels Kuznets's argument for the inverted U in the inequality- development relationship: the initial waves of migration to the city bring inequality between urban and rural areas and as the migration becomes complete, this inequality disappears.

This connection is further reinforced by the modeling exercises Kuznets used to quantitatively analyze these two parallel hypotheses. A core feature[55] of "Economic Growth and Income Inequality" is a toy model Kuznets builds that explores the possibility that the moving of population into a wealthier but more unequal sector might first generate and then reduce income inequality, under different assumptions about the relative income of the sectors. In the early version of "Economic Structure" in this volume, Kuznets includes a similar exercise where he explores the effects of changing distribution of migrants among cohorts over time on the patterns of intra-Jewish inequality, under different assumptions about the relative wages of the cohorts. The similarities between these are striking. Both consider a discrete number of sectors, assume various relative incomes in the sectors, allow shares of population allocated to the sectors to vary over time, and trace the implications for the path of income inequality (in the latter case both absolute and relative to the rest of the population). So that the reader may judge for herself the stylistic and substantive connections between these, both tables are shown on the following page.

(p. 12) The implications can be brought out most clearly with the help of a numerical illustration (see Table I). In this illustration we deal with two sectors: agriculture (A) and all others (B). For each sector we assume percentage distributions of total sector income among sector deciles: one distribution (E) is of moderate inequality, with the shares starting at 5.5 per cent for the lowest decile and rising 1 percentage point from decile to decile to reach 14.5 per cent for the top decile; the other distribution (U) is much more unequal, the shares starting at 1 per cent for the lowest decile, and rising 2 percentage points from decile to decile to reach 19 per cent for the top decile. We assign per capita incomes to each sector: 50 units to A and 100 units to B in case I (lines 1-10 in the illustration) ; 50 to A and 200 to B in case I1 (lines 11-20). Finally, we allow the proportion of the numbers in sector A in the total number to decline from 0.8 to 0.2.

[55] Kuznets, Simon S. 1955. Economic Growth and Income Inequality. *The American Economic Review* 45 (1):1-28; p. 13.

Table 1. Percentage Shares of 1st and 5th Quintiles in the Income Distribution for Total Population Under Varying Assumptions concerning Per Capita Income Within the Sectors, Proportions of Sectors in Total Number, and Intrasector Income Distributions[56]

	Proportion of Number in Sector A to Total Number						
	0.8 (1)	0.7 (2)	0.6 (3)	0.5 (4)	0.4 (5)	0.3 (6)	0.2 (7)
I. Per Capita Income of Sector A=50; of Sector B=100							
1. Per capita income of total population	60	65	70	75	80	85	90
Distribution (*E*) for Both Sectors							
2. Share of 1st quintile	10.5	9.9	9.6	9.3	9.4	9.8	10.2
3. Share of 5th quintile	34.2	35.8	35.7	34.7	33.2	31.9	30.4
4. Range (3-2)	23.7	25.9	26.1	25.3	23.9	22.1	20.2
Distribution (*U*) for Both Sectors							
5. Share of 1st quintile	3.8	3.8	3.7	3.7	3.8	3.8	3.9
6. Share of 5th quintile	40.7	41.9	42.9	42.7	41.5	40.2	38.7
7. Range (6-5)	36.8	38.1	39.1	39.0	37.8	36.4	34.8
Distribution (*E*) for Sector A, (*U*) for Sector B							
8. Share of 1st quintile	9.3	8.3	7.4	6.7	6.0	5.4	4.9
9. Share of 5th quintile	37.7	41.0	42.9	42.7	41.5	40.2	38.7
10. Range (9-8)	28.3	32.7	35.4	36.0	35.5	34.8	33.8
II. Per Capita Income of Sector A=50; of Sector B=200							
11. Per capita income of total population	80	95	110	125	140	155	170
Distribution (*E*) for Both Sectors							
12. Share of 1st quintile	7.9	6.8	6.1	5.6	5.4	5.4	5.9
13. Share of 5th quintile	50.0	49.1	45.5	41.6	38.0	35.0	32.2
14. Range (13-12)	42.1	42.3	39.4	36.0	32.6	29.6	26.3
Distribution (*U*) for Both Sectors							
15. Share of 1st quintile	3.1	2.9	2.7	2.6	2.6	2.7	3.1
16. Share of 5th quintile	52.7	56.0	54.5	51.2	47.4	44.1	40.9
17. Range (6-5)	49.6	53.1	51.8	48.6	44.8	41.4	37.9
Distribution (*E*) for Sector A, (*U*) for Sector B							
18. Share of 1st quintile	7.4	6.2	5.4	4.7	4.2	3.9	3.8
19. Share of 5th quintile	51.6	56.0	54.6	51.2	47.4	44.1	40.9
20. Range (9-8)	44.2	49.8	49.2	46.5	43.2	40.2	37.2

56 For methods of calculating the shares of quintiles, see p. 12 and fn. 6 of Kuznets, Simon S. 1955. Economic Growth and Income Inequality. *The American Economic Review* 45 (1):1-28.
The numerical illustration is only a partial summary of the calculations, showing

the shares of the lowest and highest quintiles in the income distribution for the total population under different assumption.[6] The basic assumptions used throughout are that the per capita income of sector B (nonagricultural) is always higher than that of sector A; that the proportion of sector A in the total number declines; and that the inequality of the income distribution within sector A may be as wide as that within sector B but not wider.

(fn. 6) The underlying calculations are quite simple. For each case we distinguish 20 cells within the total distribution-sets of ten deciles for each sector. For each cell we compute the percentage shares of both number and income in the number and income of total population, and hence also the relative per capita income of each cell. The cells are then arrayed in increasing order of their relative per capita income and cumulated. In the resulting cumulative distributions of number and countrywide income we establish, by arithmetic interpolation, if interpolation is needed, the percentage shares in total income of the successive quintiles of the country's population.
Some differences will not check because of rounding.

Table 2.
Illustrative Calculations of the Effect of "Recency of Entry Mix" on Movement of Average Income and Income Dispersion, Jews in the U.S.A., 1900-1950[57]

		Assumption I			Assumption II	
	Index of Average Income (1900=100) (1)	Index of Absolute Dispersion (1900=100) (2)	Relative Dispersion (Absolute Av. Income) (3)	Index of Average Income (1900=100) (4)	Index of Absolute Dispersion (1900=100) (5)	Relative Dispersion (Absolute Av. Income) (6)
1900	100	100	0.32	1001	100	0.43
1905	95	97	0.34	94	98	0.45
1910	94	102	0.34	93	100	0.46
1915	102	97	0.31	102	98	0.40
1920	113	86	0.25	117	86	0.31
1925	120	87	0.23	126	87	0.29
1930	130	65	0.16	139	65	0.20
1935	138	33	0.08	150	33	0.09
1940	137	39	0.09	148	39	0.11
1945	140	24	0.05	152	24	0.07
1950	140	24	0.05	152	24	0.07

Assumption I — Ratio of average income of groups by years of residence:
0-5 —1; 6-10 —1.5; 11-20 —2.0; over 20 —3.0.
Assumption II— Ratio of average income of groups by years of residence:
0-5 —1; 6-10 —2.0; 11-20 —3.0; over 20 —5.0.

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[57] This table appears in the early version of "Economic Structure" (page 99 of that draft), and is also included in this volume

The connections between Kuznets's understanding of Jewish and broader inequality is further reinforced at least weakly by the apparent temporal coincidence of "Economic Structure and Life of the Jews" and "Economic Growth and Income Inequality". The former was available in a fairly polished draft in April 1956[58] and the latter was given at the American Economic Association annual meeting at the end of 1954.[59] Presumably, given that it was not likely his highest work priority, Kuznets had been working on his article on Jewish economics for several years. Thus, it seems plausible that his insight into the relationship between income inequality and development, as well as the right way to model this interaction, actually arose from his work on the history of Jews. At least, his work on international income inequality seems to have been instrumental in allowing him to understand the evolution of Jewish economic structure; at most, his thinking about the economics of American Jewry may have led him to the broader connections between development and inequality.

Development and Culture

Kuznets's second objection to the initial thrust of development theory was his critique of the doctrine that developing countries could or should follow the development paths of presently developed countries. Kuznets was skeptical about how much might be learned about the future of the developing countries by studying the past of developed countries. In his book *Modern Economic Growth*,[60] the eponymous 1973 article, and several other articles, he lays out a variety of reasons why the development path of currently underdeveloped countries may differ fundamentally from the past of the developed nations.

Some of these differences were what would seem fairly obvious and conventional economic and technological distinctions. These are of less interest for my argument, but were not well-understood by economists at the time so I briefly summarize them here. Most currently underdeveloped countries have lower per-capita output than the Western nations, even before their industrialization, and are not great political powers, as were most wealthy nations during their period of development. Furthermore, consumer preferences have, to some extent, leapfrogged over early industrial goods. Service goods are a growing share of modern economies, making global demand faced by developing nations different from that in the 19th century.[61] Where currently developed countries existed at or near the technological frontier during much of their process of development, currently underdeveloped nations linger in a sort of limbo. The wide availability of certain technologies has rapidly improved standards of living in developing nations. Vaccinations, television and other consumer goods have become increasingly available to citizens of poor nations, extending the length and quality of life.

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58 It was sent to David Landes as a draft, which I have a copy of, on that date. Kuznets, Simon S. 1956. Economic Structure and Life of the Jews.

59 Kuznets, Simon S. 1955. Economic Growth and Income Inequality. *The American Economic Review* 45 (1):1-28.

60 Kuznets, Simon S. 1966. *Modern Economic Growth: Rate, Structure, and Spread*: New Haven: Yale University Press.

61 Kuznets, Simon S. 1966. *Modern Economic Growth: Rate, Structure, and Spread*: New Haven: Yale University Press; pp. 435-6.

At the same time, basic productive technologies, particularly in transport and capital goods, have failed to filter across national borders. This strange combination of consumerism without industrialism puts poor countries in a distinctly different technological state than that facing the West before its industrialization.[62]

More innovative was the emphasis Kuznets put on non-economic distinctions, such as institutions and culture. These were uncommon topics for study in economics in any form and thus Kuznets's focus on them was itself an important contribution. The first and probably least controversial of these heterodox factors was institutional. Most, though not all, currently developed countries reached that state during periods of growing democratic participation and under governments checked by the demands of individual rights and liberties. They also had developed, modern legal systems, largely professional civil services, and other modern governance institutions. To a large extent these institutions are weak or absent in many, if not most, developing nations. In addition, most developing nations had a far less benign experience with colonization than did the few currently developed nations that were at one time colonies. Their populations are largely the colonized, rather than the colonizers. As an exception that proved the rule on the plight of most developing countries, Kuznets in his work on Israel emphasizes the institutions that developed to deal with the state of constant war and the status of colonizer rather than colonized.

Compounding these problems for most developing countries is the fact that colonialism, as well as the presence of a developed global market outside the country, means that many sources of significant wealth, far beyond the usual productive capacity of the country, are available to select internationalized elites. This exacerbates problems of income and wealth inequality that may have been less severe in Europe during its development. Consequently, if institutions play an important role in economic development, as it seems likely they do, then it would be surprising if the development paths of currently developing countries were similar to the past of currently developed nations.[63]

More controversially, Kuznets highlights the cultural contrasts between currently backward nations and the past of wealthy nations. Unlike other divergences, he has little data to formalize these distinctions. Religious differences, absence of Western cultural heritage, and "colonial hangover" all make the cultures of developing nations systematically different from those of developed nations at their time of industrialization. Kuznets concedes that little is known about the relationship between such cultural factors and economic growth and therefore that the implications of such differences may or may not be important. But he emphasizes that it is worth keeping in mind the role of such cultural elements may play in supporting an entrepreneurial society by facilitating risk sharing and informal trade, efficiently allocating resources to new endeavors, and fostering a focus on the educational and intellectual culture important to developing the human capital .[64]

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62 Kuznets, Simon S. *Nobel Prize Speech* 1971.

63 Kuznets, Simon S. 1966. *Modern Economic Growth: Rate, Structure, and Spread*: New Haven: Yale University Press.

64 Kuznets, Simon S. 1966. *Modern Economic Growth: Rate, Structure, and Spread*: New Haven: Yale University Press; 458-60.

While certainly not opposed to the use of economic history to learn about the economic future, Kuznets was strongly skeptical of simplistic, de-contextualized extrapolation from a hazy Western economic past.[65] In moving economic theory beyond such "linear" and purely economic theories of growth, Kuznets helped give birth to modern development economics, which has focused on understanding the economics of currently developing countries on their own terms. At the same time, Kuznets was not, like some of his more radical colleagues such as Albert Hirschman[66], opposed to economic theorizing or committed to the notion that development policy should be based on purely "case-based" or "pragmatic" considerations.[67] Rather, Kuznets argued for a vision of development economics that worked to develop generalizing theories, but theories that took into account and understood the most dramatic and important distinctions while abstracting from less important differences. Thus, beyond the narrower point of difference between past and future development, Kuznets's emphasis on culture and institutions was revolutionary within development economics and has had a large and lasting impact on the field.

Many of these distinctions between currently developing nations and the past of developed nations parallel the distinctions he draws between Jewish and Gentile economic structures in his work on the economic history of the Jews. Most prominent among these parallels is structural. In his analysis of Jewish and Gentile economic structure, Kuznets primarily stresses the broadest distinctions and most theoretically-justified distinctions between the economics of a small minority within a country and that of the majority, eschewing Jewish-specific explanations.[68] This parallels Kuznets's later belief in the utility of theories addressing the broad sweep of developing countries, rather than considering development on a case-by-case basis, while at the same time emphasizing the distinction between the current state of developing nations and the past of developed nations.[69] The basic approach, in both cases, is one of carefully complicating theory one level at a time and of avoiding a rush either to overgeneralization or to a purely case-based, infinitely flexible anti-theoretical analysis. This parallel is further reinforced by the differing "development paths" that he envisions small (immigrant) minorities following relative to the majorities within the same country. Small minorities, unwedded to majority customs, are likely to participate most heavily in the fastest growing technological sectors of the economy, paralleling the possible technological and product-space "leapfrogging" that Kuznets suggests may be possible for developing nations.[70]

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65 Ibid., 433-5.

66 See, for example, Hirschman, Albert O. 1959. *The Strategy of Economic Development*: New Haven: Yale University Press.

67 Kapuria-Foreman, Vibha, and Mark Perlman. 1995. An Economic Historian's Economist: Remembering Simon Kuznets. *The Economic Journal* 105 (433):1524-1547.

68 Kuznets, Simon S. 1960. Economic Structure and Life of the Jews. *The Jews: their History, Culture and Religion*, 1597-1666; pp. 1600-4.

69 Kuznets, Simon S. 1963. Notes on the Takeoff. *The Economics of Takeoff into Self-Sustained Growth*.

70 Ibid., 1601.

Yet, perhaps the greatest relationship between Kuznets's thinking about history of Jews and development economics comes in his emphasis of cultural and social factors. Of all the foci Kuznets suggested, these are perhaps the most controversial within the economics community, which tends to view such claims as vague at best and culturally deterministic (even crypto-racist) at worst. Despite this widespread hostility within the economic community, Kuznets was deeply committed to the importance of culture, as is perhaps most dramatically demonstrated by the title of the journal he helped found and make prominent, *Economic Development and Cultural Change*. Kuznets's interest in culture and society as driving forces in economic development likely had roots in the continual dialogue he maintained in his personal life with academics of widely varying fields, particularly sociologist and anthropologists,[71] but was also tightly connected to his understanding of the distinctive cultural and social structure of the Jewish community that underlay its economic success and more general economic structure.

In fact, the first time, as far as I know[72], that Kuznets discusses the relationship between culture and economics is in "Economic Structure." After the publication of that article, it explodes into a primary theme in his research interests. While the founding of *Economic Development and Cultural Change* predates his completion of a draft of "Economic Structure" by four years, it seems plausible that Kuznets's interest in the cultural factors underpinning economic development was reinforced, if not spurred, by his study of Jewish history.

A major theme of "Economic Structure" is the notion that economic patterns of the Jewish community might be explained by the desire within the community to maintain cultural cohesion and that Jews might be willing to sacrifice substantial economic advantage in order to work in sectors of the economy where other Jews work. Kuznets also emphasizes that Jewish urbanity may be seen as an outgrowth of the greater anonymity afforded by cities, allowing for increased cultural cohesion without excessive fear of backlash from majority population. Furthermore, in the context of Israel/Palestine, although he implies this may be a feature of Jewish economic structure more broadly, he emphasizes the importance of "social capital" that allowed informal social insurance and efficient allocation of financial resources for investment within the Jewish community.

Kuznets's interest in the connection between Jewish cultural and social conditions to

Jewish economic structure is further highlighted the speech he gave, later in his career, at the home of the President of Israel, which appears in this volume on pages ???-???. In particular, Kuznets stresses the cultural inheritance that appears to spur Jews toward the aggressive pursuit of education, leading to their eventual prominence in the highly trained professional and academic sectors of the American economy (volume ?. pages ???-???).

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71 Kuznets, Paul. *Personal Interview: May 3* 2007.

72 The most comprehensive bibliography of his work was compiled by Robert W. Fogel, Marilyn Coopersmith, and Kathleen McCauley and edited and supplemented by Edith Kuznets to be published in a book of posthumous essays in 1989. I will refer to this simply as Kuznets et al. (1989).

Interestingly, he also stresses the tendency of Jewish intellectuals to be more radically left-wing than intellectuals at large. He argues that, given that more Jews are intellectuals in the first place, this fact has important implications for the political, and eventually economic, composition
of the Jewish workforce. In particular, he feared that the increasing trend of radical intellectuals "dropping out" of school, the workforce and mainstream society in general might lead Jews to follow this misguided trend particularly zealously. In understanding the economic position of Jews in the United States after immigration from Eastern Europe, Kuznets emphasizes the fact that Jews had much stronger family ties and were much more likely to bring their entire family along when they immigrated than were other immigrants to the United States.[73] He also makes fairly vague references to the selectivity of Jewish history for intelligence and a culture focused on education, an argument controversial anywhere, but particularly among economists.
Finally, Kuznets emphasizes the potential economic inefficiencies and inhibitions of development that ethnic division in developing countries might create.[74] Considerations of the importance of such ethnic conflict dates to years before those writings, however, when he first took up this theme in his discussion of the economic structure of the Jewish minority, in fact of any ethnic minority.[75] Beyond his writing on Jewish history, Mark Perlman[76] also emphasizes more direct connections between Kuznets's, and other Jewish economists', past of separateness and youth in Russia and his hesitance to see the past of developed countries as an appropriate model for current developing countries.

Jewish Immigration and the Population Debate

The 1960's were a time of ferment for neo-Malthusian worries about exploding world population, culminating in Paul Ehrlich's famously alarmist and hugely influential *The Population Bomb*. In the economics community, too, population problems became a focus, including the topic of Joseph Spengler's 1965 Presidential Address to the American Economics Association. The dominant view of academics outside economics followed broadly Ehrlichian lines: population growth threatened a Malthusian implosion of living standards. Economics was somewhat more optimistic, but still concerned; Solow's neo- classical growth model indicated that increased population growth would reduce the level, but not the growth rate, of per-capita incomes.

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73 Kuznets, Simon S. 1975. Immigration of Russian Jews to the United States: Background and Structure. *Perspectives in American History*, 35-124; pp. 97-100.

74 Kuznets, Simon S. 1966. *Modern Economic Growth: Rate, Structure, and Spread*: New Haven: Yale University Press; 454-6.

75 Kuznets, Simon S. 1960. Economic Structure and Life of the Jews. *The Jews: their History, Culture and Religion*, 1597-1666; pp. 1602-3.

76 Perlman, Mark. 1996. Jews and Contributions to Economics: A Bicentennial Review. *The Character of Economic Thought, Economic Characters and Economic Institutions*.

Kuznets influentially[77] took a different perspective, beginning with his article "Population Change and Aggregate Output".[78] Kuznets argues that population growth could actually be an important source of per-capita income growth, as population growth offered increased opportunities for specialization and, more importantly, greater numbers of people meant greater numbers of rare geniuses who advanced technological progress, accelerating economic development. Inspired by and drawing upon Kuznets's work, Edmund Phelps[79] summarized this argument eloquently:

One can hardly imagine how poor we would be today if it were not for the rapid population growth of the past to which we owe the enormous number of advanced technologies we enjoy today...If I could redo the history of the world, halving population from the beginning of time on some random basis, I would not do it for fear of losing Mozart in the process.

Phelps's argument is perhaps nowhere more palpable and present than in the Jewish community: how much richer would today's world be if the Jewish intellectuals murdered in the Holocaust had survived? Kuznets was also particularly skeptical about the more limited and widely accepted claim that developing countries could not afford their rapid rates of population growth. In a 1967 piece for the *Proceedings of the American Philosophical Society*, "Population and Economic Growth," Kuznets sought to bring a more balanced perspective to the broader academic community's understanding of the costs and benefits of population growth.[80] Kuznets argues that current technology, if simply applied to developing countries, would be more than sufficient to supply food for not only all current inhabitants but also all projected future inhabitants for at least forty years without any increase in arable land.[81] The capacity of population growth to be supported by the adoption of new technology is clearly echoed in Kuznets's comments on the high rates of population growth within the Jewish community. These have hardly retarded Jewish economic advance, given the expansion of Jewish human capital through education to support these greater numbers.[82] However, in his typical style, Kuznets was exceedingly cautious in advancing these arguments beyond the bounds of what is clear from data.

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77 Perhaps the most popularly-known pro-population economist Julian Simon (2003. *A Life Against the Grain: The Autobiography of an Unconventional Economist*: New Brunswick, NJ: Transaction Publishers) attributed many of his ideas to Kuznets and even asked Kuznets to write the introduction to his 1977 *The Econoimcs of Population Growth*. Kuznets, in characteristically non-confrontational fashion, demurred. I thank Pierre Desrochers for pointing me to this story.

78 Kuznets, Simon S. 1960. Population Change and Aggregate Output. *Demographic and Economic Changes inDeveloped Countries*; pp. 326-30.

79 Phelps, Edmund S. 1968. Population Increase. *The Canadian Journal of Economics* 1 (3):497-518; pp. 510-3.

80 Kuznets, Simon. 1967. Population and Economic Growth. *Proceedings of the American Philosophical Society* 111 (3):170-193.

81 Ibid., p. 185.

82 Kuznets, Simon S. 1975. Immigration of Russian Jews to the United States: Background and Structure. *Perspectives in American History*, 35-124.

For example, he writes on page 184 of “Population and Economic Growth”, “intellectual caution and modesty should compel one to stop right here—with this confession that economic analysis alone is inadequate in dealing with such a fundamental aspect of economic growth as its relationship to population increase.” Thankfully, Kuznets did not stop right there, instead expressing the cautious insights he had gained from a lifetime of studying population and development.

Kuznets’s emphasis on the role of immigration in economic development also manifests itself in his work on “Israel’s Economic Development”. Section 3 of that article (???-???) is devoted to arguing that half or more of the excessive growth of Israel compared to other developing nations is due to the combination of immigration and the young nation’s astonishing ability to raise the torrent of immigrants consistently to the level of income of those who immigrated earlier. This success, and the astonishingly rapid economic growth he documents and argues it fostered, contrasts favorably even compared to the impressive track record of Jews in the United States and likely represents one more force that drew the migrant-friendly Kuznets’s affections towards the blossoming new state.

From a careful review of his bibliography,[83] it appears that Kuznets’s inclination in favor of immigration in his academic work begins with in his pioneering research, jointly with Ernest Rubin, on the subject.[84] On page 1 they write, “The growth of a national economy may be stimulated by the increase of its population, which strongly affects consumer demand and the size of the labor force … In the United States population growth has traditionally been regarded as a source of strength and a sign of material progress.” Yet, as they discuss, in the 1920’s views on population policy reversed sharply: the titanic wave of immigration to the United States that carried the Eastern European Jewish Diaspora to the United States ended with the Immigration Act and National Origins Quota of 1924. Opposition to such policies was one of the few political issues about which generally apolitical Kuznets was passionate, believing that immigration was the foundation of American success.[85] This is unsurprising, given that Kuznets barely made it into the country before the restrictions were imposed.

While such restrictions were almost certainly motivated more deeply by racist and eugenicist popular sentiment in the country, they were often justified publically, and gained crucial support from (even Jewish) organized labor, by arguments about the excessive overcrowding and wage depression caused by immigration.[86] Kuznets and Rubin argue, again on page 1, that while these “interests (may have been) acting in supposed accordance with their economic advantage” they were likely misled due to a lack of “carefully considered…scientific research in the national interest,” research they hope to provide.

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83 Kuznets et al. 1989.

84 Kuznets, Simon, and Ernest Rubin. 1954. Immigration and the Foreign Born. *National Bureau of Economic Research Occasional Papers* (46).

85 Stein, Judith. 2010. Personal Communication, February 10, 2010.

86 Goldin, Claudia. 1994. The Political Economy of Immigration Restriction in the United States, 1890 to 1921. *The Reuglated Economy: a Historical Approach to Political Economy*.

Kuznets (1960) goes on to emphasize, in a strikingly theoretical article by his standards, the importance of allowing free migration and communication of intellectuals in order to achieve maximal rates of technical progress. What he saw as mistaken Malthusian views of immigration clearly struck Kuznets close to home.

Work with Milton Friedman

Superficially Milton Friedman and Simon Kuznets do not seem like the most natural collaborators. Arch-free-marketeer and adherent of the Keynesian mainstream, father of modern Neo-Marshallianism and persistent skeptic of simple models, bold public intellectual and ever-cautious empiricist: Friedman and Kuznets had very different professional inclinations. Nonetheless, their lives overlapped significantly for many years. They shared a common mentor and advisor, Wesley Clair Mitchell, who taught them both empirical economics; moreover, Friedman became Kuznets's assistant during the war years.

Eventually, the pair published the bulk of Friedman's dissertation, first as an article in 1939[87]and then as a book in 1945,[88] both as *Income from Independent Professional Practice*.

This work typified the Mitchell-Kuznets school of empirical work: it was several hundred pages devoted overwhelmingly to the dispassionate tabulation of patterns of income earned by professionals in various careers. The book sowed the seeds of two ideas that, largely through Friedman's advocacy of them, were to be central concerns of labor economics for the following half century: first, occupational licensure as a means of reducing competition, and, second, modeling educational choices as investment in "human capital".

The breakthrough idea of Friedman and Kuznets regarding occupational licensure was typified by a quote they include from Harold Rypins on page 12 of their book, who noted, "In all the professions there has developed in the last few years an aristocratic, or at least restrictive movement which, in a sense, is reminiscent of the medieval guilds." Morris Kleiner[89] cites Kuznets and Friedman's work as having major influence on views among economists; particularly influential was the idea, much espoused by Friedman, that occupational licensure may and often does act as a anticompetitive barrier to entry .[90]

87 Friedman, Milton, and Simon Kuznets. 1939. Income from Independent Professional Practice. *National Bureau of Economic Research Bulletin* (72-73).

88 Friedman, Milton, and Simon S. Kuznets. 1945. *Income from Independent Professional Practice*: New York: National Bureau of Economic Research.

89 Kleiner, Morris M. 2000. Occupational Licensing. *The Journal of Economic Perspectives* 14 (4):189-202; p. 190.

90 Friedman, Milton, and Rose D. Friedman. 1962. *Capitalism and Freedom*: Chicago: University of Chicago Press Chicago; 137-160. Friedman, Milton. 1976. Autobiography.

Prominent citations of this argument by Theodore Schultz[91] and Gary Becker[92] confirm this view. In fact, this view was so controversial at the time of publication of the volume that it caused a five-year delay in the publication of the work due to objections of a National Bureau of Economic Research board member affiliated with the American Medical Association. While I am not aware of any work on this history of this contentious proposition, I think most casual readers, including myself, would initially assume this argument was likely of Friedman, the libertarian, not Kuznets, the moderate leftist's, invention.

While I have no clear proof that this view is mistaken, several elements of Kuznets work suggest that it may be. First, it should be recalled that at this time, Friedman's ardent free-market views were just developing.[93] Second, the medieval guild system was hardly an interest of Friedman's and therefore the Rypins quote is unlikely to have caught his eye among the myriad of other references from which the pair chose. On the other hand, Kuznets, eventually in 1960 and more extensively in 1975,[94] wrote on the guild system and and Structure.

Its destructive impact on Jewish life in Eastern Europe. While it is unclear when in his career this interest began, an unpublished, handwritten manuscript that I discovered in the Kuznets archive, "The Doctrine of Usury in the Middle Ages," indicates that Kuznets had an abiding interest in medieval professional and economic regulation. I include a version of these notes, transcribed by my co-editor Stephanie Lo, on my website http://www.glenweyl.com.

While the manuscript is classic Kuznets in concealing its motivation and (perhaps partly due to the Bureau's censorship) ultimate conclusions, it stands out from the rest of the corpus of Kuznets's work in several ways. First, it is one of the only writings of his I have encountered with absolutely no quantitative dimension. Second, it is purely a piece of intellectual history, tracing the evolution of the doctrine of usury through the Middle Ages. This is, as far as I know, the only intellectual history work Kuznets ever did. Finally, the piece is exceptional among treatments of usury in that it makes no mention whatsoever of the Jews that ended up filling the money lending roles proscribed to Christians.

This omission seems particularly odd given that it seems apparent that the connection to Jewish economic regulation must have played an important role in motivation the manuscript. Of course, it is hard to know whether this was the beginning of an academic paper (as the fact that the paper shows signs of having been edited throughout), a set of personal notes (as the fact that he never after referred to or built upon it suggests) or somewhere in between. Furthermore, while the positioning of the manuscript in the archive indicates that it was from his early career, I have not been able to associate a date to the paper with any certainty (i.e. before or after his work with Friedman).

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91 Schultz, Theodore W. 1961. Investment in Human Capital. *American Economic Review* 51 (1):1-17; p. 14.

92 Becker, Gary S. 1962. Investment in Human Capital: A Theoretical Analysis. *Journal of Political Economy* 70 (S5):9-49; p.10.

93 Ebenstein, Lanny. 2007. *Milton Friedman: A Biography*: New York: Palgrave Macmillan; pp. 31-52.

94 Kuznets, Simon S. 1975. Immigration of Russian Jews to the United States: Background *Perspectives in American History*, 35-124; pp. 56-7.

Regardless, it seems clear that Kuznets, not Friedman, was the primary student of the economic and professional system of the old world. In fact, a thorough review of a bibliography of Friedman compiled by Niels Thygesen[95] indicates that Friedman's only explicit research on history through 1977 was his celebrated work with Anna Schwartz on money in the United States.

The second idea for which the book is famous sprung from the authors' effort to understand the first. Friedman and Kuznets tried,[96] and failed,[97] to explain the income differentials between professional and non-professional careers as a return on capital investments necessary to enter the professions. Their failure led them to conclude that occupational licensure and other barriers made professionals a "noncompeting group" (p. 93). Their method of accounting for the fair market return of such "human" capital investments, which improved on earlier work by J. R. Walsh[98], became the foundation of an enormous literature on the returns to education.

In fact, the pioneers of the theory of human capital, Yoram Ben-Porath[99], and to a lesser extent Jacob Mincer,[100] Theodore Schultz,[101] and Gary Becker[102], attribute the genesis of their ideas to Friedman and Kuznets's book; for instance, Ben-Porath establishes in his opening paragraph the importance of "[t]he development by Friedman and Kuznets[103] of the theory…of…human capital." Friedman carried the idea of human capital developed in his work with Kuznets forward to his classic theory of

95 Thygesen, Niels. 1977. The Scientific Contribution of Milton Friedman. *Scandinavian Journal of Economics* 79 (1):56-98.

96 Friedman, Milton, and Simon S. Kuznets. 1945. *Income from Independent Professional Practice*: New York: National Bureau of Economic Research; pp. 83-4.

97 Ibid., pp. 84-6.

98 Walsh, J. R. 1935. Capital Concept Applied to Man. *Quarterly Journal of Economics* 49 (2):255-285.

99 Ben-Porath, Yoram. 1967. The Production of Human Capital and the Life Cycle of Earnings. *Journal of Political Economy* 75 (4):352-365; p. 352. Admittedly, Ben-Porath was not as central of a figure to this developing literature as the others, but he does offer the clearest articulation of the role played by Friedman and Kuznets. A more important figure was Barry Chiswick (Becker, Gary S. and Barry R. Chiswick. 1966. "Education and the Distribution of Earnings," *American Economic Review* 56: 358–69.), especially given that he has become something of an heir to Kuznets in his interests in immigration and the economic history of Jews.

100 Mincer, Jacob. 1958. Investment in Human Capital and Personal Income Distribution. *Journal of Political Economy* 66 (4):281-302; p. 284.

101 Schultz, Theodore W. 1961. Investment in Human Capital. *American Economic Review* 51 (1):1-17; p. 14.

102 Becker, Gary S. 1962. Investment in Human Capital: A Theoretical Analysis. *Journal of Political Economy* 70 (S5):9-49; p. 10.

103 Friedman, Milton, and Simon S. Kuznets. 1945. *Income from Independent Professional Practice*: New York: National Bureau of Economic Research.

permanent income,[104] the fundamental ideas of which he attributed to his work with Kuznets in his Nobel autobiography.[105]
Friedman's interest in education and its implications for income continued throughout his career, albeit somewhat obliquely through his interest in lifetime, as opposed to temporary, income,[106] another idea[107] he attributed to his work with Kuznets, and reform of the educational system.[108] Likely through his influence, including his role as Becker's advisor, human capital became a dominant theme of the Chicago school, occupying much of the attention of scholars such as Becker, Schultz and Ben-Porath. Thus, there is little doubt that, despite its relative obscurity, *Income from Independent Professional Practice* set off a quiet revolution in labor economics.

Yet, from where did its emphasis on human capital originate? The most I can do is speculate as I found no information concerning the process of writing the work. However, the connections to Jewish economic history, and Kuznets's understanding of it, could hardly be more apparent. Perhaps the primary focus of virtually all of Kuznets's work on the economic history of the Jews[109] was their outstanding educational attainment and the role this played in accounting for their outstanding differential economic advance beyond the position of the general immigrant and native population. It is widely known that education and (religious) study were central values of Judaism at least since the advent of Christianity and Kuznets documented quantitatively the universal popular perception that this translated into far higher Jewish educational attainment in the United States than among other immigrant or native groups.

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104 Friedman, Milton, and National Bureau of Economic Research. 1957. *A Theory of the Consumption Function*: Princeton: Princeton University Press.

105 Friedman, Milton. 1976. Autobiography.

106 Friedman, Milton, and National Bureau of Economic Research. 1957. *A Theory of the Consumption Function*: Princeton: Princeton University Press.

107 Stein (2009) suggests Kuznets may also have been the genesis of the emphasis of the Chicago school on home production, having emphasized in his work during the war that national income accounts should consider women's work in the home, a fight he lost. Given that I can find no references to this in any written work by Kuznets or Friedman it remains a speculation and given that it is not directly connected to my thesis I will not explore it further.

108 Friedman, Milton. 1955. The Role of Government in Education. *Econoimcs and the Public Interest*, 123-144. Friedman, Milton. 1955. The Role of Government in Education. *Econoimcs and the Public Interest*, 123-144.

109 Kuznets, Simon, S. 1960. Economic Structure and Life of the Jews. *The Jews: their History, Culture and Religion*, 1597-1666. Kuznets, Simon S. 1972. Economic Growth of U.S. Jewry. *Papers of Simon Smith Kuznets, 1923-1985 (inclusive), 1950-1980 (bulk), Correspondence and other papers relating to Jewish studies, ca.1959-1977 , Box 1, in folder \em Economic Structure of U.S. Jewry. Call Number: HUGFP88.25*.
Kuznets, Simon S. 1972. *Economic Structure of U. S. Jewry: Recent Trends*: Jerusalem: Institute of Contemporary Jewry, Hebrew University of Jerusalem.

For example, Kuznets[110] found that Jews of Eastern European descent completed college at twice the rate of the general American population.

Any direct connection between Jewish educational attainment and the human capital theory of Kuznets's work with Friedman is at best speculative. Nonetheless, it seems a plausible potential source of motivation for that important research. Furthermore, it is not just its connection to Jewish economic history that is hard to draw out of *Income*. In typical Kuznets style, the book is written in highly technical and concrete style that entirely masks both the motivation for its writing as well as the broad generalizations based on the research that Friedman and others obviously took away from it. For example, the most influential passage of the book, the basis of subsequent interest in licensure as a barrier to entry (page 93) reads:

The inference from this analysis is that professional workers constitute a 'non-competing' group…Our data suggest that this group is sufficiently small to lead to underinvestment…that in the absence of…limitations on entry, incomes in the professions would exceed incomes in other pursuits by less than they do now. The limitations of the data and the speculative character of our analysis make this conclusion tentative.

This bears comparison with Friedman's later writing, in *Capitalism and Freedom* on occupational licensure on pages 141-142:

Licensure therefore frequently establishes essentially the medieval guild kind of regulation in which the state assigns power to the members of the profession ... the problem of licensing of occupations is something more than a trivial illustration of the problem of state intervention, that it is already in this country a serious infringement on the freedom of individuals to pursue activities of their own choice, and that it threatens to become a much more serious one with the continual pressure upon legislatures to extend it.

The reserve, modesty and scientific demeanor with which Kuznets expressed his claims means that any hopes of understanding the sources of his ideas must be somewhat indirect. The most we may hope for in understanding the motivation behind this work is a series of circumstantial, mutually reinforcing connections between Kuznets's understanding of Jewish history and various areas of his mainstream economics.

Kuznets, Simon S. 1975. Immigration of Russian Jews to the United States: Background and Structure.
Perspectives in American History, 35-124.

110 Kuznets, Simon S. 1972. Economic Growth of U.S. Jewry. *Papers of Simon Smith Kuznets, 1923-1985 (inclusive), 1950-1980 (bulk), Correspondence and other papers relating to Jewish studies, ca.1959-1977 , Box 1, in folder \em Economic Structure of U.S. Jewry. Call Number: HUGFP88.25.*

The Cautious Empiricist of the Eastern European Jewish Diaspora

While it certainly carries its frustrations for the historian, Kuznets's reticence about the personal causes and consequences of his work is key to understanding him and his contribution. When Bertil Ohlin presented Simon Kuznets, his committee's selection as 1971 Nobel laureate in Economics, he said, "Kuznets, of course, makes use of models which demonstrate the connections between strategic elements in the economic system, but he shows a very limited sympathy for abstract and generalizing models which provide few opportunities of empirical testing. He chooses and defines concepts which (sic) correspond as closely as possible to what can be observed and statistically measured." Fogel[111] discusses extensively about Kuznets's careful, humble, empirical approach to economics.

His hesitance to extrapolate from data or propose hypotheses not directly based in observation is apparent throughout his research. I consider a couple of examples. The conclusion of his famous AEA Presidential Address in which he proposed the inverted U hypothesis begins "In concluding this paper, I am acutely conscious of the meagerness of reliable information presented. This paper is perhaps 5 per cent empirical information, 95 per cent speculation, some of it possibly tainted by wishful thinking." The apology for this, one of the most empirically based presidential addresses for many years, continues for almost half a page. His extreme caution applied even to the most mundane extrapolations from data. On page 21 of "Economic Growth of U. S. Jewry" he ends a paragraph of apologies for the assumptions he was forced to make in order to generate the first estimates of a time series of American Jewish population with "We shall have to rest content with these rough approximations." To the jaded reader who is accustomed to daily encounters with the most complex contortions of structural econometrics, it is astonishing[112] to see such fervent caution about steps of data collection that would probably not even be reported in most contemporary papers.

Kuznets's painstaking effort to separate conjecture from fact reflects a related, but broader, set of dualities that pervaded his life and work: between his work on Jewish history and its motivation in his past, between that work and his professional life as an economist and between his loyalty to his heritage and the strict American life he built for his family. To gain a richer perspective on Kuznets as a thinker and as a person, it is useful to consider each of these, briefly, in turn.

It could hardly be more apparent that Kuznets's past and identifications led him to do his research on Jewish economic history. In fact, in a 1973 letter to Martin Feldstein, which we have published on page ??? of this volume, Kuznets writes "I did this paper (and other in the series) because of my interests and associations as a Jew (I frankly doubt that were it not for these interests and associations, I would have, as a general economist, devoted much thought or effort to this topic)."

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111 Fogel, Robert W. 1987. Some Notes on the Scientific Methods of Simon Kuznets.

112 In fact Kuznets's extreme care is likely a good part of the reason why, despite his enormous contribution to economics, Kuznets has few contemporary followers. In an age where fights over empirical methodology are between an "atheoretical" camp using instrumental variables and regression discontinuity analysis and a "structural" camp advocating complex models of entire industries, it is hard to imagine where a skeptic of even multiple linear regression, as Fogel (1987, 16-17) describes Kuznets as being, could fit in.

However, absolutely no sense for such motivations, or even any mention of his past, appears in any of Kuznets's scholarly work on the history of the Jews. His first article on the Jews[113] begins in typically universalistic fashion, "The economic structure and life of any group, within a given historical epoch, is largely a matter of its natural and social environment." In the most informal and personal of his writing on the history of the Jews, a speech he gave at the home of the President Zalman Shazar of Israel,[114] Kuznets touches on a wide range of topics very close to his life, yet never explicitly betrays the slightest personal interest or emotion. When he discusses the forcing of Jews in Eastern Europe, like his parents, towards a limited range of professions,[115] when he analyzes the cultural inheritance of Jews and the role it plays in their success,[116] when he discusses the difficulties immigrants faced with language,[117] when he analyzes the constraints on occupational choice imposed by anti-Semitism[118] and even when he notes the overwhelming preponderance of Jews among Ivy League faculty,[119] he never mentions his own or his family's experience nor lapses into any sort of discernable emotion.

Even with motives so carefully absent, Kuznets worried that his research on the economic history of the Jews was too personal to constitute real professional work. He therefore sought to separate it entirely from his mainstream work in economics. In fact, of the half dozen colleagues and students of Kuznets's I interviewed for this project, not a single one ever remembers discussing with him about any for his work on the history of the Jews, despite all of their being of Eastern European Jewish descent themselves! When Martin Feldstein asked in 1973 to include his unpublished "Economic Growth of U. S. Jewry" in a Harvard Departmental working paper series, Kuznets[120] replied, after noting as above his personal motivation in writing the paper, "I would deem it inappropriate to (publish the paper in the series)…[O]bjective as the tools employed may be, the very choice of topic reveals a concern with, and interest in, a highly specialized aspects (sic). I would feel differently if this were a paper on trends in the structure of several ethnic minorities in the United States."[121]

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113 Kuznets, Simon S. 1960. Economic Structure and Life of the Jews. *The Jews: their History, Culture and Religion*, 1597-1666.

114 Kuznets, Simon S. 1972. Economic Growth of U.S. Jewry. *Papers of Simon Smith Kuznets, 1923-1985 (inclusive), 1950-1980 (bulk), Correspondence and other papers relating to Jewish studies, ca.1959-1977 , Box 1, in folder \em Economic Structure of U.S. Jewry. Call Number: HUGFP88.25.*

115 Ibid., pp. 11-2.

116 Ibid., pp. 12-4.

117 Ibid., pp. 14-6.

118 Ibid., p. 18. Rosovsky reports that anti-Semitism played a role in Kuznets's residential choice in Philadelphia.

119 Ibid., pp. 26-7.

120 Kuznets, Simon S. 1973. Personal letter to Martin Feldstein. *Papers of Simon Smith Kuznets, 1923-1985 (inclusive), 1950-1980 (bulk), Correspondence and other papers relating to Jewish studies, ca.1959-1977 , Box 1, in folder \em Correspondence, Tables and Worksheets on Jewish Economics. Call Number: HUGFP88.25.*

Kuznets ensured his past was, in fact, two steps removed from his profession. It was not only his interested in Jewish history that Kuznets clearly separated from this professional life and relationships, but also the entirety of his personal views and opinions. Rosovsky,[122] an advisee of Kuznets and one of this close friends and colleagues, reports that all throughout the 1960's, perhaps the most political moment of US history, he remembers Kuznets as being perhaps the only member of the Harvard department who expressed no political views he could recall. In fact, none of the dozens of colleagues and family members of Kuznets's I interviewed had a recollection of *any* strong political views (other than on immigration as described above) held by Kuznets and almost all described him as apolitical. While Rosovsky also attests that Kuznets was also one of the few Jews at Harvard that made no attempt to conceal his background, he made no attempt to discuss any aspect of his personal background or views professionally. The separation between his past and his present extended beyond work, back another level, into a separation between his private past and the future he built for his family.

Unlike the fabled and stereotypical first-generation Eastern European Jewish immigrant, but typically for Jewish fathers of his generation, Simon Kuznets taught his children almost nothing of the "old world" he had left behind. He never spoke with them in Yiddish nor Russian, never forced or even encouraged them to attend synagogue or remember their Jewish heritage, never cooked them Russian food nor played them Russian or Yiddish music.[123] While he maintained a personal interest in contemporary Russian literature and affairs, as many accounts attest, he never imposed these interests on his family. Kuznets took Judah Leib Gordon's *maskilim* mantra "Be a Jew in and a man in the street" to an extreme: he was a fervent (cultural) Jew in his heart but a man to all the world.

Thus, I hope, the full portrait of Kuznets I wish to paint has come into view. He was a consummate inductive empiricist whose interpretation of facts that confronted him was shaped by the categories of his past and his struggle to understand it. He was a passionately dispassionate analyst of the history of an interesting ethnic minority, which happened to be his own people. He was an apolitical fervent supporter of the state of Israel from the day of its birth,[124] making regular trips to the Falk Institute there and becoming a fixture of the Israeli economics community[125]. The unifying theme of his life and work was a series of dualities and apparent contradictions, a straightforward enigma: the cautious empiricist of the Eastern European Jewish Diaspora.

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121 See page ??? of this volume.

122 Rosovsky, Henry. *Personal Interview: January 28* 2010.

123 Kuznets, Paul. *Personal Interview: May 3* 2007. Stein, Judith. 2010. Personal Communication, February 10, 2010.

124 Kuznets, Paul. *Personal Interview: May 3* 2007.

125 Rosovsky recalls that every time one came to visit Cambridge they would make a mandatory pilgrimage to the Kuznets residence on Francis Avenue, just a block and a half from my current apartment.

Eastern European Jews and Modern Economics

What interests me in Kuznets's story is not its idiosyncrasy or quirkiness, but rather how it takes to a logical extreme a broader story of the Jews of Eastern European descent who played such a crucial role in transforming economics in the twentieth century. That the Eastern European Jewish Diaspora was at the center of creating Economics, as we understand it today, can hardly be doubted. Some simple statistics may be instructive.

According to data collected by jinfo.org[126] and systematized for this article by Yanislav Petrov[127], since 1969 when the Economics prize was first given, 50% of economics Nobel laureates have been Jews. This compares with 29% in Physics and 27% in Chemistry over the same time frame. Similarly, since the awards began at similar times in the late 1940's and early 1950's, 63% of recipients of the John Bates Clark medal have been Jews, compared to 27% of the comparable Fields medal in mathematics.

Table 3.
Jewish accomplishments in economics and other scientific fields

	Percentage of Jewish recipients
Nobel Prizes:	
Economics (1969-2009)	42.2%
Chemistry (1969-2009)	28.4%
Physics (1969-2009)	27.6%
John Bates Clark Medal (Economics) (1947-2009)	62.5%
Fields Medal (Mathematics) (1936-2006)	27.1%

Sources: The Jewish Contribution to World Civilization", http://www.jinfo.org/; "All Laureates in Economic Sciences", http://nobelprize.org/nobel_prizes/economics/laureates/; "John Bates Clark Medal", http://www.vanderbilt.edu/AEA/clark_medal.htm; and "International Mathematical Union: Fields Medal", http://www.mathunion.org/general/prizes/fields/details/ (All accessed 10 February 2010).

These statistics are particularly striking given their contrast with history. During the 19th century, economics had few, if any, Jews and was in fact dominated by Christian activists; almost 40% of those who founded the American Economic Association in 1885 were either ordained ministers or lay religious activists.[128]

126 *Jews in Economics*. JInfo.org 2009.

127 Petrov, Yanislav. 2010. Data on Jewish Accomplishments in Economics and Other Scientific Fields. http://www.people.fas.harvard.edu/~weyl/JewsinScience.xls.

128 Fogel, Robert W. 2000. Simon S. Kuznets: April 30, 1901-July 9, 1985; pp. 3-4.

Also, anti-Semitism was common in the profession, as discussed in Melvin Reder[129] and immortalized in the famous story, recounted by Richard Swedberg,[130] of Paul Samuelson's decision to found an economics department at the Massachusetts Institute of Technology after being rejected for an assistant professorship at Harvard despite having written one of the best dissertations of the century.

The cold statistics are very much visible in the everyday life of the profession. My hair has always had the characteristically tight Jewish curls, but despite growing up in heavily Jewish communities my whole life I had never met so many fellow Jewish curly-heads as I did when I came to Harvard's economics department. And the trend is even more pronounced if one focuses even more narrowly than the leaders and prizewinners in the field on the few figures who were truly revolutionary in building the framework of modern economics.

Simon Kuznets built the accounting methodologies underlying most of modern empirical economics. Paul Samuelson, father of the dominant algebraic-computational school of modern economic theory, was the son of Polish Jewish immigrants living in Indiana.[131] Kenneth Arrow, father of the other main geometric-mathematical strain of economic theory, was born to a New York Jewish family in the early 1920's. Two of the three founders of the Neo-Marshallian second Chicago School, Milton Friedman and Gary Becker, were respectively the son of very recent Jewish immigrants from Hungary[132] and the son of an Eastern European Jewish immigrant mother.[133] Jacob Marschak, founder of modern structural econometrics, who died before he could be awarded the Nobel Prize, was a Jewish immigrant[134] from Kiev. Many of the other heroes of any account of the forging of the modern quantitative, empirical-mathematical Neo-Classical economics, such as that given by Roy Weintraub,[135] are of Eastern European Jewish extraction. Of course there are many exceptions: John Hicks in theory, George Stigler in the Chicago School, Trygve Haavelmo and Tjalling Koopmans in econometrics. Nonetheless, it is astonishing that a group representing less than three in every hundred people in the United States and less than two in every *thousand* worldwide was the overwhelming force in the development of modern economics, far beyond even the outsized role they played in physics, mathematics and other fields.

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129 Reder, Melvin W. 2000. The Anti-Semitism of Some Eminent Economists. *History of Political Economy* 32 (4):833-856.

130 Swedberg, Richard. 1991. *Schumpeter: A Biography*: Princeton: Princeton University Press; p. 139.

131 Weinstein, Michael M. 2009. Paul A. Samuelson, Economist, Dies at 94. *New York Times* (December 13).

132 Theroux, David J. 2006. Milton Friedman (1912-2006).

133 Becker, Gary S. *Autobiography* 1992.

134 According to a correspondence between Jacob Viner and Joseph Schumpeter reviewed by Amartya Sen, Marschak was nearly barred from becoming one of the first fellows of the Econometric Society because Schumpeter believed he was "both a Jew and a socialist".

135 Weintraub, E. Roy. 2002. *How Economics Became a Mathematical Science*: Durham, NC: Duke University Press.

Why? The most straightforward and essentialist answer, one that borders dangerously on standard anti-Semitic images of Shylock the moneylender, is that there is some inherent connection (perhaps through occupational restrictions in the old country and their legacy) between the Jewish cultural inheritance and the questions in which economists take interest.

Equally speculative, but more plausible to me, is a story suggested by Kuznets's own life: that there was something that placed the generation of Jews that arrived in the United States between 1880 and 1920, and their children and grandchildren, in an ideal position to lead a revolution in economics. I conclude by exploring a possible causal mechanism for this conjecture. Any attempt to actually provide evidence for it, to test it against alternative hypothesis, or even to formulate such alternatives, is left squarely to future research.

Perhaps the most striking feature of the revolutions wrought by the great economists of Eastern European Jewish extraction was their fundamentally methodological nature. Kuznets, Samuelson, Arrow, Friedman, Becker and Marschak certainly added important substantive insights to the field. But what they are overwhelmingly remembered for was the methodological lenses (empirical, mathematical, statistical and "price theoretic") they made central to the discipline. None of these had any discernible connection to anything Jewish; in fact by stripping away historicist and institutionalist traditions, they represented a forceful universalizing push within the discipline. As Friedman's quote with which I began this paper suggests, the sources of this revolution must be sought elsewhere than in their formal writings as these sources themselves impelled them to hide their tracks.[136] To paraphrase Chaim Weizmann's (who also hailed from Pinsk, 1949) famous dictum, the great Eastern European Jewish Diaspora economists of the twentieth century were just like any other economists, only more so.

On the "demand side", the universalizing thrust of "scientific"[137] economics offered a natural defense against anti-Semitic hostility to Jewish influence in the more culturally- implicated humanities and social sciences. This made economics a unique outlet for Jewish political and social thinkers.

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136 In fact, Kevin Hoover pointed out to me that Friedman's quote parallels a distinction Hans Reichenbach (1938. *Experience and Prediction: An Analysis of the Foundations and Structure of Knowledge*: Chicago: University of Chicago Press) dwelled on between psychology and epistemology, between the historical and logical origins of an idea. Ronald Giere (1999. *Science without Laws*: Chicago: University of Chicago Press; p. 228) argues that this distinction was important to Reichenbach, and perhaps by extension to Friedman, precisely because of its connection to the anti-Semitic attempt to discredit many modern scientific ideas as "Jewish" science. This highlights the "demand side" cause of the universalizing, methodological thrust of the Eastern European Jewish contribution to modern economics that I discuss below.

137 David Hollinger (1996. *Science, Jews and Secular Culture*: Princeton: Princeton University Press) makes a similar argument regarding the sciences and public intellectual culture more broadly. Steven Beller (1989. *Vienna and the Jews: 1867-1938*: Cambridge, UK: Cambridge University Press.) suggests such demand side factors were the driving forces in establishing the dominantly Jewish professions in Vienna prior to German annexation, while also emphasizing, along the lines of my argument, the importance of heterogeneous and often surprising Jewish reactions to Jewishness.

Furthermore, Eastern European Jews' past prepared them with the skills for which modern economics called, but had not prepared them for the problems it would pose, leaving them with fresh eyes. Derek Penslar's[138] impressive recent book, *Shylock's Children: Economics and Jewish Identity in Modern Europe*, traces the history of modern Jewish economic thinking in Western Europe and the lack thereof in Eastern. Penslar argues that Jewish learning through the early *Haskalah* focused overwhelmingly on the natural sciences, neglecting social sciences given the lack of Jewish influence over or interest in the policies of Gentile host societies.[139] While Jewish politico-economic thinking developed over the course of the early 19th century, it was confined almost entirely to (a radical fringe of) German Ashkenazi and especially Western European Sephardic Jewry.[140]

The aspiration of Eastern European Jewish students remained firmly religious or, if secular, natural scientific. Cut-off from political influence, concern and learning by repression, Eastern European Jews came to the United States with extraordinary training in and devotion to the study of natural scientific method but with an equal political naïveté.

Yet the rapid succession of emancipation, immigration to democratic America and the rise of political anti-Semitism in Germany and economic catastrophe worldwide quickly forced them to come to terms with social affairs. Rapidly upwardly mobile, powerfully organized through unions given their professional concentration in America, finally offered a voice through American free speech and universal franchise, Jews rapidly emerged as a political force in the United States. A select, but disproportionate, few of these immigrants and immigrants' children had extraordinary, rigorous scientific and mathematical training.

Free from the cultural burden of a long-standing political tradition, application of these tools to those social problem through a science of economics[141] they helped build must have seemed the most natural and accessible means of confronting academically the new range of challenges they were invited to address. While it was socially sophisticated Western European Jews like Albert Einstein, Wolfgang Pauli[142], and Niels Bohr who helped make modern physics, it was the unwashed but upwardly mobile easterners that made modern economics.

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138 Penslar, Derek J. 2001. *Shylock's Children: Econonomics and Jewish Identity in Modern Europe*: Berkeley and Los-Angeles: University of California Press.

139 Ibid., p. 56.

140 Ibid., pp. 81-4.

141 Of course there is no reason why economics should have assumed such a dominant role compared to other quantitative social sciences. Thus, a natural implication of my hypothesis is that Eastern European Jews should have had a similarly transformative quantifying impact on other potentially quantitative social sciences, such as political science and sociology. Paul Lazarsfeld is a leading example that would seem to confirm this conjecture, as founder of modern quantitative sociology, but neither quantitative evidence of the form made possible by the awards nor a strong personal knowledge of the field make it possible for me to test this hypothesis. It therefore remains as an interesting direction for future research.

142 Pauli's father converted to Catholicism before his birth, but came from a prominent Jewish family.

More than any of those pioneers, Simon Kuznets typified that spirit. I have argued that what he brought to economics was, to a large extent, *not* a series of substantive political, economic or social commitments. Rather, he arrived from Kharkov with rigorous training in statistical and empirical methods and an earnest desire to understand the forces that had shaped and were shaping his life. His beloved cultural inheritance was an ability to see the economy and his own past with a *tabula* close to *rasa*: a rigorous empirical lens unburdened by preconceived theory. That, I think, is something of the resolution to the enigma of his life and work. He was committed to, inspired by, and grateful for, his past *precisely for the rigorous, scientific and universalistic perspective it lent him*.

And it is precisely this commitment that interested me in his story. Born to two atheist, culturally assimilated Jewish parents, I always resented the social expectations accompanying my Judaism, seeking always a secular universalist vision of my identity. Yet, I have come to realize the inevitability, and intellectual attraction, of my Jewish heritage as I found so many of my fellow travelers in that struggle for universalism to be themselves born to atheist, culturally assimilated Jewish parents.

Of course, the story I have just told is explicitly and disproportionately shaped by my experience and by Kuznets's story, through which I have come to understand it. It is at best a provocative reflection and at worst self-indulgent speculation. Yet, I hold out some hope that it can be more the former than the latter. I believe that the story of the rebirth of economics as a mathematical science in the twentieth century cannot be, as it has in the past been, easily separated from the story of the Eastern European Jewish immigrants' struggle to understand political, social and economic affairs. Perhaps someday the pogroms, the great wave of Jewish immigration at the turn of the 20^{th} century, the rise of German anti-Semitism, and the birth of the state of Israel will be seen as rivaling the Great Depression in having shaped modern economic thought. Only through future scholarship on this important neglected subject will we be able to tell.

Delay Differential Neoclassical Growth Model*

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Abstract

This paper develops a continuous-time neoclassical growth model with time delay. Despite of its simple structure, the resulting dynamic system shows emergence of erratic fluctuations in the capital accumulation process when the production function is unimodal and the delay in production is explicitly considered. It complements the seminal paper of Day (1982) in which a discrete-time neoclassical growth model displayed chaotic behavior for some configurations of the propensity to save, the growth rate of labor and the capital depreciation rate. Our analysis has at least two implications. First, nonlinearities and delay matter for a birth of aperiodic fluctuations of national product in the continuous-time model. Second, comparing the effects caused by two different time delays, fixed and continuously distributed time delays, reveals that the continuous-time model with the former can generate complex fluctuations while the one with the latter generates simple fluctuations when the shape parameter of the density function is small and complex fluctuations when large.

Keyword: Neoclassical growth model, continuously distributed time delay, fixed time delay, complex dynamics, period-doubling bifurcation

JEL: E13, E32, C62

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*The authors highly appreciate financial supports from the Japan Society for the Promotion of Science (Grant-in-Aid for Scientific Research (C) 21530172), Chuo University (Joint Research Grant 0981) and the Japan Economic Research Foundation. They are also grateful to K. Ishiyama and M. Kobayashi for their supporting computational studies. The paper was prepared when the second author visited the Department of Science and Engineering of Chuo University. He appreciated its hospitality over his stay. The usual disclaimer applies.

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1 Introduction

In his seminal papers, Day (1982, 1983) examines two conventional dynamic models, one of neoclassical economic growth and the other of productivity and population growth, to show emergence of complex behavior from a quite simple economic structure when sufficient nonlinearities are present. Both models are constructed in discrete-time and have a mound-shaped production function representing the negative effects of pollution caused by increasing capital. It is then analytically and numerically demonstrated that the models generate not only periodic cycles but also chaotic behavior for certain values of the shape parameter of the production function. Each paper has been one of the most influential papers in the study of nonlinear economic dynamics for the last three decades. Since the publications of these papers, a lot of effort has been devoted to deepen the understanding of economic complexity including chaotic behavior, multistability, basins of attractions and so on. Overviews of early contributions and recent developments in nonlinear economic dynamics are found in, to name a few, Day (1994), Puu (2003) and Bischi *et al.* (2010).

In discrete-time analysis, there is a clear and simple criteria such as the "period-three condition" of Li and Yorke (1975) to detect chaos, which can be applied to first-order nonlinear difference equations. In consequence, discrete-time chaos has numerous applications to economic models, as shown, for example, in the papers collected in Rosser (2004). On the other hand, the economic applications of continuous-time chaos are so far limited. This may be due to the facts that there are no general criteria to establish the presence of chaos in continuous-time analysis and that a continuous-time system must be at least three dimensional. As a result, advanced analytical and numerical considerations are required to find chaos in continuous-time models.

Despite of this inconvenience, this paper aims to reconsider the emergence of erratic and unstable fluctuations in a delay continuous-time neoclassical growth model for the following four reasons. The first reason is concerned with continuous time scales and is due to Gandolfo (1997) who raises a question on the choice of continuous or discrete time scales in the construction of dynamic economic models and gives eight reasons to be in favor of the use of continuous-time models including the following: it is natural to treat economic phenomena as continuous, provided that the variables to be examined in the dynamic models are the outcomes of a great number of decisions taken by different agents at different points of time. This is in particular the case in the aggregate model. The second is concerned with a delay in the production process. In the case of the most models discussed earlier in the literature, it is assumed that each economic agents has instantaneous information about its own as well as its rivals' behavior. This assumption is mathematically convenient but does not fully describe real economic situations in which there are always time delays between the times when information is obtained and the times when the decisions are implemented. The third is concerned with nonlinearities. If the analysis is confined to a neighborhood of a stationary point, then linear systems are appropriate. However if certain "global" phenomena are considered, then nonlinear behavior is responsible for the endogenous dynamics. Finally the fourth reason is concerned with the use of the neoclassical model and similar to the argument of Day (1983). Namely, it is the simplest possible framework and still provides a convenient starting point for an investigation of continuous complex dynamics.

In addition to these, we believe it is of interest to find economic circumstances under which continuous-time models give rise to similar dynamic behavior as discrete-time models. Similarly to the spirit and functional form of the discrete-time neoclassical model of Day (1982), continuous time scales, nonlinearity and delay are assumed in this paper.

The structure of the paper is as follows. Section 2 gives a brief overview of the neoclassical growth model and discusses the fact that the model is globally stable if its dynamics is constructed in continuous time while the discrete-time model possesses the possibility of chaotic outcomes when the stationary state is locally unstable. The continuous-time neoclassical growth model can be transformed into a delay differential system which is also capable of producing complex dynamics. Section 3 introduces a continuously distributed time delay and Section 4 replaces it with a fixed time delay. Section 5 compares the effects caused by these two different types of delays. Finally some concluding remarks are given in Section 6.

2 Neoclassical Growth Model

This section serves as a short summary of the neoclassical aggregate model of growth, which can be traced back to Solow (1956) and Swan (1956). An enormous amount of work carried out since then. The main interest has been on two questions: whether full employment growth is possible and whether such growth is stable. Affirmative answers are given under various conditions. The turning point is Day's (1982) model showing the occurrence of chaos.

This section is divided into two parts: First the continuous-time version is briefly discussed to show asymptotic stability of the steady state and then the discrete-time version is abstracted. Erratic fluctuations are demonstrated when the steady state is locally unstable and the production function involves strong nonlinearities.

2.1 Continuous Time Scales

In its simplest version, the neoclassical growth model assumes that the economy has one sector producing national product Y with a production function,

$$Y(t) = F(L(t), K(t)) \tag{1}$$

where $L(t)$ is labor, $K(t)$ is capital and t denotes time. F is assumed to be conventional implying that it is constant returns to scale (i.e., $F(\alpha L, \alpha K) = \alpha F(L, K)$ for $\alpha > 0$) and the two factors are indispensable for production (i.e., $L(t) > 0$ and $K(t) > 0$). Denoting consumption by $C(t)$ and investment by $I(t)$, the equilibrium in the output market is described by the supply-equals-demand relation

$$Y(t) = C(t) + I(t). \tag{2}$$

We assume that the amount of consumption is a constant fraction of net income,

$$C(t) = (1 - s)(Y(t) - \mu K(t)) \tag{3}$$

where $s \in (0, 1)$ is the average propensity to save and $\mu \in (0, 1]$ is the depreciation ratio of capital. There are two sources of dynamics: investment and labor supply. The amount of gross investment $I(t)$ is equal to the sum of the change in capital stock and the amount of capital depreciation at each instant of time,

$$I(t) = \dot{K}(t) + \mu K(t), \tag{4}$$

where the dot over variables denotes time derivatives, e.g., $\dot{K}(t) = dK(t)/dt$. We further assume that labor grows at a (exogenously determined) positive constant rate $n > 0$,

$$L(t) = L_0 e^{nt} \tag{5}$$

where L_0 is the amount of labor available at $t = 0$.

Substituting equations (1) and (4) into (2) gives

$$F(L(t), K(t)) = C(t) + \dot{K}(t) + \mu K(t).$$

Dividing both sides by $L(t) > 0$ and introducing the per-capita variables, $c(t) = C(t)/L(t)$ and $k(t) = K(t)/L(t)$, we obtain

$$f(k(t)) = c(t) + \frac{\dot{K}(t)}{L(t)} + \mu k(t) \tag{6}$$

where the assumption of constant returns to scale implies that we can write the production function in the "intensive" or "per-capita" form

$$f(k) = F\left(1, \frac{K}{L}\right).$$

Dividing both sides of equation (3) by $L(t)$ gives

$$c(t) = (1 - s)(f(k(t)) - \mu k(t)).$$

Differentiating $k(t)$ with respect to t and noticing that the growth rate of labor is n, we obtain, after arranging terms,

$$\frac{\dot{K}(t)}{L(t)} = \dot{k}(t) + nk(t).$$

Combining the last two equations with (6), we have the fundamental dynamic equation of the neoclassical aggregate growth model:

$$\dot{k}(t) = sf(k(t)) - \delta k(t) \text{ where } \delta = n + s\mu. \tag{7}$$

We now impose the conventional assumptions (i.e., Inada conditions) on the per-capita production function $f(k)$:

Assumption 1.

(1) $f'(k) > 0$ and $f''(k) < 0$ for all $k \geq 0$;

(2) $f(0) = 0$;

(3) $\lim_{k \to 0} f'(k) = \infty$ and $\lim_{k \to \infty} f'(k) = 0$.

Under these conditions, the continuous-time neoclassical growth model (7) has a steady state $k^* > 0$ which is the unique positive solution of equation

$$sf(k^*) = \delta k^*.$$

Outside this steady state we have two possibilities:

$$\dot{k} > 0 \text{ for } k_0 < k^* \text{ or } \dot{k} < 0 \text{ for } k_0 > k^* \tag{8}$$

with $k_0 \neq k^*$ and $k_0 \neq 0$. The directions of inequalities of $\dot{k}$ in (8) imply the local asymptotical stability of the positive steady state. This classical result is well known from the literature.

2.2 Discrete Time Scales

In this section, we transform the continuous model to the discrete time model where t now refers to period t. The behavioral specifications (1), (2) and (3) are not affected by changing time scales. On the other hand, dynamic specifications (4) and (5) should be modified accordingly. The labor supply equation is changed to

$$L(t) = (1+n)^t L_0 \tag{9}$$

where the notation n is used to denote the growth rate of labor. A law of motion for the capital stock in discrete time is obtained by replacing $\dot{K}(t)$ in equation (4) by $K(t+1) - K(t)$, so

$$K(t+1) - K(t) = I(t) - \mu K(t). \tag{10}$$

Substituting (3) and (10) into the equilibrium relation (2) and re-arranging terms yield

$$sY(t) = K(t+1) - (1 - s\mu)K(t).$$

Dividing both sides by $L(t)$ and using relations $y(t) = f(k(t))$ and $L(t+1)/L(t) = 1+n$ yield the discrete-time fundamental dynamic equation of capital:

$$k(t+1) = \frac{1}{1+n}(sf(k(t)) + (1 - s\mu)k(t)) \equiv g(k(t)). \tag{11}$$

Given the initial condition k_0, this first-order difference equation determines the entire time path of the capital stock. The conventional neoclassical assumptions (i.e., Assumption 1) ensures that there is a positive and stable steady state $k_s > 0$ such that $k_s = g(k_s)$ and all trajectories monotonically converge to k_s as $t \to \infty$ for all $k_0 > 0$. As a benchmark, given the well known Cobb-Douglas production function $F(L, K) = AK^a L^{1-a}$ with $A > 0$ being the technology parameter and $a \in (0, 1)$, the elasticity parameter, we have its intensive form $f(k) = Ak^a$. In this case it can be confirmed that the steady state is $k_s = [sA/(n + s\mu)]^{1/1-a}$ which is locally asymptotically stable.

Day (1982) reconstructs this neoclassical theory of capital accumulation under circumstances in which productivity of the production function is reduced by a pollution effect caused by increasing concentration of capital. Applying the nonlinear dynamic theory, he demonstrates that erratic behavior exhibiting wandering and sawtooth patterns can emerge when sufficient nonlinearities are present. In particular, in contrast to the conventional neoclassical assumptions, the per-capita production function is modified to have the form

$$f(k) = Ak^a(1-k)^b \tag{12}$$

with $A > 0$ and $f'(k) < 0$ for large k. The factor $(1-k)^b$ reflects the influence of pollution on per-capita output. When the capital intensity increases, pollution increases as well. However for a small enough value of b, the pollution effect is so small that the modified production function behaves like the conventional production function. When b is relatively large, the pollution effect reduces national output. Substituting the new production function into (11), we have the modified fundamental equation of capital accumulation:

$$k(t+1) = \frac{sA}{1+n}k(t)^a(1-k(t))^b + \frac{1-s\mu}{1+n}k(t).$$

Consider a simplification by assuming that $a = b = 1$. Introducing the new variable $z = Bk$ with

$$B = \frac{sA}{1+(A-\mu)s}$$

reduces the fundamental equation to the logistic map,

$$z(t+1) = \theta z(t)(1-z(t)) \tag{13}$$

where

$$\theta = \frac{1+(A-\mu)s}{1+n}.$$

A positive fixed point

$$z_s = \frac{\theta - 1}{\theta},$$

is obtained for $\theta > 1$. The dynamics of $k(t)$ generated by (11) with the modified production function is equivalent to the dynamics of $z(t)$ generated by the logistic map (13). The dynamic structure of (13) has been extensively studied and it is now well-know that equation (13) can generate a wide variety of dynamics ranging from monotonic convergence to chaotic behavior, depending on the value of θ. The factor θ usually varies between zero and four. In summary, the discrete-time neoclassical growth model (13) can generate the following dynamics:

(1) monotonic convergence to zero for $0 < \theta \le 1$,
(2) monotonic growth and convergence to z_s for $1 < \theta \le 2$,
(3) oscillatory convergence to z_s for $2 < \theta \le 3$,
(4) continued oscillations around z_s for $3 < \theta \le 4$.

3 Growth Model with a Continuously Distributed Time Delay

It is demonstrated in the first half of the last section that the continuous-time neoclassical growth model is definitely stable. It is also demonstrated in the latter half that the discrete-time neoclassical model may induce erratic dynamics in the capital-labor ratio under strong nonlinearity presented by the larger value of θ. There is a sharp difference between dynamics generated by the continuous and discrete models. To discuss emergence of irregular fluctuations in the continuous model, it may be helpful to take account two facts; one is that a simple nonlinear delay differential equation such as the Mackey-Glass equation (1977) may display complex dynamics involving chaos and the other is that there is always a time delay due to the decision making process and the decision implementation. In this and next sections, we will confine our attention to delays in the capital accumulation process. Using the mound-shaped production function as, for example, in the case of the modified production function (12), we take a close look at destabilizing effects caused by delays in production.

3.1 Model and General Stability Condition

If a production delay is present and the delay is known or certain, then the production function is written as

$$y(t) = f(k(t-\tau))$$

where $\tau \geq 0$ is the delay. We will examine the case of known delays in the next section and inquire into the case of unknown delay in this section. A convenient way to introduce unknown or uncertain delay into the model is offered by considering it as a random variable and replacing it by its expected value. If the expected per-capita capital stock at time t is denoted by $k^e(t)$ and is based on the entire history of the actual capital stock from zero to t, then the continuous-time dynamic system (7) with delay in production can be rewritten as a Volterra-type integro-differential equation:

$$\begin{aligned} \dot{k}(t) &= -\alpha k(t) + \beta k^e(t)(1-k^e(t)), \\ k^e(t) &= \int_0^t \omega(t-\sigma, T, m) k(\sigma) d\sigma, \end{aligned} \tag{14}$$

where $\alpha = \delta$, $\beta = sA$ and the weighting function is defined as

$$\omega(t-\sigma, T, m) = \begin{cases} \dfrac{1}{T} e^{-\frac{t-\sigma}{T}} & \text{if } m = 0, \\ \dfrac{1}{m!}\left(\dfrac{m}{T}\right)^{m+1} (t-\sigma)^m e^{-\frac{m(t-\sigma)}{T}} & \text{if } m \geq 1 \end{cases} \tag{15}$$

with $\tau = t - \sigma$. Here the shape parameter m is a nonnegative integer and T is a positive constant, which is associated with the length of the delay. If $m = 0$, then weights are exponentially decreasing with the largest weight given to the most current data. If $m \geq 1$, then the weighting function has a bell-shaped curve implying that the most current data has zero weight, the weight increases to a maximal value at $t - \sigma = T$ and decreases thereafter. A steady state of (14) is obtained by setting $\dot{k}(t) = 0$ and $k(t) = k^e(t) = k^*$,

$$k^* = \frac{\beta - \alpha}{\beta} < 1.$$

To ensure positivity of the steady state, we make the following assumption:

Assumption 2. $\beta - \alpha > 0$.

To examine local dynamics of the above system in the neighborhood of the steady state, we consider its linearized version. The approximated nonlinear term evaluated at the steady state is

$$\begin{aligned}\beta \frac{d}{dk^e} k^e(t)(1 - k^e(t)) &= \beta(1 - 2k^*)k_\delta(t) \\ &= (2\alpha - \beta)k_\delta(t),\end{aligned}$$

where $k_\delta = k - k^*$ denotes deviation from the steady state. The linearization yields

$$\dot{k}_\delta(t) = -\alpha k_\delta(t) + (2\alpha - \beta)\int_0^t w(t - \sigma, T, m)k_\delta(\sigma)d\sigma. \tag{16}$$

At the steady state, we look for the solution in the usual exponential form

$$k_\delta(t) = e^{\lambda t}u$$

which is substituted into the linearized equation (16)

$$\lambda e^{\lambda t}u = -\alpha e^{\lambda t}u + (2\alpha - \beta)\int_0^t w(t - \sigma, T, m)e^{\lambda\sigma}ud\sigma.$$

Notice that $\tau = t - \sigma$, then

$$\begin{aligned}\int_0^t w(t - \sigma, T, m)e^{\lambda\sigma}d\sigma &= \int_t^0 w(\tau, T, m)e^{\lambda(t-\tau)}(-d\tau) \\ &= e^{\lambda t}\int_0^t w(\tau, T, m)e^{-\lambda\tau}d\tau\end{aligned}$$

By letting $t \to \infty$, the integral part converges to

$$\int_0^\infty w(\tau, T, m)e^{-\lambda\tau}d\tau = \left(1 + \frac{\lambda T}{q}\right)^{-(m+1)}$$

with

$$q = \begin{cases} 1 & \text{if } m = 0, \\ m & \text{if } m \geq 1. \end{cases}$$

Substituting this expression into the integral part of the linearized equation, dividing both sides of the resultant expression by $e^{\lambda t}u$ and arranging terms, we obtain the characteristic equation

$$-(2\alpha - \beta) + (\alpha + \lambda)\left(1 + \frac{\lambda T}{q}\right)^{m+1} = 0. \tag{17}$$

It has been already shown that the continuous-time neoclassical model without time delay is locally stable. By continuity, it can be supposed that the delay model is locally stable if the delay is small enough. We start our examination with a general result which generates local asymptotical stability regardless of the size of the delay and the shape of the weighting function.

Theorem 1 *The steady state k^* of the distributed delay differential equation (14) is locally asymptotically stable if $|2\alpha - \beta| \leq \alpha$, regardless of the values of m and T.*

Proof. Notice first that $\lambda = 0$ is not an eigenvalue under Assumption 2. Assume that $\lambda \neq 0$ and $\mathrm{Re}\,\lambda \geq 0$. Then

$$|\lambda + \alpha| > \alpha \text{ and } \left|1 + \frac{\lambda T}{q}\right| > 1$$

implying that

$$\left|(\alpha + \lambda)\left(1 + \frac{\lambda T}{q}\right)^{m+1}\right| > \alpha.$$

This relation and condition $|2\alpha - \beta| \leq \alpha$ imply that the two terms in the left hand side of the characteristic equation (17) have different absolute values, thus $\lambda \neq 0$ with $\mathrm{Re}\,\lambda \geq 0$ cannot be an eigenvalue. ■

Theorem 1 implies that the time delay is harmless to stability if $\beta \leq 3\alpha$. In order to examine a possibility of stability loss, we make the following assumption, which makes Assumption 2 redundant:

Assumption 3. $\beta > 3\alpha$.

The characteristic equation (17) is a polynomial equation with degree $n = m + 2$ having the form

$$a_0\lambda^n + a_1\lambda^{n-1} + ... + a_{n-1}\lambda + a_n = 0$$

with all real coefficients. Suppose that a_0 is positive. Then the Routh-Hurwitz theorem states that necessary and sufficient conditions for all roots of this polynomial equation to have negative real parts are given by

(1) all coefficients are positive, $a_k > 0$ for $k = 0, 1, 2, ..., n$,

(2) $\Delta_1 > 0,\ \Delta_2 > 0, ...,\ \Delta_n > 0$,

where $\Delta_1,\ \Delta_2, ...,\ \Delta_n$ are the leading principal minors of the Routh-Hurwitz matrix

$$\begin{bmatrix} a_1 & a_3 & a_5 & a_7 & ... & 0 \\ a_0 & a_2 & a_4 & a_6 & ... & 0 \\ 0 & a_1 & a_3 & a_5 & ... & 0 \\ 0 & a_0 & a_2 & a_4 & ... & 0 \\ 0 & 0 & a_1 & a_3 & ... & 0 \\ ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & 0 & ... & a_n \end{bmatrix}.$$

Since general stability results cannot be obtained analytically, we will specify the values of parameters and consider special cases.

3.2 Spcial Cases

The first case represent the model, when no delay is present, and then different values of m will be considered which corresponds to different shapes of the weighting function.

Case 1 $T = 0$.

$T = 0$ corresponds to the model without delay. Taking $T = 0$ in the characteristic equation gives the unique characteristic root,

$$\lambda = \alpha - \beta$$

which is negative under Assumption 3. Local stability under the general form of the production function satisfying Assumption 1 is already shown in (8). Here we have local asymptotic stability under the specific production function (12) without time delay. This result implies that a strong nonlinearity alone cannot be a source of complex dynamics in the continuous-time model. This shows a sharp contrast to dynamic results in nonlinear discrete-time models, in which a strong nonlinearity itself is a source of complex dynamics.

Case 2 $T > 0$ and $m = 0$.

By substituting $m = 0$ and $q = 1$ into (17), we obtain the quadratic characteristic equation

$$T\lambda^2 + (1 + \alpha T)\lambda + \beta - \alpha = 0. \tag{18}$$

Since $a_0 = T > 0$, $a_1 = 1 + \alpha T > 0$ and $a_2 = \beta - \alpha > 0$, then $\Delta_1 > 0$ and $\Delta_2 = a_1 a_2 > 0$. Due to the Routh-Hurwitz criterion, we have local asymptotic stability when the weighting function is exponentially declining.

Case 3 $T > 0$ and $m = 1$.

In this case, the characteristic equation becomes cubic in λ,

$$T^2\lambda^3 + T(2 + \alpha T)\lambda + (1 + 2\alpha T)\lambda + \beta - \alpha = 0. \tag{19}$$

All coefficients are positive, $a_0 = T^2 > 0$, $a_1 = T(2+\alpha T) > 0$, $a_2 = (1+2\alpha T) > 0$ and $a_3 = \beta - \alpha > 0$. Then the stability condition is that the leading principle minors are positive,

$$\Delta_1 = a_1 > 0,$$

$$\Delta_2 = a_1 a_2 - a_0 a_3 > 0,$$

$$\Delta_3 = a_3(a_1 a_2 - a_0 a_3) > 0.$$

The first and third inequalities are satisfied, given the second. We, therefore, have asymptotical stability if and only if $\Delta_2 > 0$, that is,

$$\begin{aligned} a_1 a_2 - a_0 a_3 &= T(2 + \alpha T)(1 + 2\alpha T) - T^2(\beta - \alpha) \\ &= T\left\{2\alpha^2 T^2 + (6\alpha - \beta)T + 2\right\} > 0. \end{aligned}$$

Let us denote the expression in the parentheses by $\varphi(T)$. It is quadratic in T and its discriminant is

$$D = (\beta - 2\alpha)(\beta - 10\alpha).$$

Under Assumption 3, the first factor is positive, so we can identify the following three cases.

(i) If $3\alpha < \beta < 10\alpha$, then $D < 0$ indicating that $\varphi(T)$ has no real roots. Consequently $\Delta_2 = T\varphi(T) > 0$ holds for all $T > 0$, implying asymptotic stability of the steady state.

(ii) If $\beta = 10\alpha$, then $D = 0$ so $\varphi(T)$ has only one root (with multiplicity two)

$$T^* = -\frac{6\alpha - \beta}{4\alpha^2} = \frac{1}{\alpha} > 0.$$

Thus $\Delta_2 > 0$ holds for all $T \neq T^*$, implying asymptotic stability for all T except a configuration of (α, β, T) such that $\alpha T = 1$ and $\beta = 10\alpha$.

(iii) If $\beta > 10\alpha$, then $D > 0$ indicating that $\varphi(T)$ has two real roots, T_1^* and T_2^* such as $0 < T_1^* < T_2^*$ where

$$T_{1,2}^* = \frac{\beta - 6\alpha \pm \sqrt{(\beta - 2\alpha)(\beta - 10\alpha)}}{4\alpha^2}.$$

It is clear that $\Delta_2 > 0$ holds for $T < T_1^*$ or $T > T_2^*$ and $\Delta_2 < 0$ for $T_1^* < T < T_2^*$. Therefore we have asymptotic stability if $T < T_1^*$ or $T > T_2^*$ and local instability if $T_1^* < T < T_2^*$. So stability is lost at $T = T_1^*$ and is regained at $T = T_2^*$. Notice that $(\alpha T_1^*)(\alpha T_2^*) = 1$.

We visualize the analytical results obtained above in Figure 1. Given a value of α, the whole region of (β, T) is divided into two parts by the partition curve, $\Delta_2 = 0$; the gray region is the instability region with $\Delta_2 < 0$ and the white region is the stability region with $\Delta_2 > 0$. The red point on the $\Delta_2 = 0$ curve implies that the characteristic roots are multiple for $\beta = 10\alpha$ and $T = 1/\alpha$. From the above analysis, we can draw the following conclusion in the case of $m = 1$. If $\beta < 10\alpha$, then the stationary point is locally asymptotically stable regardless of the value of T. Returning to the definitions, the value of $\alpha = n + s\mu$ depends on model parameter values and $\beta = sA$ is the product of the marginal propensity to save and the technology parameter. Since we need a large value of the technology parameter to reverse this inequality, the high technological progress may be a source of instability. Assume next that $\beta > 10\alpha$. The delay has to be sufficiently small or sufficiently large to guarantee local asymptotical stability. For example, when $\beta = 12\alpha$, the threshold values of T are

$$T_1^* = \frac{3 - \sqrt{5}}{2\alpha} < \frac{1}{\alpha} \text{ and } T_2^* = \frac{3 + \sqrt{5}}{2\alpha} > \frac{1}{\alpha},$$

both of which are indicated by the two black points on the separation curve $\Delta_2 = 0$ in Figure 1. The stationary point is locally unstable for T in the shaded region (i.e., $T_1^* < T < T_2^*$) and stable for T in the white region (i.e., $T < T_1^*$ or

$T > T_2^*$).

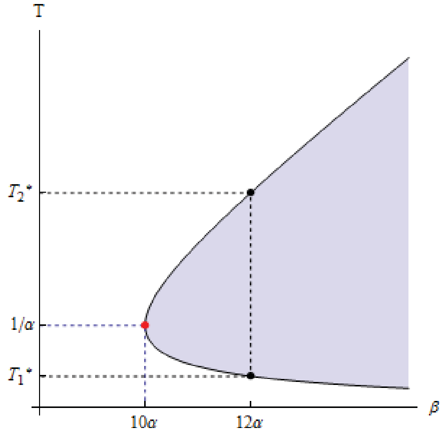


Figure 1. Stability and instability regions when $m = 1$

We now return to the characteristic equation (19) and show the possibility of the birth of a limit cycle at the threshold values $T = T_1^*$ and $T = T_2^*$ by applying the Hopf bifurcation theorem. According to the theorem, we can establish the existence of a cyclic solution if the cubic characteristic equation has a pair of pure imaginary roots and the real parts of these roots change signs with a bifurcation parameter. At the critical values where $\Delta_2 = 0$ or $a_3 = a_1 a_2 / a_0$, the characteristic equation can be rewritten as

$$a_0\lambda^3 + a_1\lambda^2 + a_2\lambda + \frac{a_1 a_2}{a_0} = 0,$$

which factors as

$$\left(\lambda + \frac{a_1}{a_0}\right)\left(a_0\lambda^2 + a_2\right) = 0$$

showing that one of the characteristic roots is real and negative,

$$\lambda_1 = -\frac{a_1}{a_0} = -\frac{2 + \alpha T}{T} < 0$$

and the other two are pure imaginary,

$$\lambda_{2,3} = \pm i\sqrt{\frac{a_2}{a_0}} = \pm i\sqrt{\frac{1 + 2\alpha T}{T^2}} = \pm i\gamma. \tag{20}$$

In order to apply the Hopf bifurcation theorem, we need to check whether the real part of the conjugate complex root changes its sign as the bifurcation parameter passes through its critical value. Selecting T as the bifurcation parameter, assuming $\lambda = \lambda(T)$ and then differentiating the characteristic equation with respect to T, we obtain, after arranging terms, the derivative of $\lambda(T)$,

$$\frac{d\lambda}{dT} = -\frac{2\left(T\lambda^3 + (1 + \alpha T)\lambda^2 + \alpha\lambda\right)}{3T^2\lambda^2 + 2(2T + \alpha T^2)\lambda + (1 + 2\alpha T)}.$$

Using the facts that the terms with λ are pure imaginary and the constant terms are real gives

$$\begin{aligned}\left[\frac{d(\mathrm{Re}\,\lambda)}{dT}\right]_{\lambda=\pm i\gamma} &= \mathrm{Re}\left[-\frac{2\left(T\lambda^3+(1+\alpha T)\lambda^2+\alpha\lambda\right)}{3T^2\lambda^2+2(2T+\alpha T^2)\lambda+(1+2\alpha T)}\right]_{\lambda=\pm i\gamma} \\ &= \frac{2(1+T_i^{*2}\gamma^2)}{(1+2\alpha T_i^*-3T_i^{*2}\gamma^2)^2+(4T_i^*+2\alpha T_i^{*2})^2\gamma^2}(1-\alpha T_i^*),\end{aligned}$$

where both the numerator and the denominator of the first factor are positive. Therefore we have

$$\left[\frac{d(\mathrm{Re}\,\lambda)}{dT}\right]_{\lambda=\pm i\gamma} \gtrless 0 \text{ if } 1 \gtrless \alpha T_i^* \text{ for } i=1,2.$$

These inequalities imply that the roots cross the imaginary axis at $i\gamma$ from left to right as T increases from $T_1^* < 1/\alpha$ (i.e., stability is lost) and from right to left as T increases from $T_2^* > 1/\alpha$ (stability is regained). In Figure 1, the stability loss occurs when T enters the gray unstable region from below and the stability regain occurs when T re-enters the white stable region from below.

In order to find what kind of dynamics is generated for $T \in (T_1^*, T_2^*)$, we perform numerical simulations. To this end, we transform the integro-differential dynamic system to the three-dimensional dynamic system of ordinary differential equations:

$$\begin{aligned}\dot{k}(t) &= -\alpha k(t) + \beta k^e(t)\left[1-k^e(t)\right], \\ \dot{k}^e(t) &= \frac{1}{T}\left(x(t)-k^e(t)\right), \\ \dot{x}(t) &= \frac{1}{T}\left(k(t)-x(t)\right).\end{aligned}$$

The first equation is the same as the first equation of (14). With $m = 1$, the weighting function is

$$k^e(t) = \int_0^t \left(\frac{1}{T}\right)^2 (t-\sigma)e^{-\frac{t-\sigma}{T}}\,d\sigma,$$

and the second equation is obtained by differentiating both sides of the above equation with respect to t and introducing the new variable

$$x(t) = \int_0^t \frac{1}{T}e^{-\frac{t-e}{T}}k^e(\sigma)d\sigma.$$

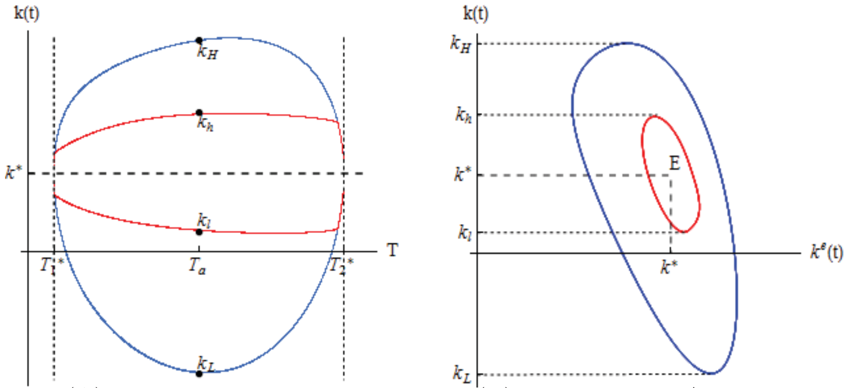
The third equation is obtained by differentiating $x(t)$ with respect to t. Needless to say, the characteristic equation of this 3D system is identical with (19).

Returning to Figure 1, taking $\beta = 12\alpha$ with $\alpha = 1$ and increasing T from a little bit smaller value than T_1^* to a little bit larger value than T_2^* yield the bifurcation-like diagram as is presented in Figure 2(A). It is shown that the stationary point k^* is asymptotically stable for $T < T_1^*$ and $T > T_2^*$ and it bifurcates to cyclic behavior if T is between T_1^* and T_2^*. For each value of T, the dynamic 3D system is solved and we measure a time series of $k(t)$, $k^e(t)$

and $x(t)$ for $3000 \leq t \leq 4000$. The extreme values (i.e., local maximums and local minimums) of the time series $k(t)$ are plotted in blue over each T. The two (upper and lower) blue curves mean that a periodic cycle generated for each $T \in (T_1^*, T_2^*)$ has one maximum and one minimum. In Figure 2(B) we take $T_a = (T_1^* + T_2^*)/2$ and depict the periodic cycle in blue in the phase plane of $k^e(t)$ and $k(t)$, which describes the dynamic flow based on the measured data. It can be seen that points k_H and k_L are the maximum and minimum values of the cycle when $T = T_a$. Two observations can be made: first, there are two points for each $T \in (T_1^*, T_2^*)$ implying that the trajectories converge to a periodic cycle having one maximum and one minimum; second, some segments of the trajectories enter the negative region of k. One way to avoid such economically unfavorable phenomena that the capital dynamics might lead to negative quantities is to modify the capital dynamic equation in such a way that

$$\dot{k}(t) = -\alpha k(t) + \beta Max \{0, k^e(t) [1 - k^e(t)]\}.$$

The simulation results with this modified dynamic equation are obtained in the same way and are depicted as the red curves in Figure 2(A) in which the trajectories of capital stay in the positive region for $T \in (T_1^*, T_2^*)$. The red cycle in Figure 2(B) is in the first quadrant of the phase plane and has the maximum k_h and the minimum k_l. The point k_j for $j = H, h, \ell, L$ in Figure 2(A) corresponds to the same point in Figure 2(B). We will return to the second observation in the final section.



(A) Bifurcation diagram (B) Periodic cycles for $T = T_a$

Figure 2. Birth of cyclic behavior when stability is lost

Case 4 $T > 0$ and $m = 2$.

Substituting $m = 2$ into the characteristic equation (17) yields the following fourth-order equation

$$a_0 \lambda^4 + a_1 \lambda^3 + a_2 \lambda^2 + a_3 \lambda + a_4 = 0$$

where

$$a_0 = T^3 > 0, \ a_1 = T^2(6 + \alpha T) > 0, \ a_2 = 6T(2 + \alpha T) > 0,$$

$$a_3 = 4(2 + 3\alpha T) > 0 \text{ and } a_4 = 8(\beta - \alpha) > 0.$$

All coefficients are positive. In the case of a fourth-order equation, the basic Routh-Hurwitz matrix is

$$\begin{pmatrix} a_1 & a_3 & 0 & 0 \\ a_0 & a_2 & a_4 & 0 \\ 0 & a_1 & a_3 & 0 \\ 0 & a_0 & a_2 & a_4 \end{pmatrix}$$

with leading minors

$$\Delta_1 = a_1 > 0,$$

$$\Delta_2 = 2T^3(3T^2\alpha^2 + 18T\alpha + 32) > 0,$$

$$\Delta_3 = 8T^3\left\{64 + 12T(14\alpha - 3\beta) + 12T^2\alpha(6\alpha - \beta) + T^3\alpha^2(10\alpha - \beta)\right\} \gtreqless 0,$$

$$\Delta_4 = \Delta_3 a_4 > 0 \text{ provided that } \Delta_3 > 0.$$

Therefore $\Delta_3 > 0$ suffices to achieve local asymptotic stability. Notice that $\Delta_3 = 0$ if the cubic equation in T in the braces is equal to zero. The discriminant of this cubic equation is

$$\begin{aligned} D &= -4A_1^3A_3 + A_1^2A_2^2 - 4A_0A_2^3 + 18A_0A_1A_2A_3 - 27A_0^2A_3 \\ &= 442368\alpha^3(\beta - 2\alpha)^2(\beta - 6\alpha) \end{aligned}$$

where $A_0 = 64$, $A_1 = 12(14\alpha - 3\beta)$, $A_2 = 12\alpha(6\alpha - \beta)$ and $A_3 = \alpha^2(10\alpha - \beta)$. The cubic equation $\Delta_3 = 0$ can be reduced to $4(-2 + \alpha T)^2(4 + \alpha T) = 0$ if $\beta = 6\alpha$. It is then clear that the cubic equation has the unique root with multiplicity two for $\beta = 6\alpha$ and $T = 2/\alpha$. By taking the same procedure as in the case of $m = 1$, we can show that the dynamic system with $m = 2$ generates cyclic behavior when the local stability is violated, as illustrated in Figure 3. It is possible to examine the case with $m \geq 3$. However, calculations becomes longer and more clumsy.

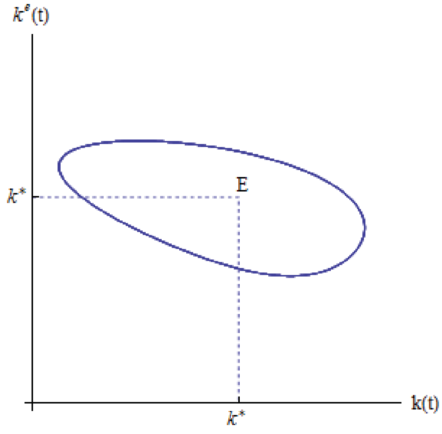


Figure 3. Birth of a limit cycle when $m = 2$, $\alpha = 1$, $\beta = 7$ and $T = 1$

4 Growth Model with a Fixed Time Delay

A fixed time delay is an appropriate approach to deal with the case in which the delay is certain. If τ represents the time delay inherent in the production process, then the per-capita production function reflecting the influence of pollutions is described by

$$f(k(t-\tau)) = Ak(t-\tau)((1-k(t-\tau)).$$

By replacing $k^e(1-k^e)$ in the first equation of (14) by this expression, we obtain a fixed delay differential equation

$$\dot{k}(t) = -\alpha k(t) + \beta k(t-\tau)(1-k(t-\tau)). \tag{21}$$

The dynamics of this equation is examined henceforth. A stationary point of the capital accumulation dynamics is obtained by solving $\dot{k}(t) = 0$ with $k(t) = k(t-\tau) = k$. A simple calculation shows that it is identical with k^* obtained under the continuously distributed time delay.

4.1 Model and Stability Conditions

Following the tradition of the study on fixed delay equations, we start from the local stability analysis of the stationary state and then consider the question of *stability switching* of the fixed delay equation (21). The stability of the stationary state depends on the location of the roots of the associated characteristic equation. The stability analysis concerns with whether all roots lie in the left half of the complex plane. The location of the roots, in turn, depends on the value of the delay. The stability switching analysis concerns with whether roots cross the imaginary axis when the delay changes.

For the first purpose, we make a linear approximation of the nonlinear delay equation (21) to obtain its variational equation,

$$\dot{k}_\delta(t) = -\alpha k_\delta(t) + \beta(1-2k^*)k_\delta(t-\tau)$$

where $k_\delta(t) = k(t) - k^*$ is the deviation from the stationary state. This is a first-order delay differential equation of retarded type. We look for the solution in the usual exponential form $k_\delta(t) = e^{\lambda t}u$. Substituting this expression into the variational equation and noticing that $\beta(1-2k^*) = 2\alpha - \beta$, we have the corresponding characteristic equation,

$$(\lambda + \alpha) - (2\alpha - \beta)e^{-\lambda\tau} = 0. \tag{22}$$

Without time delay (i.e., $\tau = 0$), the characteristic root is $\lambda = -(\beta - \alpha) < 0$. Thus the steady state k^* is a stable fixed point which is attained for any nonzero initial point. We can also obtain the following stability result for the fixed delay model with $\tau > 0$, which corresponds to Theorem 1 obtained in the case of distributed-delay models:

Theorem 2 *The steady state k^* of the fixed delay dynamic equation (21) is locally asymptotically stable if $|2\alpha - \beta| \leq \alpha$, regardless of the value of the fixed time delay τ.*

Proof. Assume to the contrary that for an eigenvalue, $\operatorname{Re}\lambda \geq 0$. Then

$$\begin{aligned} e^{-\lambda\tau} &= e^{-(\operatorname{Re}\lambda)\tau} e^{-i(\operatorname{Im}\lambda)\tau} \\ &= e^{-(\operatorname{Re}\lambda)\tau} \left(\cos((\operatorname{Im}\lambda)\tau) - i\sin((\operatorname{Im}\lambda)\tau)\right). \end{aligned}$$

So $\left|e^{-\lambda\tau}\right| = \left|e^{-(\operatorname{Re}\lambda)\tau}\right| \leq 1$. Notice that since $\lambda \neq 0$,

$$|\lambda + \alpha| > \alpha$$

and

$$\left|(2\alpha - \beta)e^{-\lambda\tau}\right| \leq |2\alpha - \beta|.$$

Under condition $|2\alpha - \beta| \leq \alpha$, the absolute values of the two terms of the characteristic equation are different and thus $\lambda \neq 0$ with $\operatorname{Re}\lambda \geq 0$ cannot be an eigenvalue. This completes the proof. ■

Theorem 2 ensures that the fixed time delay is harmless if $\beta \leq 3\alpha$. Even if this sufficient condition is violated, it can be supposed, due to continuity, that k^* can be still stable for a smaller value of τ. It is also expected that as the length of the delay increases, stability of the stationary state may change (i.e., stability switch). In order to understand this phenomena, it is crucial to determine a threshold value of τ for which the characteristic equation has a pair of conjugate pure imaginary roots. Since $\lambda = 0$ is not a root of the characteristic equation, we can assume without loss of generality that $\lambda = i\omega$ with $\omega > 0$ is a root. Substituting it into the characteristic equation, we can write the real and imaginary parts as:

$$\begin{aligned} \alpha - (2\alpha - \beta)\cos\omega\tau &= 0, \\ \omega + (2\alpha - \beta)\sin\omega\tau &= 0. \end{aligned} \tag{23}$$

The sum of the squares of the two equations yields

$$\omega^2 = (\beta - \alpha)(\beta - 3\alpha)$$

implying that

$$\omega_+ = \sqrt{(\beta - \alpha)(\beta - 3\alpha)},$$

which is real and positive under Assumption 3. Solving the first equation of (23) for τ and substituting ω_+ into the resultant expression yields the threshold value of time delay for which stability switching may occur

$$\tau^* = \frac{\cos^{-1}\left(\dfrac{\alpha}{2\alpha - \beta}\right)}{\sqrt{(\beta - \alpha)(\beta - 3\alpha)}} \equiv \phi(\beta, \alpha). \tag{24}$$

In order to observe stability switching, we need to determine the sign of the derivative of the real part of the purely imaginary root. We can select τ as the bifurcation parameter and consider the characteristic equation as a continuous function in terms of the delay τ. Differentiating the characteristic equation with respect to τ yields

$$\left(1 + (2\alpha - \beta)\tau e^{-\lambda\tau}\right)\frac{d\lambda}{d\tau} = -(2\alpha - \beta)\lambda e^{-\lambda\tau}.$$

For convenience, we study $(d\lambda/d\tau)^{-1}$ instead of $d\lambda/d\tau$. Then we have

$$\left(\frac{d\lambda}{d\tau}\right)^{-1} = \frac{1 + (2\alpha - \beta)e^{-\lambda\tau}\tau}{-\lambda(2\alpha - \beta)e^{-\lambda\tau}}$$

and from the characteristic equation we obtain

$$e^{-\lambda\tau} = \frac{\lambda + \alpha}{2\alpha - \beta}.$$

Thus

$$\begin{aligned}
\left[\mathrm{Re}\left(\frac{d\lambda}{d\tau}\right)^{-1}\right]_{\lambda=i\omega} &= \mathrm{Re}\left(\frac{e^{\lambda\tau}}{-\lambda(2\alpha-\beta)} - \frac{\tau}{\lambda}\right)_{\lambda=i\omega} \\
&= \mathrm{Re}\left(-\frac{1+\tau(\lambda+\alpha)}{\lambda(\lambda+a)}\right)_{\lambda=i\omega} \\
&= \mathrm{Re}\left(\frac{\omega^2(1+\tau\alpha) - \tau\omega^2\alpha}{\omega^4 + \omega^2\alpha^2}\right) \\
&= \frac{1}{\omega^2 + \alpha^2} > 0.
\end{aligned}$$

The last inequality implies that the crossing of the imaginary axis is from the left to the right as τ increases and thus results in the loss of stability. In other words, the fixed delay has a destabilizing effect since the real part turns to be positive from negative as the delay becomes longer. In summary we have the following:

Theorem 3 *Under Assumption 3, the neoclassical growth model with fixed time delay is locally asymptotically stable when $\tau < \tau^*$ and unstable when $\tau > \tau^*$ where the critical level of time delay τ^* is defined by*

$$\tau^* = \frac{\cos^{-1}\left(\dfrac{\alpha}{2\alpha - \beta}\right)}{\sqrt{(\beta - \alpha)(\beta - 3\alpha)}}.$$

4.2 Dynamic Analysis

In this section we will examine the asymptotical properties of the state trajectories of equation (21). Locally stable and unstable combinations of β and τ are graphed in Figure 4 where α is taken to be 1. The dotted vertical locus, $\beta = 3\alpha$, divides the whole (β, τ) region into two parts; the steady state k^* is definitely locally stable in the left of the locus due to Theorem 2. Further, the black-bold locus of $\tau^* = \phi(\beta, \alpha)$ is asymptotic to the $\beta = 3\alpha$ locus and the horizontal axis and divides the remaining region into two parts: the steady state is locally stable for (β, τ) in the region below this locus and unstable above. Since this partition curve between the stable and unstable regions is downward-sloping, larger (respectively smaller) values of the two variables contribute instability (respectively stability) while a trade-off between the two is necessary to preserve stability. We now turn attention to the instability region above the partition curve. It is colored and shows a bifurcation diagram with respect to β and τ.

Different colors indicate different periods of cycles up to 16. Periodic cycle with a period larger than 16 and aperiodic cycle are colored in red. A period of one cycle is defined by a number of crossing of a time-trajectory with the $k = k^*$ locus from above within one cycle (from one peak to the next peak or from one trough to the next trough). This diagram shows that the delay dynamic equation (21) generates a wide variety of dynamics ranging from a simple limit cycle to chaotic fluctuations when stability is lost. As is seen shortly, the dynamic equation gives rise a period-one cycle for a combination of β and τ in the purple region just above the separation curve, a period-two cycle for a combination in the dark purple region above the purple region, a period-four cycle in the thin blue region and chaos in the red region. The equilibrium point goes to chaos via the period-doubling cascade that displays a sequence of bifurcations in which the period of the cycle doubles as a parameter combination is changed slightly.

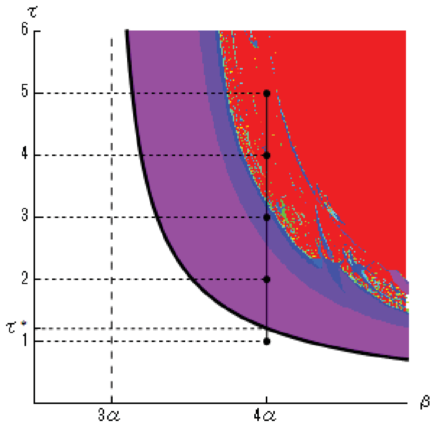


Figure 4. Two-parameter bifurcation diagram of the fixed delay equation (21)

To check what dynamics emerges in each different colored region, we take $\alpha = 1$, $\beta = 4\alpha$, pick up three dotted points, $(4\alpha, 2)$, $(4\alpha, 3)$ and $(4\alpha, 4)$ along the vertical real line in Figure 4 and then perform simulations under these specifications of the bifurcation parameters. These show a sequence of a time trajectory and $(k(t), k(t-\tau))$ plots of these time delay embeddings for three different values of τ. Notice that $k(t-\tau)$ is the ordinate in both diagrams in each part of Figure 5 to make comparison easier. When $\tau = 2$, as shown in Figure 5(A), a time trajectory regularly repeats ups and downs and intersects the $k = k^*$ locus once from above within one cycle. That is, the plot of $k(t)$ and $k(t-\tau)$ forms a simple limit cycle which we call a period-one cycle. The dynamic system with any parametric combination in the purple region exhibits the same dynamics. When τ is increased to 3, then Figure 5(B) shows that regularity of the time trajectory is distorted and there are three local maxima and three local minima within one cycle. However it is also observed that the time-trajectory intersects the $k = k^*$ locus twice from above implying that a period-two cycle was born as depicted in the $(k(t), k(t-\tau))$ plane. When $\tau = 4$, as shown in Figure 5(C), the time trajectory exhibits erratic fluctuations implying a very long period or chaos. Accordingly the return map also displays chaotic motions. Although it is not presented here, we have numerically confirmed that the dynamics with parameters in the red region have initial point dependency so that arbitrarily close two initial conditions diverge as time proceeds.

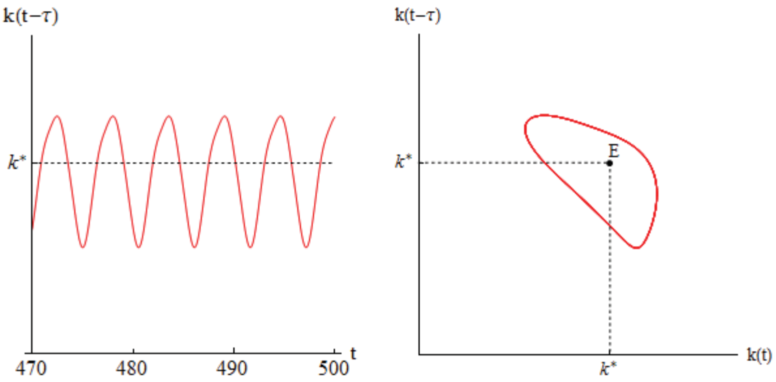


Figure 5(A). Periodic cycle with $\tau = 2$

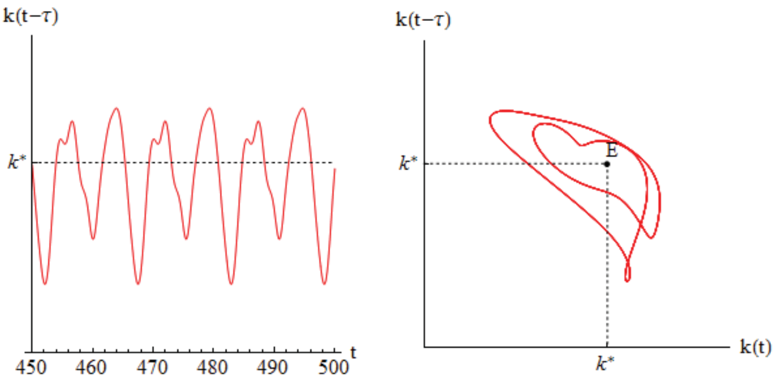


Figure 5(B). Periodic cycle with $\tau = 3$

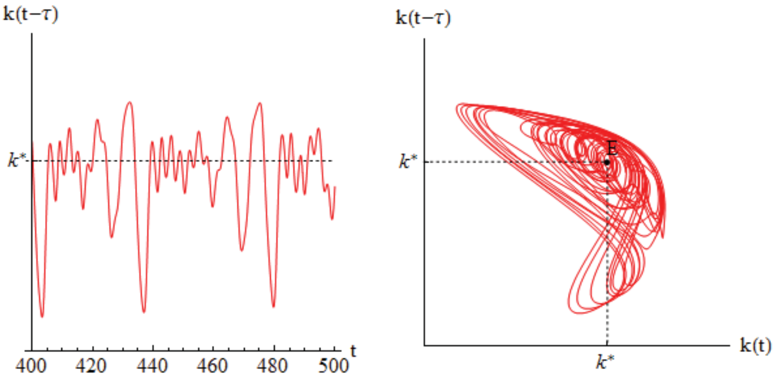


Figure 5(C) Aperiodic cycle with $\tau = 4$

To extend the above analysis, we continuously increase the value of τ from 1 to 5 along the vertical real line in Figure 4. When the value of τ passes the threshold value $\tau^* = \phi(\beta, \alpha)$, which is approximately 1.29 for the specified values of α and β, the fixed point loses its stability and bifurcates to a periodic solution. Simulation results are summarized in a bifurcation-like diagram of $k(t)$ against τ in Figure 6. It is constructed in the following way. For each value of $\tau > \tau^*$, the delay equation (21) is numerically solved for $0 < t \leq 400$, given an initial function $\varphi(t) = 0.5$ for $t \in [-\tau, 0]$. The local maximums and minimums of the time trajectory for $t \in [300, 400]$ are plotted above the selected value of τ. This diagram exhibits a number of local maximum and local minimum within one cycle so it is more likely to the Poincare section. As is already seen above, a time-trajectory has one local maximum and one local minimum for $\tau = 2$ so that there are two points over $\tau = 2$. Two points bifurcates to four extrema that turn to six extrema, shortly after two more points come in. We have already checked that the period-two cycle has six extrema when $\tau = 3$ as shown in Figure 6 where the vertical dotted line at $\tau = 3$ crosses six time the red colored graphs. Although the bifurcation of the extremes does not follow exactly period-doubling, this process repeats itself to generate periodic cycles with higher periods and then chaotic fluctuations.

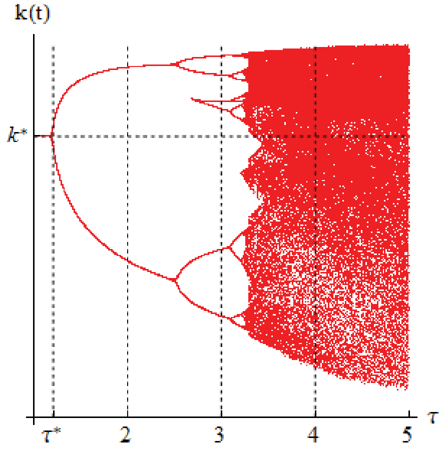


Figure 6. Bifurcation diagram with respect to τ, given $\alpha = 1$ and $\beta = 4\alpha$

5 Comparison of Continuously Distributed and Fixed Delays

No matter which time delay is introduced into the neoclassical growth model, it is found that the delay has a destabilizing effect if Assumption 3 is violated. In this section, we compare the destabilizing effect caused by the continuously distributed time delay with the one by the fixed time delay. We have obtained the instability and stability regions separated by equation (24) under the fixed time delay. We have also obtained the stability regions divided by $\Delta_2 = 0$ when $m = 1$ and $\Delta_3 = 0$ when $m = 2$ under continuously distributed time delay. Repeating the same procedure for $m = 3$ through 5, we get three more partition curves that divide the (β, T) space into the stable and unstable regions. The specified forms of these separation curves, however, are not given here because of their too-long expressions. Then given $\alpha = 1$, the $\tau^* = \phi(\beta, \alpha)$ curve colored in red and the five black partition curves with m from 1 to 5 are illustrated in Figure 7. The four red points there correspond to the combination of (β, T)

for which the partition equations have multiple roots, although it is not easy to recognize that all these separation curves have fork-shaped profiles. It can be seen that all of the partition curves with continuously distributed time delay are located in the gray region where the steady state is locally unstable under the fixed time delay and approaching the partition curve with fixed time delay from the right as the value of m increases.

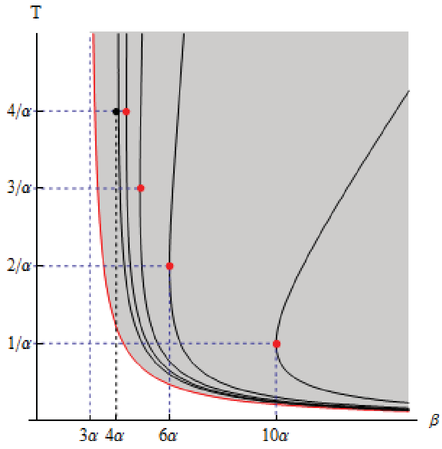


Figure 7. Partition curves of continuously distributed and fixed time delays

It is natural to guess that the distributed stable region becomes gradually smaller and converges to the fixed stable region when m tends to infinity. This is confirmed to be correct if we notice the properties of the weighting function. For $m \geq 1$, it takes a bell-shaped profile. As m increases, the function becomes more peaked around $t - \sigma$ and tends to the Dirac delta function. For sufficient large m, the weighting function may be regarded as very close to the Dirac delta function and the dynamic behavior under the continuously distributed time delay is very similar to the one under the fixed time delay. We can explain this phenomenon mathematically. Given $m \geq 1$, the characteristic equation (17) under continuously distributed time delay can be written as

$$\lambda + \alpha - \frac{2\alpha - \beta}{\left(1 + \dfrac{T}{m}\lambda\right)^m \left(1 + \dfrac{T}{m}\lambda\right)} = 0.$$

As $m \to \infty$, it converges to

$$\lambda + \alpha - (2\alpha - \beta)e^{-T\lambda} = 0$$

which is the characteristic equation (22) of the fixed time delay if T is replaced with τ. In short, under continuously distributed time delay, although we comprehensively use the delayed or past data of capital, the stability region is sensitive to the shape of the weighting function. Increasing the value of m strengthens the destabilizing effect caused by the distributed delay in a sense that the stability region becomes smaller. It then converges to the fixed delay stability region as $m \to \infty$.

We now turn our attention to similarity and dissimilarity of dynamics generated under the fixed time delay to those generated under the continuously distributed time delay. To this end, we set $\alpha = 1$, $\beta = 4$ and $\tau = 4$ which corresponds to the black point in Figure 7 and Figure 4. Given this configuration of the parameters, complex dynamics under the fixed time delay is obtained as illustrated in Figure 6 in which many points remain on the vertical dotted line $\tau = 4$ or in Figure 5(C) in which the same dynamics is plotted in the $(k(t), k(t-\tau))$ plane and seems to be very high-order cyclic or aperiodic. In order to see what dynamics the distributed model can generate under the same parametric condition, we reduce the integro-differential equation (14) to a system of ordinary differential equations. Since the equivalence is already shown in the case of $m = 1$, we assume that $m > 1$. The expected value of capital is given by

$$k^e(t) = \int_0^t \frac{1}{m!}\left(\frac{m}{T}\right)^{m+1} (t-\sigma)^m e^{-\frac{m(t-\sigma)}{T}} k(\sigma) d\sigma$$

and for all $\ell = 0, 1, ...m$, introduce the functions

$$x_\ell(t) = \int_0^t \frac{1}{\ell!}\left(\frac{m}{T}\right)^{\ell+1} (t-\sigma)^\ell e^{-\frac{m(t-\sigma)}{T}} k(\sigma) d\sigma$$

where $x_m(t) = k^e(t)$. Then by differentiation,

$$\dot{k}^e(t) = \frac{m}{T}\left(x_{m-1}(t) - k^e(t)\right) \text{ for } \ell = m,$$

$$\dot{x}_\ell(t) = \frac{m}{T}\left(x_{\ell-1}(t) - x_\ell(t)\right) \text{ for } \ell = 1, ...m-1$$

and

$$\dot{x}_0(t) = \frac{m}{T}\left(k(t) - x_0(t)\right) \text{ for } \ell = 0.$$

Combining these $m+1$ equations with

$$\dot{k}(t) = -\alpha k(t) + \beta k^e(t)(1 - k^e(t))$$

yields the $m+2$ dimensional system of ordinary differential equations for $k(t)$, $k^e(t)$ and $x_\ell(t)$, $\ell = 0, 1, ..., m-1$. Specifying the value of m, we perform simulations. When $m = 10$, the ODE system exhibits a periodic cycle with two extremes as shown in Figure 8(A). When m increases to 30, a periodic cycle with six extremes appears, as shown in Figure 8(B). When m becomes more than 50, erratic behavior may occur and the aperiodic behavior with $m = 100$ is represented in Figure 8(C).

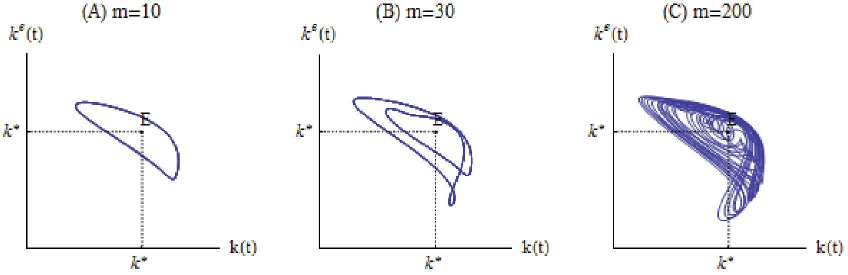


Figure 8. Return maps under continuously distributed time delay

Although Figures 8(A), (B) and (C) look similar to Figures 5 (A), (B) and (C), there is a distinctive difference: the values of α, β and τ are fixed and m is altered in Figure 8 while the value of α and β are fixed and τ is changed in Figure 5. Notice however the similarity between Figure 5(C) and Figure 8(C), both are obtained under the same values of the parameters α, β and τ and $m = 200$. So the following result is confirmed in a different way: the distributed delay model with high value of m generates similar dynamics as the fixed delay model. To measure the complexity of these dynamics generated by the ODE system, we calculate the maximum Lyapunov exponent against various values of m and the results are plotted in Figure 9 where ML on the ordinate stands for Maximum Lyapunov:

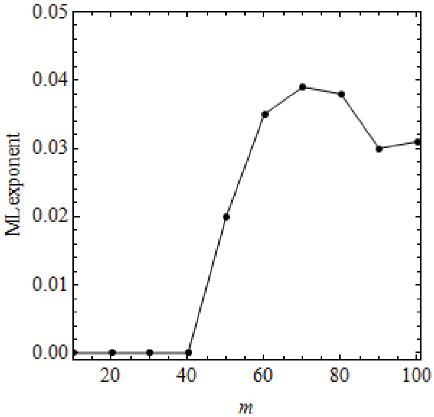


Figure 9 Maximum Lyapunove exponent againts m

The ODE system has zero maximum Lyapunov exponent for m below 40 and a positive maximum Lyapunov exponent for m greater than 50. That is, chaotic dynamics are born for some values of m between 40 and 50. The fixed delay model (14) can give rise to chaos as numerically confirmed in Figure 6. It looks like a simple equation, however, it is actually infinite dimensional dynamic system. If the distributed delay model with higher value of m can approximate dynamics generated by the fixed delay model, it can be safe to say that chaos generated by the fixed delay model is thought to be high dimensional chaos since the distributed delay model can generate the similar complex dynamics only when m takes a large value. With these simulations and the results obtained in Section 3, we summarize the main points of the dynamic model with continuously distributed time delay:

Theorem 4 *Under Assumption 3, dynamics in the neoclassical growth model with continuously distributed time delay is the following:(1) it is locally asymptotically stable when the shape parameter m is zero or unity; (2) it generate periodic cycles for* $2 \leq m \leq 40$*; (3) it gives rise to aperiodic fluctuations for* $m \geq 50$*; (4) it converges to the neoclassical growth model with fixed time delay as m goes to infinity.*

6 Concluding Remarks

The neoclassical growth model is developed in this paper with continuous time scales and two kinds of time delay. It complements the pioneering work of Day (1982) in which discrete-time and a production lag are assumed. A special functional form is introduced to consider pollution in production, which results in the dynamic model equivalent to that induced by the logistic map. Our continuous-time model is similar in spirit and functional form to Day's discrete-time model.

The capital accumulation process has been formulated in a way that allows for both continuously distributed and fixed time delays. They are a clear consequence of delays in information gathering in the production process and in investment decisions. We start with the continuous model without time delay and confirm the well-known result that the steady state k^* is always asymptotically stable if $\alpha < \beta$. Our first concern in this paper is on the preservation of stability in the delay models. Theorems 1 and 2 show that the same state k^* in the delay model is always asymptotically stable, regardless of the length of the time delay, if $\beta \leq 3\alpha$. This condition holds if the technology parameter A is sufficiently small. In addition to this, Case 2 with $T > 0$ and $m = 0$ implies asymptotic stability of k^*. Thus, our first conclusion is that the delay becomes harmless to stability of the steady state if the technology parameter takes a small value or the weighting function is exponentially declining.

Our second concern turns to the destabilizing effect caused by time delay when $\beta > 3\alpha$. In the case of a continuously distributed time delay, the capital accumulation process becomes a set of integro-differential equations and under specific conditions, local instability may occur. The possibility of the birth of limit cycles is analytically verified based on the Hopf bifurcation theorem. The shape of the separation curves depends on the value of the shape parameter m of the weighting function. The separation curve takes a fork-shaped profile for a small values of m, indicating that the time lag has to be sufficiently small or sufficiently large to guarantee local asymptotical stability as depicted in Figures 1 and 4. Further, numerical simulations show that some segments of the cyclic trajectories enter into the non-feasible negative region (Figure 2). This cannot happen if deviations of time delay from its critical values are small enough as shown in Figure 1 and Figure 3. However this is not the case with large deviations. In order to force the trajectory to move back to the positive region from zero values, we impose the non-negativity constraint on output in the capital dynamic equation. This is a simple way to prevent trajectories from being negative. In a more sophisticated way, the mound-shaped production function has to be specified more properly. In our next paper this issue will be further elaborated and the corresponding dynamic model investigated, where the economic variables remain non-negative without any nonnegative constraints. Our second conclusion is that the model with a continuously distributed time delay exhibits various dynamics including stability, periodic-cycles and aperiodic-cycles, depending on the value of m as shown in Figure 8.

Next we examine the destabilizing effect when the dynamic system moves to fixed delay from distributed delay. We analytically derive the critical value of the length of the fixed delay at which the stability switch occurs and then numerically demonstrate that increasing the length of the fixed delay from the critical value alters movement of the dynamics from stable steady state, through periodic cycle, to chaos. The simulation result suggests that a longer fixed delay inducing complex behavior involving chaos becomes more likely. Our third conclusion is essentially the same as our second: there is a period-doubling cascade to chaos if τ increases beyond its critical value τ^*.

An important relation between continuously distributed and fixed time delays has been pointed out in this paper. We have shown that the characteristic equation of the distributed case converges to that of the fixed delay model when the shape parameter of the weighting function tends to infinity. This interesting property of the growth model has been also illustrated by the convergence of the stability regions. This means that when the distributed delay becomes more narrow, cyclic and chaotic behavior are more pronounced. The destabilizing effect can be summarized as follows: a time delay can change the dynamics significantly implying that the steady state bifurcates to chaos through period-doubling cascade if the time length τ increases in the fixed delay model while the same phenomenon occurs if the value of m increases in the distributed delay model. The main result of this paper is to show the birth of limit cycles and possible chaotic behavior in the continuous-time neoclassical growth model if time delays ar taken into account.

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DIMENSIONS, LOGARITHMIC FUNCTION, COBB-DOUGLAS FUNCTION AND CURVE FITTING PRACTICE IN ECONOMICS: MAINTAINING SIMON KUZNETS' EMPIRICAL TRADITION

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Abstract

This paper is to examine the proper use of dimensions and curve fitting practices in view of Simon Kuznets' empirical tradition. Section 2 deals with logarithmic function, and shows that the argument in the function must be a dimensionless number, otherwise it is nonsensical. Section 2 also deals with this analytical fallacy and presents unfortunate examples of this analytical error including several Nobel prize winners in economics. Section 3 then investigates a criterion of data set under which any functional transformation (including logarithmic transformation) from the original data is superior to regression specification of a linear form. Section 4 deals with the standard Cobb-Douglass function, proving that the operational meaning cannot be obtained for capital or labor within the Cobb-Douglas function. Section 4 also deals with "curve fitting fetishism" by resorting two famous mathematical theorems, i.e., the Weierstrass Approximation Theorem and the Luzin Theorem. Section 5 concludes this paper with several epistemological issues in relation to dimensions and curve fitting practices in economics.

1. Introduction

Simon Kuznets is well known as a meticulous scientist concerning use of mathematical models (see for example, Georgescu-Roegen 1976; Fogel 2001; Weyl 2010). Kuznets indicates that many different mathematical models, varying in complexity and structure, may give fairly good statistical fits to a given body of data. However, Kuznets "shows a very limited sympathy for abstract and generalizing models which provide few opportunities of empirical testing" (cited by Wely 2010). The purpose of this paper is to examine the proper use of dimensions and curve fitting practices in view of Simon Kuznets' empirical tradition. Section 2 deals with logarithmic function, and shows that the argument in the function must be a dimensionless number, otherwise it is nonsensical. Section 2 also deals with this analytical fallacy and presents unfortunate examples of this analytical error including several Nobel prize winners in economics. Section 3 then investigates a criterion of data set under which any functional transformation (including logarithmic transformation) from the original data is superior to regression specification of a linear form. Section 4 deals with the standard Cobb-Douglass function, proving that the operational meaning cannot be obtained for capital or labor within the Cobb-Douglas function. Section 4 also deals with "curve fitting fetishism" by resorting two famous mathematical theorems, i.e., the Weierstrass Approximation Theorem and the Luzin Theorem. Section 5 concludes this paper with several epistemological issues in relation to dimensions and curve fitting practices in economics.

2. The logarithmic function and dimensions: a fatal analytical fallacy

Any normal person understands the meaning and implications of dimensional homogeneity: two numbers with different dimensions cannot be added, thus the sum "10 kg" plus "20 m^2" does not make any sense (Mayumi and Giampietro 2010). A corollary of this principle is that it is meaningless to put a dimensional argument in the logarithmic function. Yet it is quite surprising to see that many economists violate this fundamental principle of arithmetic to be shown in this section. In particular, many economists put dimensional arguments in a logarithmic function.

Let's start with the following expression,

$$\log_e(1+x) = \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} +,,,+ \frac{(-1)^{n-1}x^n}{n} +,,,, \quad (1)$$

where $-1 < x \le 1$ (2).

Replacing x by $-x$ in relation (2) produces the following,

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} -,,,- \frac{x^n}{n} +,,,, \quad (3).$$

Combining these two expressions (1) and (3) we have the following,

$$\ln\left(\frac{1+x}{1-x}\right) = 2(x + \frac{x^3}{3} + \frac{x^5}{5} +,,,+ \frac{x^{2m-1}}{2m-1} +,,,) \quad (4).$$

Therefore, a unique value of x ($-1<x<1$) exists corresponding to z which is positive, as shown in Figure 1. Thus, for every positive real number z, we can safely define the logarithmic function as follows using the relation: $x = \dfrac{z-1}{z+1}$

$$\ln z = 2\{\left(\frac{z-1}{z+1}\right) + \frac{1}{3}\left(\frac{z-1}{z+1}\right)^3 + \frac{1}{5}\left(\frac{z-1}{z+1}\right)^5 +,,,+ \frac{1}{2m-1}\left(\frac{z-1}{z+1}\right)^{2m-1} +,,,\} \quad (5)$$

It is obvious that if the value of z is expressed in US$, this operation will create both "a square dollar" and "a cubic dollar", both of which are nonsensical, let alone "higher order dollars". Putting dollar values in the logarithmic function is analytically absurd as shown in Figure 2 and Figure 3.

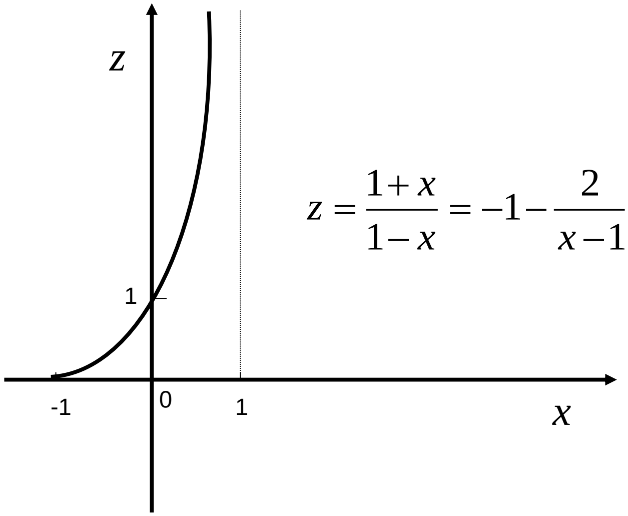

Figure 1. The monotonically increasing function z(x) for -1<x<1.

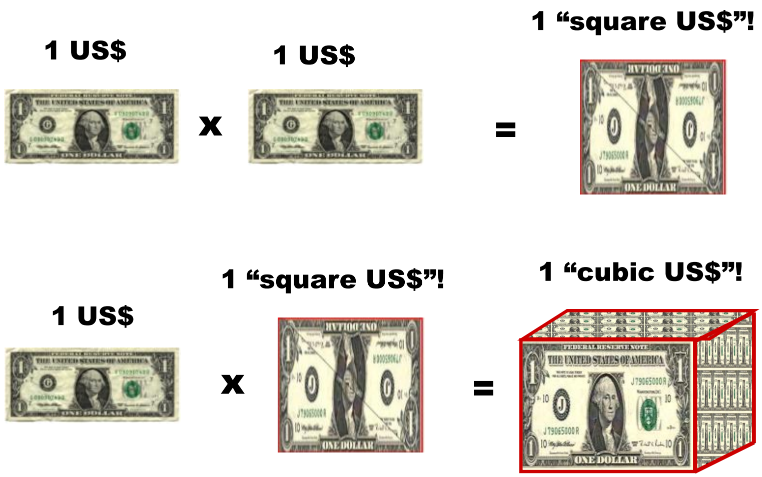

Figure 2. Square and Cubic Dollars created by Economists

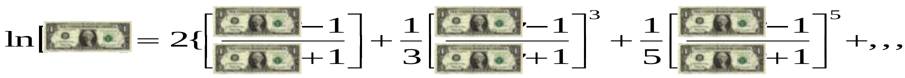

Figure 3. Putting 1 US Dollar in the Logarithmic Function

Before reviewing examples of this analytical fallacy committed by researchers including several Nobel prize winners in economics, we have to reveal our own error with respect to dimensions, which was made when using a logarithmic function (Pastore et al. 2000). In that paper, after introducing the biophysical indicator of socioeconomic development BEP (Bio-Economic Pressure), we tried to check whether or not BEP is well correlated with other traditional economic indicators. BEP is the energy consumption within a particular year divided by the labor hours allocated to the productive sectors of the economy. The dimensions of BEP is (energy unit per year)/(work hours). In spite of this fact, we mistakenly put this quantitative assessment of BEP into a logarithmic function. Of course this operation cannot be accepted.

The second example is Arrow et al (1961). Arrow et al. tried to investigate the substitution between capital and labor within the neoclassical production theory. In Section I of that paper they used regression analysis incorporating the following variables (Arrow et al. 1961, p. 227, please note that the definitions of these variables are stated by themselves):
V: value added in thousands of U.S. dollars
L: labor input in man-years
W: wages (total labor cost divided by L) in dollars per man-year

They statistically *tested* the following two simple relations using these three variables:

$$\frac{V}{L} = c + dW + \eta \qquad (6)$$

$$\log \frac{V}{L} = \log a + b \log W + \varepsilon \quad (7)$$

Of course relation (7) cannot be used judging by the dimensions *V/L* and *W* which they used.

Similarly macroeconomics often uses the logarithmic specification. Consider three papers of Lobert Lucas, Jr. that we happened to encounter during our writing of this paper, since he can be regarded an important representative of the macroeconomics field.

In the paper, "Making A Miracle" (Lucas 1993), *perhaps without any doubt* Allan D. Searle's result (1945), shown in Lucas' paper as Figure 1, is cited. According to Lucas, "Searle plotted man-hours vessel against number of vessels completed to date in that yard *on log-log paper* (Lucas 1993, pp. 259-260, italics added).

In another paper, "Macroeconomic Priorities" (Lucas 2003), Lucas states that "[u]sing annual U.S. data for the period 1947-2001, the standard deviation of *the log of real per capita consumption* about a linear trend is 0.0032" (Lucas 2003, p. 4, italics added).
In yet another paper, "Trade and the Diffusion of the Industrial Revolution" (Lucas 2009), he mentions that we "consider a world of one sector "AK" economies in which an economy's *GDP per capita* is proportional to its stock of human capital, knowledge capital, or whatever term you like" (Lucas 2009, p. 5). At this moment, we put aside the issue of measuring the amount of "knowledge capital" in concrete terms, which is itself a formidable task for any human beings. Lucas created four figures (Figure. 11, Figure 13, Figure 14 and Figure 15 in that paper) all of which have the same horizontal axis, *Log per capita GDP*. All these figures are nonsensical according to what has been said thus far.

At this moment we think that it is crucially important to note the following point. Suppose that an argument a, for instance per capita GDP, is represented in US dollars and we transform a into b represented in Japanese yen where $b=ea$ and e is the exchange rate (yen/US dollar). Taking the natural logarithm on both sides (supposing this operation makes sense), we have

$$\ln b = \ln ea = \ln e + \ln a \quad (8).$$

The readers must be convinced that the principle of dimensional homogeneity is totally violated, since the exchange rate e is transformed into $\ln e$ and added to $\ln a$.
Is it possible, therefore, for us to make an international comparison in Macroeconomics of per capita GDP if we transform per capita GDP into a logarithmic scale? Of course, not!

It is very interesting to investigate when this unfortunate practice of putting dimensional arguments in the logarithmic function started. Our "educated guess" is that this analytical fallacy started with the publication of the classic article written by Christensen et al. (1973). Consider their original formulation. They assumed that there are two outputs—consumption (C) and investment (I)—and two inputs—capital (K) and labor (L). The corresponding prices are $q_C, q_I, q_K,$ and q_L. They call F the production frontier in the following formulation,

$$\begin{aligned}\ln(F+1) = {} & \alpha + \alpha_C \ln C + \alpha_I \ln I + \alpha_K \ln K + \alpha_L \ln L + \alpha_A \ln A + \ln C(\frac{1}{2}\beta_{CC} \ln C + \\ & \beta_{CI} \ln I + \beta_{CK} \ln K + \beta_{CL} \ln L + \beta_{CA} \ln A) + \ln I(\frac{1}{2}\beta_{II} \ln I + \beta_{IK} \ln K + \beta_{IL} \ln L + \beta_{IA} \ln A) + \\ & \ln K(\frac{1}{2}\beta_{KK} \ln K + \beta_{KL} \ln L + \beta_{KA} \ln A) + \ln L(\frac{1}{2}\beta_{LL} \ln L + \beta_{LA} \ln A) + \ln A(\frac{1}{2}\beta_{AA} \ln A)\end{aligned} \quad (9),$$

where, *according to the authors, A* is an index of technology.

It is not clear how to properly create this index. However, they use the price frontier as follows,

$$\ln(P+1) = \alpha_0 + \alpha_C \ln q_C + \alpha_I \ln q_I + \alpha_K \ln q_K + \alpha_L \ln q_L + \alpha_A \ln A + \ln q_C (\frac{1}{2}\beta_{CC} \ln q_C + \beta_{CI} \ln q_I + \beta_{CK} \ln q_K + \beta_{CL} \ln q_L + \beta_{CA} \ln A) + \ln q_I (\frac{1}{2}\beta_{II} \ln q_I + \beta_{IK} \ln q_K + \beta_{IL} \ln q_L + \beta_{IA} \ln A) + \ln q_K (\frac{1}{2}\beta_{KK} \ln q_K + \beta_{KL} \ln q_L + \beta_{KA} \ln A) + \ln q_L (\frac{1}{2}\beta_{LL} \ln q_L + \beta_{LA} \ln A) + \ln A (\frac{1}{2}\beta_{AA} \ln A) \quad (10).$$

Since they clearly state that the "corresponding *prices* are q_C, q_I, q_K, q_L" (Christensen et al. 1973, p. 33, italics added), this specification cannot be used both in relation (9) and in relation (10).

3. A logarithmic specification

It is worthwhile to investigate the issue of whether or not using a transformation including logarithmic function as a dependent variable really improves the least square norm. We compare two regressions for the same data set as follows (assuming that *Y* is dimensionless pure number):

$$Y = \alpha_1 + \beta_1 X + u_1 \quad (11)$$

$$h(Y) = \alpha_2 + \beta_2 X + u_2 \quad (12)$$

where u_1 and u_2 are regression error terms.

The corresponding least square norms are:

$$R_1^2 = \frac{\{n(\sum_1^n X_i Y_i) - (\sum_1^n X_i)(\sum_1^n Y_i)\}^2}{\{n(\sum_1^n X_i^2) - (\sum_1^n X_i)^2\}\{n(\sum_1^n Y_i^2) - (\sum_1^n Y_i)^2\}} \quad (13)$$

$$R_2^2 = \frac{\{n(\sum_1^n X_i h(Y_i)) - (\sum_1^n X_i)(\sum_1^n h(Y_i))\}^2}{\{n(\sum_1^n X_i^2) - (\sum_1^n X_i)^2\}\{n(\sum_1^n (h(Y_i)^2) - (\sum_1^n h(Y_i))^2\}} \quad (14)$$

We derive the condition on values of data set (X_i, Y_i) for which $R_2^2 > R_1^2$ is satisfied.

First we consider the following quadratic form:

$$n(\sum_{1}^{n} X_i Y_i) - (\sum_{1}^{n} X_i)(\sum_{i}^{n} Y_i) = {}^t X \begin{pmatrix} (n-1) & -1,,, & -1 & -1 \\ -1 & (n-1),,, & -1 & -1 \\ ,,, & ,,, & ,,, & ,,, \\ -1 & ,,, & -1 & (n-1) \end{pmatrix} Y = {}^t XAY \quad (15)$$

where $X = {}^t(X_1, X_2,,, X_n)$ and $Y = {}^t(Y_1, Y_2,,, Y_n)$ (t signifies transpose)

Since $\det(\lambda E - A) = \lambda(\lambda - n)^{n-1}$ where E is the unit matrix, there are two eigenvalues of A, namely, 0 and n.
An eigenvector associated with the eigenvalue 0 is ${}^t(1,1,,,,1)$.
(n-1) eigenvectors associated with the eigenvalue n are ${}^t(\delta_{1k}, \delta_{2k},,,, \delta_{(n-1)k}, -1)$ where δ_{ik} is the Kroneker delta and k = 1,2,,,,(n-1).

By utilizing the Gram-Schmidt orthogonalization process (e.g., Roman 2008, p. 214), we can construct the following normalized orthogonal matrix P (i.e., ${}^tPP = E$) from the n eigenvectors obtained above:

$$P = (P_1, P_2,,,, P_n) \quad (16)$$

where

$$P_1 = {}^t(\frac{1}{\sqrt{2}}, 0, 0,,,,, 0, \frac{-1}{\sqrt{2}})$$

$$P_2 = {}^t(\frac{-1}{\sqrt{2^2+2}}, \frac{2}{\sqrt{2^2+2}}, 0, 0,,, 0, \frac{-1}{\sqrt{2^2+2}})$$

$$P_3 = {}^t(\frac{-1}{\sqrt{3^2+3}}, \frac{-1}{\sqrt{3^2+3}}, \frac{3}{\sqrt{3^2+3}}, 0, 0,,, 0. \frac{-1}{\sqrt{3^2+3}})$$

$$P_4 = {}^t(\frac{-1}{\sqrt{4^2+4}}, \frac{-1}{\sqrt{4^2+4}}, \frac{-1}{\sqrt{4^2+4}}, \frac{4}{\sqrt{4^2+4}}, 0, 0,,, 0, \frac{-1}{\sqrt{4^2+4}})$$

$$\vdots$$

$$P_{n-1} = {}^t(\frac{-1}{\sqrt{(n-1)^2+(n-1)}}, \frac{-1}{\sqrt{(n-1)^2+(n-1)}},,,, \frac{-1}{\sqrt{(n-1)^2+(n-1)}}, \frac{n-1}{\sqrt{(n-1)^2+(n-1)}})$$

$$P_n = {}^t(\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}},,,,, \frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}})$$

Therefore, after transforming X and Y into the following forms

$$Z = {}^tPX \quad (17)$$

$$W = {}^tPY \quad (18)$$

$$T = {}^tPh(Y) \quad (19)$$

where $Z = {}^t(Z_1, Z_2,,, Z_n)$ and $W = {}^t(W_1, W_2,,, W_n)$. For notational purpose we represent ${}^t(h(Y_1), h(Y_2),,, h(Y_n))$ as $h(Y)$.

Then we obtain

$$n(\sum_1^n X_i^{\ 2}) - (\sum_1^n X_i)^2 = n\sum_i^{n-1} Z_i^2 \quad (20),$$

$$n(\sum_1^n Y_i^{\ 2}) - (\sum_1^n Y_i)^2 = n\sum_1^{n-1} W_i^2 \quad (21) \quad \text{and}$$

$$n(\sum_1^n X_i Y_i) - (\sum_1^n X_i)(\sum_1^n Y_i) = n\sum_1^{n-1} Z_i W_i \quad (22)$$

It should be noted that since the eigenvalue corresponding to ${}^t(1,1,,,,1)$ is zero, the n[th] term does not show up in (20), (21) and (22).

Thus we obtain

$$R_1^2 = \frac{(\sum_1^{n-1} Z_i W_i)^2}{n(\sum_1^{n-1} Z_i^2)(\sum_1^{n-1} W_i^2)} \quad (23)$$

In the same way, we obtain

$$R_2^2 = \frac{(\sum_1^{n-1} Z_i T_i)^2}{n(\sum_1^{n-1} Z_i^2)(\sum_1^{n-1} T_i^2)} \quad (24)$$

Let θ_1 be the angle between $Z^1 = {}^t(Z_1, Z_2,,,, Z_{n-1})$ and $T^1 = {}^t(T_1, T_2,,,, T_{n-1})$ and let θ_2 be the angle between $Z^1 = {}^t(Z_1, Z_2,,,, Z_{n-1})$ and $W^1 = {}^t(W_1, W_2,,,, W_{n-1})$.

These angles are measured counterclockwise from *Z*.

We are interested in the region (θ_1, θ_2) that satisfies the following inequality:

$$\frac{R_2^2}{R_1^2} = \frac{\{\|Z^1\|\|T^1\|\cos\theta_1\}^2 \times \|W^1\|^2\}}{\{\|Z^1\|\|W^1\|\cos\theta_2\}^2 \times \|T^1\|^2\}} > 1 \quad (25)$$

where $\|Z^1\| = \sqrt{\sum_1^{n-1} Z_i^2}$, $\|T^1\| = \sqrt{\sum_1^{n-1} T_i^2}$, and . $\|W^1\| = \sqrt{\sum_1^{n-1} W_i^2}$

Therefore, relation (25) becomes

$(\cos\theta_1 + \cos\theta_2)(\cos\theta_1 - \cos\theta_2) > 0$ (26)

In Figure 4 we show the region (in black) where relation (26) is satisfied.

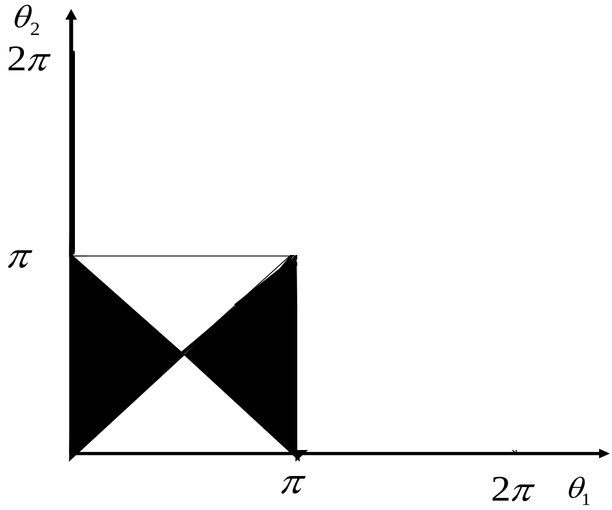

Figure 4. Regions for the two angles for $R_2^2 > R_1^2$

It is easy to check whether or not relation (26) is satisfied with a given data set on (X_i, Y_i). That is, *it is possible to produce an algorithm to judge which specification (11) or (12) is superior in terms of R^2 norm just for the purpose of curve fitting*.

Since $\cos\theta_1 = (\sum_1^{n-1} Z_i T_i)/(\sqrt{\sum_1^{n-1} Z_i^2}\sqrt{\sum_i^{n-1} T_i^2})$

and $\cos\theta_2 = (\sum_1^{n-1} Z_i W_i)/(\sqrt{\sum_1^{n-1} Z_i^2}\sqrt{\sum_i^{n-1} W_i^2})$, these two values are easily calculated from relations (20), (21) and (22). The important point here is that only the relationship between the two angles measured counterclockwise from $Z^1 = {}^t(Z_1, Z_2,,,, Z_{n-1})$ characterizes the superiority of the two regressions in terms of least square norm, not necessarily the overall spatial distribution of the data. So, setting a dependent variable in any functional form does not necessarily improve the regression in terms of least square norm. The analysis presented above can be easily generalized and formulated for cases where other forms of independent variables are included.

4. **The Cobb-Douglas function and curve fitting fetishism in economics**

We have examined how ridiculous it is to put dimensional arguments into the logarithmic function based on the dimensional homogeneity. However, we should also note that there are cases where certain types of algebraic operations on dimensional arguments become meaningless, as already shown in Figure 2. For the same reason we also examine whether or not each term represented in the Cobb-Douglas function has an operational meaning without any analytical fallacy like those we have identified in the case of the transcendental function, in particular the logarithmic function, before thoroughly discussing the curve fitting practice in economics.

We start with the standard Cobb-Douglas function as follows,

$$Y = AK^{\alpha}L^{1-\alpha} \quad (27)$$

Suppose that *K*, *L*, and *Y* are represented in terms of the US dollar. Since $\alpha + (1-\alpha) = 1$, the dimension of the left-hand side, the US dollar, is compatible with that of *the right-hand side as a whole* if A is a dimensionless pure number. However, each term on the right-hand side, i.e., K^{α} and $L^{1-\alpha}$, does not make any sense unless $\alpha = 0$ or 1. Suppose $\alpha = 1/2$, is there any operational meaning of $\sqrt{100USdollar}$, for example?

Thus we are at a loss to understand the true reason why the Cobb-Douglas specification is often used in economic science. However, in fairness to Cobb and Douglas, the following fact must be emphasized. When we carefully read Cobb and Douglas' important classic paper (1928), one remains awed by their meticulous attitude. They devoted almost half of their paper to the task of how to create *the indices for capital and labor, not the prices*. They were also very careful about avoiding the generation of pseudo measures with the inconsistent ranking order of capital and labor indices.

In relation to curve fitting practices in economics, Georgescu-Roegen once aptly remarked (Georgescu-Roegen 1966, p. 277, italics added), "econometricians seem to ignore the fact that a better fit obtained by *adding a new variable* does not mean at all that the formula is also a better law. For a formula to represent a law it is not sufficient that it should fit well the available observations: the acid test is the fit for all other observations". The present situation for econometric analyses seems to have greatly worsened due to the increasing computational power of computers and programming techniques.

In mathematics there is a famous theorem called the Weierstrass Approximation Theorem: a real-valued continuous function can be approximated *uniformly* over a given domain by a polynomial (e.g., Randolph 1968). The uniform convergence means that for any given positive number ε (however small it may be) it is possible to create an approximate polynomial such that the absolute value of the distance (the norm) between the real-valued continuous function and the approximate polynomial can be less ε for a given domain.

For illustrational purposes we construct a polynomial series (the Bernstein polynomial) that uniformly converges to a continuous function *f*(*x*). The *n*th Bernstein polynomial for *f*(*x*) is constructed as follows,

$$B_n(x) = \sum_{k=0}^{n} f\left(\frac{k}{n}\right) {}_nC_k x^k (1-x)^{n-k} \quad (28),$$

where

$$ {}_nC_k = \frac{n!}{k!(n-k)!} \quad (29). $$

Suppose the following continuous function,

$$ f(x) = (1+\frac{x}{8})e^x \sin \pi x - \cos \pi x \qquad (30) \quad \text{where } x \in [0,1]. $$

Figure 5 shows a uniform convergence of $B_i(x)$ into $f(x)$. Raising the power of polynomials corresponds to Georgescu-Roegen's sense of adding new variables (or adding new parameters) in the analytical representation. So, it is rather easy to have a polynomial approximation that can fit perfectly well to past data using computer programming. However, the situation facing economists is much more formidable. The "true function" $f(x)$ cannot be known in advance, especially if we seriously consider the evolutionary nature of the economic process! The resulting curve fitting is a series of approximations that is supposed to be a real "law". Unfortunately $f(x)$ itself is simply *a formal representation of the perceived behavior of a system created by a modeler*. Therefore, this formal representation is based on: (i) the relevant system narrative adopted by the modeler; and (ii) the data observed in the system and based on the perception of the modeler.

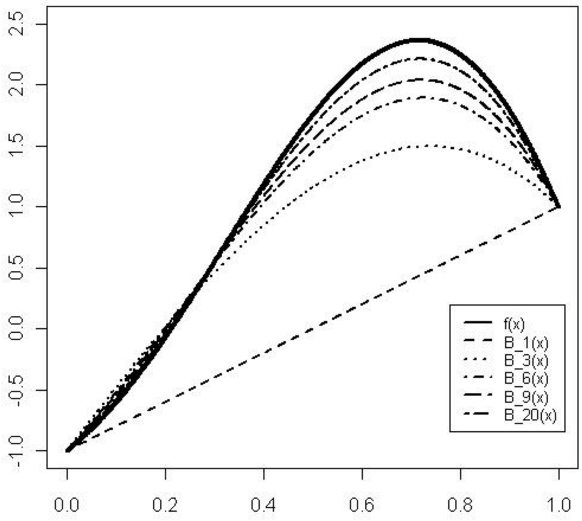

Figure 5. An Illustration of Weierstrass Approximation Theorem

At this moment perhaps the vast majority of readers of this paper might argue that polynomials do not cover many functions that can be conceived in economic analysis. So it is better to explain without getting into mathematical technicalities why we consider the Weierstrass approximation theorem here. In mathematics there is a class of functions called measurable functions. Measurable functions cover almost any function used in econometrics. For this class of function there is a theorem (Luzin theorem, see e.g., Randolph 1968) that essentially states: for any measurable function there exists a continuous function over *almost everywhere* within the closed domain of the measurable function. That is to say, we can construct a continuous function that is almost identical to the original measurable function and the domains for both functions (the constructed continuous function and the original measurable function) are also almost identical for practical purposes of econometrics. Furthermore, polynomials are *dense* in the functional space of continuous functions due to the Werstrass approximation theorem, and we can approximate any conceivable function that is practically used in economics by polynomials as accurately as possible.

6. Conclusion: the economic process and the true source of the limits of analytical representations

Concerning the issue of dimensions we have shown that it is an analytical fallacy to put the dimensional arguments in logarithmic functions and the meaningless variables in Cobb-Douglas functions. Surprisingly there is one example in which these two types of analytical fallacy simultaneously have been committed. Paul A. Samuelson wrote the following (Samuelson 1974, p. 1268): $U = \log tea + (salt)^{\frac{1}{2}}$ (31)

When addressing the dimensions issue in relation to curve fitting practices in economics, there is an important epistemological problem. Neoclassical production functions, whether for individual firms or the aggregate economy, usually assume that any factor can always be substituted for any other factor. The implication of this assumption is that an increase in the input of any factor always yields an increase in output. For neoclassical economists any factor is a jelly-like substance, so that production is carried out everywhere in the input-output space. Such a space is assumed in the classic paper by H. S. Houthakker who formally derived the Cobb-Douglas production function based on the generalized Pareto distribution (Houthakker 1955). As S. Islam aptly showed, the second law of thermodynamics excludes the possibility of obtaining production isoquants of the Cobb-Douglass type (Islam 1985). However, there is more to it. Those neoclassical economists adopting the substitution assumption have not paid due attention to the essential distinction between flows and funds in the material production process (Georgescu-Roegen 1971). This distinction leads to the heart of the issue which is the length of time horizon. It is the pre-analytical selection of a time horizon for the analysis, a descriptive domain associated with the choice of a given time scale, that defines what is produced by an economy. On a short time horizon one can decide to focus the analysis on the production of goods and services (performing an analysis of the flows). On a longer time horizon, when accounting for economic sustainability, one can decide to focus the analysis on the very processes required to produce and consume goods and services by performing an analysis of the reproduction and expansion of the funds. These two different types of analysis will provide different conclusions to the modeler and would require a different selection of models, variables and parameters. Neglecting the distinction between funds and flows (and neglecting the need of representing their production and reproduction using different attributes and models referring to different time scales) results in a systematic indifference to the biophysical foundation of economic activities. It is not surprising then that the curve fitting practice typical of aggregated production functions prevails.

Unfortunately, every time econometric models failed to predict energy demand, econometricians found a ready, yet self-defeating, excuse: "history has changed the parameters" (Georgescu-Roegen 1976). Georgescu-Roegen notes that if "history is so cunning, why persist in predicting it? What quantitative economics needs, above all, are economists such as Simon Kuznets, who would know how to pick out a small number of relevant variables, instead of relying upon the computer to juggle with scores of variables and thus losing all mental [introspective] contact with the dialectical nature of economic phenomena" (Georgescu-Roegen 1976).

The epistemological challenge associated with evolving systems is due to the mismatch between these two facts: (i) the information space used by any formal system of inference (mathematical model) must be closed, finite and discrete, otherwise it would not be possible to run such a model in finite time; (ii) the information space for describing any evolving system is open and always expanding (Giampietro et al. 2006). By "information space" we mean the formal representation of the evolving system expressed in terms of the epistemological categories required to characterize its behavior. This implies that no matter how good a given model is, the simulated behavior always depends on the validity of the initial choice of typologies used in the representation. Unfortunately for modelers, individual realizations belonging to given typologies tend to evolve in time, "becoming" something else (Prigogine 1978). Thus, the validity of any model of an evolving system is bound to expire due to two plausible reasons:
(i) semantic obsolescence - the set of relevant attributes for the observed system must change in time, since the concerns justifying the model will naturally evolve with the advancement of knowledge. Thus, the qualities monitored and the priority given to various criteria of performance, will sooner or later cease to reflect the modeler's perception of relevance to the goals and problem structure (e.g. outdating of the narratives of neoclassical economics theory).
(ii) syntactic obsolescence - the set of relevant attributes for the observed system remains the same for the concerned modeler, but the model can no longer provide an accurate prediction of the values taken by key indicators, since the observed system has become something else (outdating of the validity of the curve fitting parameters). The model is no longer able to simulate the movements of the system within its original state space.

Acknowledgements

D. C. Colander, H. Daly, V. Dogaru, M. Faber, S. Ferrari, G. Fiorito, S. Funtowicz, M. Glucina, J. Gowdy, R. Hamada, R. Horwarth, M. Lavoie, T. Luzzati, S. Managi, S. Mitra, G. Munda, T. Murota, S. Nakamura, R.B. Norgaard, J. Peet, J. Polimeni, J. Ravetz, A. Saltelli, A. J. Sutter, H. Tanikawa, and Jeroen C.J.M. van den Bergh, S. Zamagni gave us constructive criticisms on a previous version of this paper. We also really appreciate thoughtful suggestions from Prof. Deirdre N. McCloskey of University of Illinois at Chicago given to a previous manuscript. We would like to express our sincere thanks to John Polimeni for his valuable and tremendous help in improving the language in this paper. We would like to emphasize that all responsibility for the way in which we have taken advice and criticism into consideration remains with us.

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Accessed April 12 2011

CLIOMETRIC FINDINGS ON KUZNETS SWINGS

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Abstract:

That cliometrics is an indispensable tool in the study of economic cycles, swings or waves is no longer a very controversial statement. It is now generally agreed that economic theory, combined with historical, statistical and mathematical methods are necessary at the theoretical level, to formulate problems precisely, to draw conclusions from postulates and to gain insight into workings of complicated processes and, at the applied level, to measure variables, to estimates parameters and to organise the elaborate calculations involved in reaching empirical results. This article is an illustration of my belief in this principle. It summarizes my contributions to the study of Kuznets swings over the last decade. A reminder of the spectral methodology (I) is followed by successive examination of the various series chosen, the treatment of these series and the results of spectral analysis (II). The conclusion shows the prospects of this type of approach and synthesises a new major result for understanding economic dynamics in the nineteenth and twentieth centuries, i.e. the existence of a single intermediate cycle, a so-called Kuznets swing, with 15 to 20-year frequency that calls into question or even partially contradicts previous work on the topic of long term economic cycles.

Keywords:

Spectral analysis, economic cycles, comparison, cliometrics, GDP, OECD, Kuznets.

Introduction

Spectral analysis is a particularly valuable method for seeking dependences expressed as lags between different magnitudes. Its use in this article was first determined by the search for maximum objectivity in the observation of time series. The possibility of applying it to a large number of series was then examined. This twin requirement resulted from a desire to avoid the criticism generally levelled at statistical studies concerning cyclical movements of the economy. Spectral analysis is based on the theory of stochastic processes. It starts with the core hypothesis that a given time series consists of a large number of sinusoidal components with different frequencies (univariate spectral analysis). It makes it possible to divide a particular category of records into a set of oscillations of different frequencies and then to show the links between the components with the same frequency in the various series examined (cross-spectral or bivariate spectral analysis). It has had limited applications in cliometrics to date[1]. It is used here to determine the frequency of GDP series of several OECD countries. A reminder of the method (I) is followed by successive examination of the various series chosen, the treatment of these series and the results of spectral analysis (II). It is then possible as a conclusion to show the prospects of this type of approach and to synthesise a completely new major result for understanding economic dynamics in the nineteenth and twentieth centuries, i.e. the existence of a single intermediate cycle with 15 to 20-year frequency that calls into question or even partially contradicts previous work on the topic of long term economic cycles.

I. Methodology

Spectral analysis makes it possible to divide a particular categories of long-term data sets into a set of oscillations with different periods (breaking down any time series into a sum of periodic functions) and then to show the links between the components with the same frequency in the various data sets examined. Each of these stages has obvious cliometric interest. The first, by showing all the frequency components of a data set and by isolating—if it exists—the most important frequency component (and its harmonics) reveals the cyclical behaviour of a time series and shows the frequency and amplitude characteristics of this cyclical movement. The second stage makes it possible to compare the periodic movements of two series and to establish a correlation between them. The latter stage therefore seems particularly suited to the aim of this article. The mathematical nature of the data sets to which the method is applied is analysed briefly before identification of the main estimators used.

1. The nature of the time sets: generating, stationary and determinable processes

The spectral method is applied essentially to determinable stationary series. Understanding the reason requires an explanation of what we mean by generating, stationary and determinable processes.

1.1 Generating process

A given time series $x_t \quad (t = 1, 2, .., n)$ is assumed to be the accomplishment of a particular process:

$$\left(X_t, t = -\infty, ..., -1, 0, +1, ..., +\infty\right)$$

The generating process shows how the time series is formed at each instant. However, its stochastic nature means that it cannot determine the real value of the series at any moment.

[1]Interested readers should see, in particular, Ewijk (1982), Gerster (1988), Reijnders (1988) et Metz (2002).

Thus $X_t + aX_{t-1} = \varepsilon_t$ is a generating process, ε_t being a random term. This process is defined by its first and second moments. These are generally functions of time:

$$m_t = E[X_t]$$
$$\sigma_t^2 = E\left[(X_t - m_t)^2\right]$$
$$\mu(t,s) = E[(X_t - m_t)(X_s - m_s)]$$

In which $\mu(t,s)$ is the covariance of X_t and X_s.

1.2 Stationary process

A very important class of series is that given by processes whose first moments are not functions of time, that is to say:

$$E[X_t] = m$$
$$E\left[(X_t - m)^2\right] = \sigma^2$$
$$E[(X_t - m_t)(X_s - m_s)] = \mu(t-s) = \mu_\tau$$

where $\tau = t\text{-}s$ for all t,s

Such series are referred to as being stationary of the second order. They possess excellent properties including the particularly useful feature of making it possible to estimate the different moments by means in time instead of overall means.

In other words:

$$\bar{x} = \frac{1}{n}\sum_{t=1}^{n} x_t \qquad s^2 = \frac{1}{n}\sum_{t=1}^{n}(x_t - \bar{x})^2$$
$$c_t = \frac{1}{n-\tau}\sum_{t=1}^{n-\tau}(x_t - \bar{x})(x_{t-\tau} - \bar{x})$$

provide efficient, unbiased estimators of m, σ^2, μ_τ respectively.

1.3 Determinable process

An important theorem developed by Cramer ensures that stationary series of this kind can be broken down into a set of separate sinusoidal oscillations whose characteristics, amplitudes and relative differences are random.

There will generally be a very large number or even an infinity of such oscillations, each of which is very small and the process is then said to be indeterminable. This contrasts with determinable processes consisting of a finite number of sinusoidal oscillations, each of which will have a finite non null amplitude.

2. Estimators: estimation of spectra and cross-spectra

2.1 Estimation of spectra

The amplitudes of the different oscillations (from the breakdown of stationary series) are random variables that can be defined by the scale of the values that they might have. A variance corresponds to each given period oscillation and this defines what is called the spectrum of the process envisaged. The differences between the various sinusoids are insignificant.

More precisely, the spectrum of a stochastic process $\left[X_t\right]$ is the function $f(\omega)$ defining to within a multiplicative constant the expected value of the square of the amplitude of frequency ω in the Fourier decomposition of the relations of the process. The spectrum thus describes the importance of the various frequencies in the process in question.

The calculations are as follows. For stationary processes, a bias-free estimator can be calculated from a single realisation of the process, that is to say from a particular time series X_t in which (t = 1, 2, …, T). This estimator is the periodogram and can be defined as:

$$I(\omega) = \frac{1}{2\pi} \sum_{\theta=-T+1}^{T-1} \upsilon_\theta \cos \omega\theta$$

υ_θ being approximately the empirical autocovariance, and more precisely:

$$\upsilon_\theta = \frac{1}{T} \sum_{t=1}^{T-\theta} (x_t - \bar{x})(x_{t+\theta} - \bar{x})$$

($\bar{x}$ is the empirical mean of the series).

Although it has no bias, the periodogram is not a good estimator of the spectrum because it is not convergent. This is why the erratic function $I(\omega)$ is replaced by a more regular function representing the mean trend of variations of $I(\omega)$ with ω. This is referred to as the smoothing of the periodogram. The smoothed function is then: $f'(\omega) = \frac{1}{2\pi} \sum_{\theta=-m}^{m} \left(1 + \cos \frac{\theta\pi}{m}\right) \upsilon_\theta \cos \omega\theta$. This formula is Tuckey–Hanning's estimation function (or window). There are other estimation formulas, especially Parzen's, that causes less leakage in non-adjacent frequency bands but leads to higher correlation between the successive estimated values of the spectrum.

Finally, to improve spectrum estimation, the X_t series is generally filtered beforehand by subjecting it to a transformation in such a way that it is possible *a priori* to consider that the spectrum of the filtered series is more representative. In many of these practices, filtering thus eliminates the trend of the series in such a way that the hypothesis of the stationarity of the generating process seems less coarsely imprecise (especially for economic data series). The filter chosen here is that proposed by Hodrick and Prescott (1997), for several reasons. Firstly, it is easy to apply. Secondly, abundant literature shows that its statistical properties are satisfactory. Finally, it is commonly used in the literature and in the empirical analyses performed by national bodies and international organisations (Bouthevillain, 2002).

We thus take a series $\left[X_t\right]$ that has a trend (trend component g_t) and that fluctuates around the trend (cycle component c_t), that is to say:

$$X_t = g_t + c_t$$

The idea is then that of minimising the sum of the squares of deviations of $\left[X_t\right]$ in relation to its trend g_t while conserving a smooth trend, that is to say in which g_t does not vary too much between two successive periods:

$$\min_{g_t} \sum_{t=1}^{T} \left[\left(X_t - g_t\right)^2 + \lambda\left(\left(g_{t+1} - g_t\right)\left(g_t - g_{t-1}\right)\right)^2 \right]$$

λ (the multiplier related to the size of the constraint) is defined as the weighting awarded to the fact of having a 'smooth' trend. The choice of λ depends on the frequency of observation of the data; in general λ = 100 for annual data, 400 for half-yearly data and 1600 for quarterly data. The series on which filtering is performed are usual the log of the series by level.

2.2 Estimation of cross-spectra

Cross-spectra examine the existing relations between the spectral components taken in pairs for two given time series. Generalising the case to a single variable, $\left(X_t, Y_t\right)$ are stationary if their first and second moments are both independent of time. In this case, one of the results of the theory of stationary processes is that the component centred at ω_j is independent not only of the other components of the variable, but also of the components of any other variable whether or not it is centred on ω_j. A full description of the system of relation between two stationary processes just requires knowledge of the extent to which the frequency component ω of the process $\left[X_t\right]$ is correlated with the frequency component ω of the process $\left[Y_t\right]$ and knowledge of their phase difference.

This correlation between two frequency components in two processes is given by:

$$C^2(\omega) = \frac{c^2(\omega) + q^2(\omega)}{f_x(\omega) f_y(\omega)}$$

$0 \leq C^2(\omega) \leq 1$, $c(\omega)$ is called the cospectrum and $q(\omega)$ is the quadrature. $C^2(\omega)$ is the square of coherence at ω (equal to the square of the coefficient of correlation). The measurement of the phase difference between the frequency components of the two processes is given by:

$$\Phi^{(\omega)} = \arctan\left(\frac{q(\omega)}{c(\omega)}\right)$$

It can therefore be said that the amplitude cross-spectrum of two processes $[X_t]$ and $[Y_t]$ defines the expected values of the product of the amplitudes with which each frequency ω affects $[X_t]$ and $[Y_t]$ and that the phase cross-spectrum defines the expected phase difference with which each frequency is involved in the two processes.

Spectral analysis, whose theoretical basis has thus been described briefly, is a logical extension of our work on outliers, non-stationarity tests in macroeconomic time series and that on long memory[2]. It is a particularly suitable mathematical tool for the development of our research programme and should give good results on condition that the statistical data series used are suitable for it. These series are examined below.

II. Data and the results of spectral analysis

1. Statistical data

The aim of our research is the comparison of the behaviour of the per capita GDP in 15 OECD countries. For this we use the now well-known database drawn up by Maddison (1995), expressed in 1990 US dollars for the period 1870-1994, and extended (until 2000) by EUROSTAT indicators.

2. Cliometrics

As mentioned above, the series are decomposed additively into a trend, a cycle and a random component (in which the cyclical and irregular components respect the statistical property of stationarity) to reveal the different cyclicity in the series studied. In other words, we plot the trajectory of per capita GDP between long trends and conjunctural cycles.

[2]See especially Darné and Diebolt (2004, 2005), Diebolt (2005), Diebolt and Doliger (2008), Diebolt, Guiraud and Monteils (2003).

FIGURE 1: LOGGDPT[3] SERIES

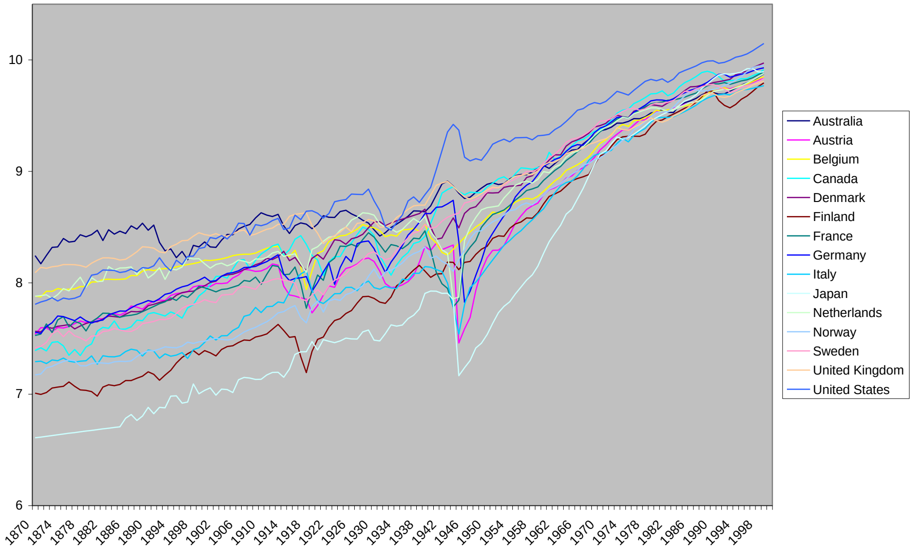
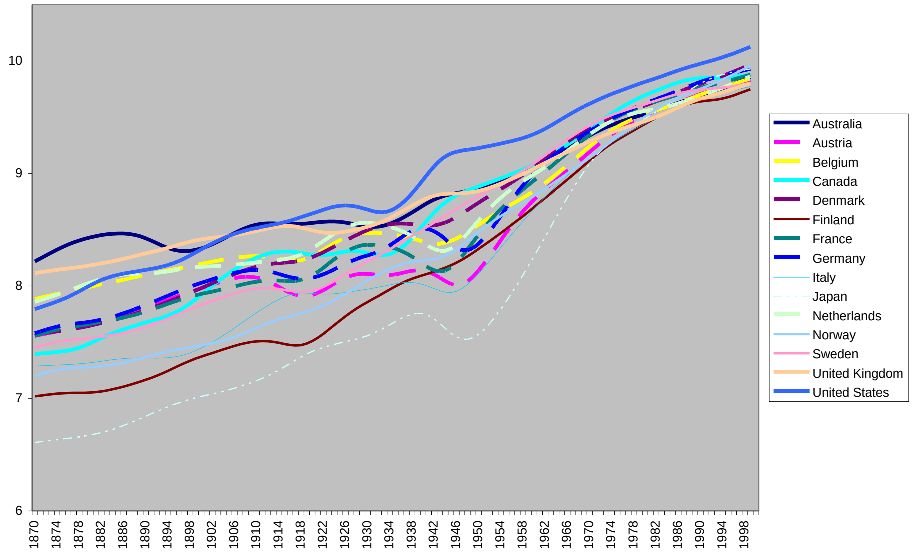


FIGURE 2: GROWTH TRENDS



Prior comparison of the trend movements of per capita GDP (Figure 2) shows overall that until 1970 growth in English-speaking countries (United States, United Kingdom, Australia and Canada) was greater than growth in the countries of continental Europe (France, Germany, Denmark, Belgium, Austria and the

[3]All the series are log-transformed.

Netherlands), which was in turn greater than that in Scandinavia (Finland, Norway and Sweden) and in Japan, which trailed at the back of all the OECD countries[4]. However, the growth trend has displayed convergence since the 1970 break. This would seem to be a sign of a catching up phenomenon as described by Barro and Sala-i-Martin (1992).

To test this, we performed a cointegration study using Johansen's test (1988) to find out whether there was a long-term stationary relation between the different series in order to confirm or invalidate our graphic observations. To set up the test, we used λ calculated from the values, λ_i, of matrix A defining the long-term relations of the model $\Delta Y_t = A_0 + AY_{t-p} + A_1\Delta Y_{t-1} + A_2\Delta Y_{t-2} + ... + A_{p-1}\Delta Y_{t-p-1} + \varepsilon_t$. The calculation is as follows: $\lambda = -n\sum_{i=r+1}^{k} Ln(1-\lambda_i)$. It obeys a probability law (similar to a chi-square) tabulated using the Johansen and Joselius simulation. The test operates by the exclusion of alternative hypotheses concerning the number of cointegration relations r. First, the null hypothesis H_0: r = 0 is tested against the alternative hypothesis r > 0. If H_0 is accepted, the test procedure stops as there are no cointegration relations. If not, the next stage is performed consisting of r = 1 against r > 1. The pattern is repeated as long as H_0 is rejected. If H_0 is rejected in the test r = k against r > k, this means that the variables are not cointegrated.

TABLE 1: JOHANSEN'S COINTEGRATION TEST

No. of CE(s)	Eigenvalue	Statistic	5 Percent	1 Percent
None	0.997166	4911.993	NA	NA
At most 1	0.992898	4161.153	NA	NA
At most 2	0.989859	3527.883	NA	NA
At most 3	0.982107	2940.208	NA	NA
At most 4 **	0.979245	2425.217	277.71	293.44
At most 5 **	0.958402	1929.224	233.13	247.18
At most 6 **	0.923238	1522.221	192.89	204.95
At most 7 **	0.900955	1193.640	156.00	168.36
At most 8 **	0.885827	897.6809	124.24	133.57
At most 9 **	0.828015	619.9159	94.15	103.18
At most 10 **	0.675564	394.5912	68.52	76.07
At most 11 **	0.598799	250.5057	47.21	54.46
At most 12 **	0.502727	133.6043	29.68	35.65
At most 13 **	0.291853	44.18128	15.41	20.04
At most 14	6.30E-05	0.008061	3.76	6.65

Our tests show clearly that the trend of per capita GDP series, that is to say national growth trends, are cointegrated. This does not mean that these trends are identical but that they are linked by linear relations, that is to say that they have common factors.

[4]Italy is an exception here. It is at the level of the Scandinavian countries.

This analysis is therefore continued with on the one had a study of the national cyclical features of the 15 OECD countries and on the other a search for common factors in these cycles.

2.1 Analysis of spectra

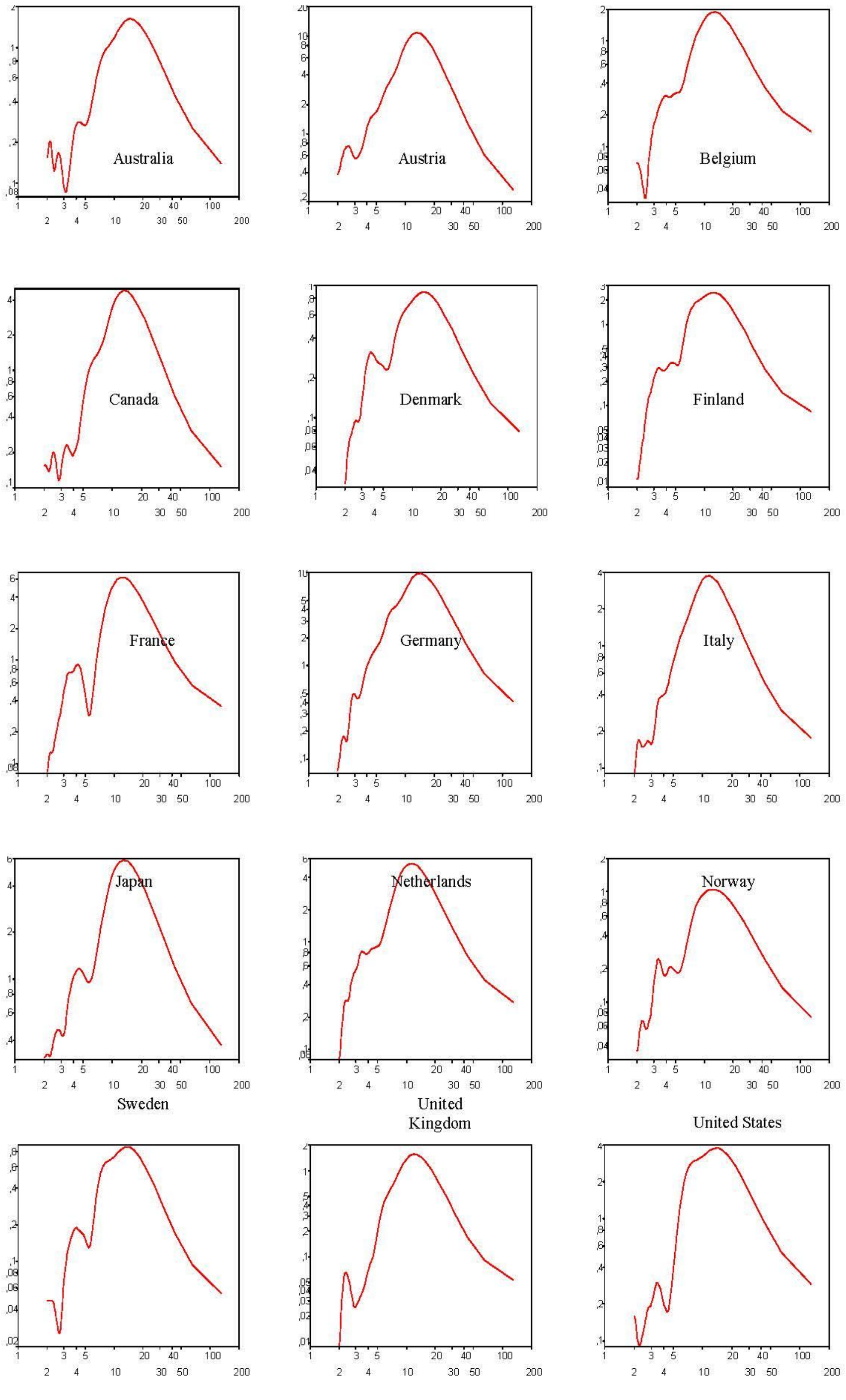
The characteristics of the national cycles are first analysed individually.

Indeed, the aim is to find out whether all the countries display the same propensity for cyclical fluctuations and the same regularity in these fluctuations. It must be possible to determine whether a country typology can be shown according to the cyclical properties as the impetus and propagation of fluctuations at the international scale can only be fully understood if this national heterogeneity is taken into account (Fayolle and Micolet, 1997). This is addressed by first analysing the graphic representations of the cyclical component and then the spectral density.

Graphic analysis of the fluctuations (see Annexes) establishes in a general manner that the different countries are characterised by relative cyclical regularity but that the features of this cyclicity are unevenly distributed at the international level. It can be seen in particular that the cycles in Scandinavian countries (Norway, Finland and Sweden) have limited amplitude but are very regular, whereas the economic cycle is more marked and displays greater amplitude in all the other countries (except in Australia, where it is similar to that of the Scandinavian countries), especially between the wars. However, these results should viewed in relative terms and subjected to spectral analysis as the cycle typology shows that duration and amplitude characteristics vary from one cycle to another and above all that cycles are similar to sets of Russian dolls and their influences can strengthen or oppose each other depending on whether the swings coincide or not or are in phase or not. Whence the advantage of spectral analysis for separating and analysing the different cycles integrated by per capita GDP and then putting forward a more perspicacious country typology in the relations of impetus and propagation of fluctuations at the international scale.

Application of the spectral density methodology described above showed an interaction between two types of cycle in each of the 15 countries analysed. Indeed, the spectral density functions (Figure 3) show that the cyclical movement can be decomposed into two distinct cyclical components. One is short and of the Kitchin type with a frequency of 3-5 years and relatively moderate amplitude and the other an intermediate phenomenon between the Juglar business cycle and a long Kondratieff type of cycle. The latter, a Kuznets type cycle with a frequency of 15-20 years and comparatively large amplitude, underlies GDP conjuncture.

FIGURE 3: SPECTRAL DENSITY GDPT



It should be noted that although the swing of short cycle is smaller than that of the intermediate cycle, it nevertheless has sufficient amplitude and volatility to inflect the overall movement of the cycle that may account for the differences observed in the graphic analyses at cycle aggregate level and thus invalidate the statistical characteristics generally used in the analysis of cycles. This justifies the use of spectral analysis and hence the analysis of cyclical international relations between fluctuations in the same categories.

With this double cyclical movement of per capita GDP combining short oscillations and slow intermediate movements, we consider that national economic fluctuations are first of all governed by storage behaviour. Here, the minor cycles reflect essentially storage and release phenomena by businesses and above all large investments involving large amounts of capital such as those for the construction industry or for transport and seen as a response to population factors as Kuznets type intermediate cycles basically govern conjuncture. The economic situation is therefore governed essentially by a 15-20-year frequency and not by short cycles of the Kitchin or Juglar type or by long movements of the Kondratieff type. We consider that the driving force underlying this frequency can be compared to the mechanisms proposed by Kuznets, that is to say an economic reaction to demographic mechanisms, for example via the movement of large investments or the labour market. Indeed, the links between the functioning of the labour market and demographic movements may be an explanation, in particular through three essential mechanisms linked to demographic factors. These are first of all a cycle linking work and employment (the productivity cycle), then an effect linking unemployment and wages (the Phillips effect) and finally a link between demand and income (the consumption function). These three mechanisms also play greater or lesser roles according to the period and the country; they can increase or reduce the cycles and hence account for the international disparities observed.

This preliminary study of the characteristics of national cycles that shows agreement between the cycle characterising the different countries is followed by a study of the impetus and propagation relations at the international scale through cross-spectral analysis.

2.2 Analysis of cross-spectra

The search for international relations between the main economic cycles is the other fundamental component of this analysis. We therefore incorporate the cycle approach in addressing the entire international economy through the multivariate spectral approach (i.e. cross-spectral analysis). This makes it possible to examine the similarities and synchronisation of the different national cycles. Here, we first analyse the *coherence*[5] between the different cycles using coherence of 0.7 or more as the

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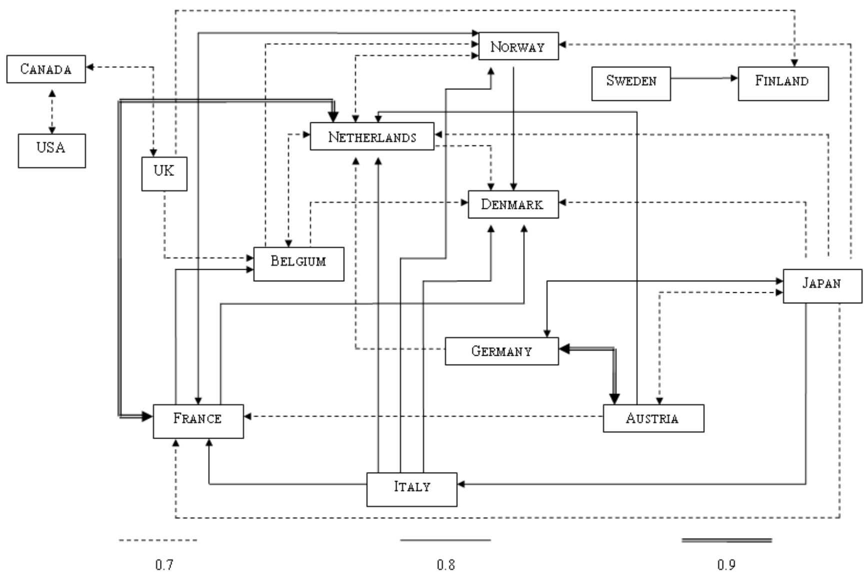
[5]Coherence makes it possible to measure the degree of linear correlation between components of the same frequency in two processes. The closer it is to 1 for a given frequency, the more the two processes move in a similar manner for this frequency or periodicity.

criterion of significance. This is followed by study of the *phases*[6] of the cyclical processes whose coherence is significant. The results of the two methods can be represented schematically as follows to make it easier to examine and interpret the results:

FIGURE 4: RELATION BETWEEN INTERMEDIATE CYCLES OF THE KUZNETS TYPE



Coherence is represented by lines (dotted, unbroken or double unbroken) indicating their degree of importance (0.7, 0.8 and 0.9 respectively), while the phases are shown by the direction of the coherence relation with the beginning of an arrow showing the process that is ahead with regard to the process at its extremity (a double arrow means that there is no lag between the processes—they are synchronous).

It is seen in a general manner that the impetus and forces governing national intermediate cycles are not independent between countries and there is thus strong interconnection between national cyclicity. This can be shown in more detail by a grouping in three geographic zones and linked to notions of similar, common cycles. Indeed, a distinction should be drawn between a similar cycle and a common cycle. A cycle is referred to as being similar when the cyclical component that drives the movement of each series is founded on the same propagation mechanism but engendered by a specific impulse series so that the cyclical components can be markedly different and desynchronised. This similar cycle can be called common when the impulse series applied to it are perfectly correlated, that is to say the cyclical

[6]The phase makes it possible to measure the time shift of a process in relation to another. A positive phase shows that the second series is ahead of the first and the opposite if it is negative. Interpretation of the phase is then strongly linked with coherence as the analysis of a lag between two processes is only meaningful if the processes are related, that is to say if their coherence is high.

components are perfectly synchronised and differ only in their amplitude (Bentoglio, Fayolle and Lemoine, 2001).

Using this, a first parallel can be made between the English-speaking countries (United States, Canada and the United Kingdom) that are linked in a joint cycle through their common historical, political and geographic context. A second, much wider parallel can be set between the European countries in the broad sense (the countries of continental Europe and Scandinavia) where the characteristic of the cycle is that it is largely similar in all the countries of this zone and sometimes common with certain neighbouring countries (e.g. Belgium and the Netherlands and Germany and Austria), explained in particular by a flexibility of the labour market and geographic mobility of labour are made easier by geographical proximity. However, the cycle in the United Kingdom displays greater similarity with that of the United States and Canada than with the European countries. Finally, Japan seems to form a zone all by itself with a cycle similar to that of all the European countries but more advanced.

In a general manner, a regime of growth and international fluctuations is defined by a combination of properties characterising the international area. The cyclicity of the economy at the international level is not independent of the hinging and compatibility of trend growths which, as has been seen above, indicates a phenomenon of catching up over a long period. This catching up is only possible through the efficiency and profitability of productive resources, whence conditioning with regard to constraints of external equilibrium and the cyclical interdependence found in all the countries studied (Fayolle and Micolet, 1997).

Conclusion

This analysis of national fluctuations and cyclical interconnections between countries with the same degree of development makes it possible to enrich our understanding of the cyclical mechanisms between nations. It gives a major, entirely new result for the understanding of economic dynamics in the nineteenth and twentieth centuries, that is to say the existence of a single intermediate cycle with a frequency of 15 to 20 years that calls into question or even contradicts (at least partially) previous work on economic cycles. This understanding of cyclical mechanisms is all the more essential today as it will make it possible to put forward more concerted and more cooperative economic policies aimed at parallel development of activities. Indeed, although growth cycles leave real but comparatively small room for intervention by the authorities on the determinants of growth, macroeconomic policies can nevertheless slow, shift, accelerate or dampen cyclical phenomena, whence the advantages of better understanding national cyclical features and their interaction at the international level.

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THE RECENT CRISIS UNDER THE LIGHT OF THE LONG WAVE THEORY

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Abstract

In this paper it is presented the secular unfolding of four economics-related agents, which when considered as a whole allow to comprehend what happened in the past in the global economy and shed some light about possible future trajectories. The four agents considered are: world population, its global output (GDP), gold price and the Dow Jones index. The joint action of these actors, in despite of being only a part of the whole, might be seen as a good depiction of the great piece representing the world economic realm. The application of analytical tools such as spectral analysis, moving averages, and logistic curves on time series data about the historical unfolding of these actors allows the demonstration that the recent global crisis seems to be a mix of a self-correction mechanism that brought the global output back to its original learning natural growth pattern, and that it carries also signals of an imminent transition to a new world economic order. Moreover it is pointed out that fingerprints of Kondratieff long waves are ubiquitous in all observed time-series used in this research and it is demonstrated that the present decade will be probably one of worldwide economic expansion, corresponding to the second half of the expansion phase of the fifth K-wave.

1 - Introduction

Since the onset of the present global financial crisis started in the fourth quarter of 2007 that at least two 'faqs' are omnipresent in the technical or amateur discussions on the unfolding of world economic affairs: *why it was not foreseen*? *And where are we presently in the framework of the long wave theory?*

It became very complex to speak about causation of this crisis; there is not a consensus about an economic theory that could explain its genesis, and much less about the hypothesis of a timely forecasting. On the other side there has been some consensus that the crisis has a pure financial and monetary policy nature and is not the consequence of any kind of overproduction as observed in previous economic shocks. Some strange names have been given to this financial turbulence: subprime crisis, real state crisis, super bubble, and more recently it was even coined as the Great Recession to differentiate from less severe 'normal' recessions of the last 80 years and from the Great Depression of the 1930s.

As usual in times of big economic recession comparisons with previous crises abounded in the technical literature. Most commonly we have seen the obvious comparisons with the Great Depression of the 1930s, but also comparisons with the worldwide panics of 1873 and of 1907 have been pointed out. But the fact is that none of these comparisons passed the necessary stringent tests. Its general character, as we will try to demonstrate in this work, seems to be unique, carrying in its structure clear symptoms either of a self-correcting mechanism or even an anomaly of the current socioeconomic system.

Strange still economists and financial analysts insist in looking at this crisis with the very narrow lenses of the current economic and financial theories and models, neglecting the potential of the overwhelming evolutionary world system approach when trying to understand the unfolding of human affairs on this planet. Economics has taken a far too narrow view not only of its modeling and assumptions, but on its reliance on definitions. Models and definitions are maintained even when they are obsolete and no more suitable.

This piece does not intend to offer an exhaustive analysis of the causes of the present crisis. Our goal relies mainly in presenting a new vision about the evolution of some economics-related agents during the last century (more exactly since 1870), which when considered as a whole allow a better comprehension on what is happening and shed some light about possible future trajectories.

2 – The four agents

Economics is above all the surface manifestation of all human activities related to the exchange of goods and services that as any other system in the universe has to follow some iron rules of nature. Humans, human activities, organizations, Earth's material resources, are all parts of the natural order. Following this line of thought we have to describe the behavior of large populations, *for which statistical regularities should emerge, just as the law of ideal gases emerge from the incredibly chaotic motion of individual molecules*, as recently stated by Bouchaud in a short paper published in Nature with the suggestive title "*Economics needs a scientific revolution*"[1]. The present author in a paper published in 1996[2] has already pointed out the same observation. The fact is that during the last twenty years we have witnessed the birth of the new science of Econophysics (a term coined by Gene Stanley in 1995, see Bouchaud[3]), which applies to economics the conceptual framework of physics and has been very successful in explaining the endogenous behavior of financial markets, demoting accepted axioms and debunking myths of mainstream economics like the rationality of agents, the invisible hand, market efficiency, etc… We will turn to this point in a later section of this article.

Socioeconomic systems are complex systems and free markets are wild markets. No framework in classical economics is able to describe wild markets. Physics' modern branch of Chaos Theory, on the other hand, has developed models that allow understanding how small perturbations can lead to wild (very big) effects. Devezas and Modelski[4] have shown that world system evolution consists in a cascade of multilevel, nested, and self-similar (fractal) processes, exhibiting power law behavior, which is also known in physics as self-organized criticality. Wild oscillations are part of the far from equilibrium chaotic behavior. In a more recent complement of this research Devezas[5] has demonstrated that the world system is prompt to a very important transition in the near future. The results described in the present paper, using other sets of data and different mathematical tools, come to reinforce this result.

It is very important to keep in mind that complex systems is perhaps a misnomer, because their manifestation and their subjacent laws are not really complex – their imperatives are very simple and usually translated in beautiful patterns like that of fractals, power laws and logistic growth curves. All that we need is to choose the suitable sets of data and apply to them simple mathematical tools. Consider that Einstein demonstrated the time dilation phenomenon using only high-school mathematics.

Let's be simple and call to the stage only four actors (agents) that, in despite of being only a part of the whole, might be seen as a good depiction of the great piece representing the world economic realm. Their historical unfolding translated by time series data represents the result of collective actions involving people, organizations, networks, nations, etc., whose interactions unfold in space and time and manifest some simple patterns that easy us to grasp recent and past economic events.

The considered agents are: the world population, the world aggregate output known as Gross Domestic Product (GDP), the historical leader of all commodities – Gold, and the still most important financial index, the DJIA (Dow Jones Industrial Average). In this paper we will examine the interplay among these agents using historical time series regarding their quantitative evolution, as well as the patterns emerging from their secular behavior when subjected to some simple analytical tools.

3 - Notes on the used sets of data

The figures for world population and GDP were taken from Maddison's historical series[6,7], which are considered to be one of the most reliable sources for economical and population data for the past 2000 years.

The macroeconomic variable - GDP - is undoubtedly a very good measure of global and region-wide economic activity, for it works as an aggregator covering the whole economy. There has been in the technical literature a hectic discussion about the validity of GDP statistics as a good measure for living standards and nation's productivity (see for instance the recent short comment on this theme by the Nobel Prize winner Joseph Stiglitz[8]). But regarding this controversial point we wish to clarify that the approach followed in the present analysis is one of comparison between countries and/or regions and moreover we compare the historical *rates of growth*, and not the absolute values of GDP estimates.

Add to that the fact that Maddison uses in his figures the *purchasing power parity* (PPPs) converters, which eliminates the inter-countries differences in price levels, so that differences in the volume of economic activity can be compared across countries, allowing a coherent set of space-time comparisons. In order to normalize the temporal variations of the used currency Maddison uses constant 1990 US dollars converted at international "Geary-Khamis" purchasing power parities (see for details reference 6, Chapter 6).

Still regarding the GDP data series it is important to point out that Maddison's figures are not complete along with the entire time span (since 1870) we want to focus in the present analysis. Maddison's tables present complete data between 1870 and 2006 only for USA, 12 Western European countries, Japan, Brazil and Indonesia. For India the numbers are complete since 1884, for Russia/USSR there are numbers for 1870, 1890, 1900, 1913, and is complete after 1928, and finally for China there are numbers for 1870, 1890, 1900, 1929-1938, and is complete since 1950. For all the other countries the figures are complete since 1950. For this reason when designing the graphs for the historical unfolding of the world GDP only a given set of countries was chosen for some given periods, as will be discussed later. Data for the most recent years of 2007 and 2008, as well as the projections for 2009 and 2010, were taken from a recent report of the International Monetary Fund[9], converted using Maddison's criteria.

The time series for the weekly Gold price since 1900 were taken from Kitco historical charts[10] and for the Dow Jones index also since 1900 from the webpage of Analize Indices[11].

4 - Spike-like growths

Graphed on a time-line of two millennia both the Earth population as well as its economic output (world GDP) presents a spike-like growth, as depicted in figures 1 and 2. Both these megaphenomena began sweeping the planet in the past century conducing nowadays to very serious concerns about materials/energy consumption, carbon dioxide concentration in the atmosphere, shortage of water, and extinction of species. These megaphenomena account for the proliferation of afflictions swamping mankind at this very onset of the 21st century. It is not exaggerated to say that humanity is presently in a very World War (or World Revolution) whose main goal is its own surviving, spending large amounts of its own GDP trying to win this war. There is already a growing planetary consciousness that some extreme measures have to be undertaken immediately if the human race intends to endure as a species.

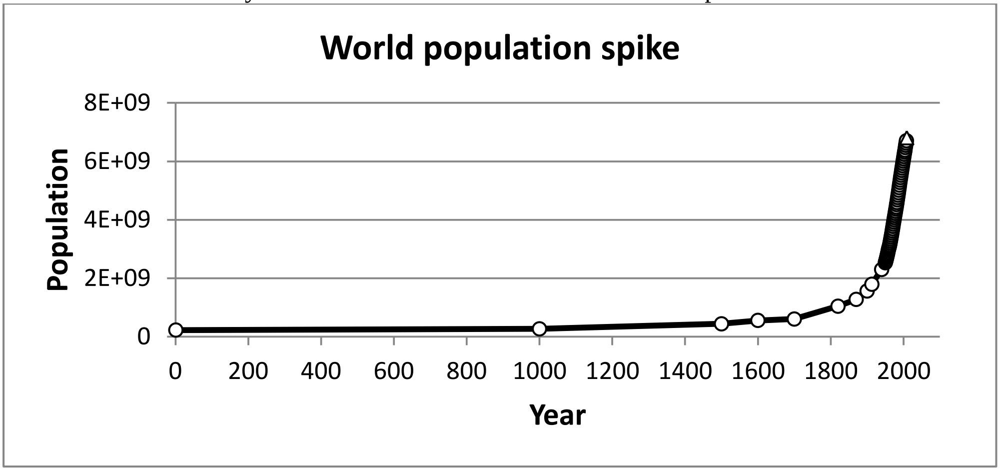

Figure 1 – Spike-like growth of the world population in the last two millennia (data from Maddison[5,6]).

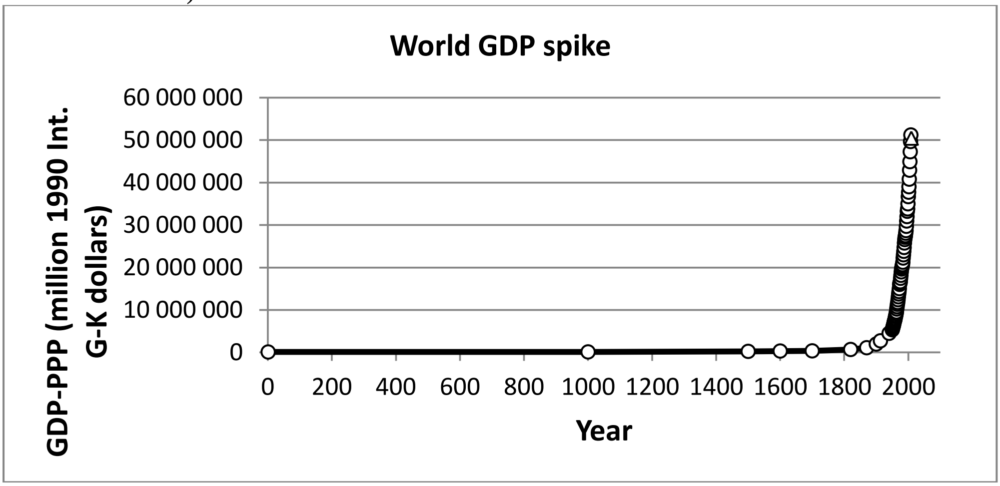

Figure 2 - Spike-like growth of the GDP in the last two millennia (data from Maddison[5,6]).

On the other side one can ask: is that really so? Is there a real menace pointing to a possible worldwide catastrophe that could definitively jeopardize human life on Earth? Another question then naturally emerges: could not Gaia as a resilient system find its own way out of this apparently imminent disaster? As will be seen in our analysis ahead in this paper, this kind of graphs evincing explosive growths are always misleading and used frequently for apocalyptic propaganda. In order to get the correct conclusions about the real trends we should look for the details hidden behind the considered growth phenomenon and this is usually done expanding the x-axis and narrowing the focus on its unfolding in shorter time spans.

We know that this is a very controversial theme of debates and equally know that there are many scientists voicing against the exaggeration of simple extrapolations of the observed trends. Our objective in this work is not properly to deliver answers about this scientific puzzle, but the fact is that the approach we are pursuing in last years and the results of our ongoing research, as well as the results of other recent investigations, point to this very concrete possibility – the World System is approaching an Era of Transition that will conduce naturally to a new order within which these troubles will be overcome. What we don’t know yet is if this transition will be a smooth one or much on the contrary, a very turbulent one as already happened in the past. We hope that the present results may help in shedding some light on the road ahead.

We have already pointed out that Devezas and Modelski[4] have demonstrated that the World System is prompt to a very important transition and demonstrated that the dominating order has already reached 80% of its millennial learning path (see figure 9 of reference 4). In another recent work Devezas at al.[12] have shown that the increasing efficiency of energy systems is following an irreversible path toward the usage of carbon free energy sources, a process that will be completed before the end of the present century (see figures 10 and 11 of reference 12).

Very recently econophysicists Johansen and Dornette[13] have given an important contribution in this direction. They have shown that, contrary to common belief, both the Earth’s human population and its economic output have growth faster than exponential, i.e., in a super-Malthusian mode. These growth rates are compatible with a spontaneous singularity occurring at the same critical time around 2050 signaling an abrupt transition to a new regime. But the abruptness of this transition might be smoothed, a fact that can be inferred from the fact that the maximum of population growth was already reached in the 1960s, in other words, a rounding-off of the finite-time singularity probably due to a combination of well-known finite-size effects and friction, suggesting that we have already entered the transition region into a new regime.

Closing this section it is shown in figure 3 the spike-like growth of the Dow Jones Industrial Average (DJIA) considered weekly from 1900 until September 2009, and in figure 4 the historical growth of gold price for the same time span, also considered weekly. In the case of gold, which will be subject of a detailed analysis ahead in this paper, we do not have what can be coined as a spike-like growth, but anyway it can be observed a spectacular growth with wild oscillations, exhibiting two very strong peaks separated by approximately 30 years.

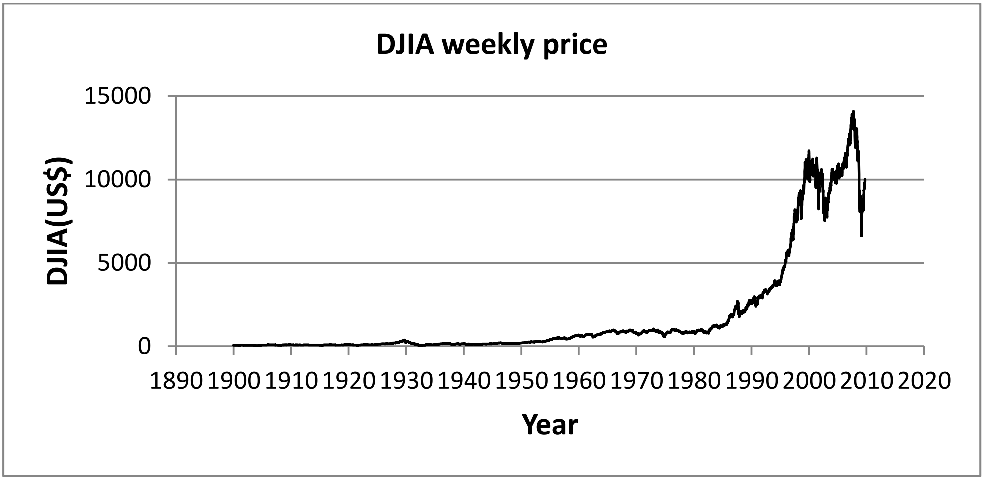

Figure 3 – Dow Jones Industrial Average weekly price since 1900 until September 2009 (data from Analize Indices[11]).

Figure 4 – Gold weekly price per troy ounce since 1900 until September 2009 (data from Kitco historical charts[10]).

5 - Signals of saturation

Let's begin looking at the evolution of the two most important agents, Earth's population and its aggregate output, but initially narrowing our observation to their recent unfolding after 1950, a period for which the most reliable data are available.

In the previous section we have already pointed out the fact that human population growth rate has already reached its maximum, as depicted in figure 5. A peak of 2.2% was reached in 1962-1963, and after this date has decreased steadily being nowadays of the order of about 1.13%. Looking another way around, the annual change in the world population peaked in the late 1980s when the world population experienced a net addition of about 88 x 10^6 individuals (obviously because the population in the 1980s was much bigger than in the 1960s). These figures were taken from the International Data Base of the US Bureau of Census[14], whose estimates for the world population in 2050 is of about 9.316 x 10^9 people.

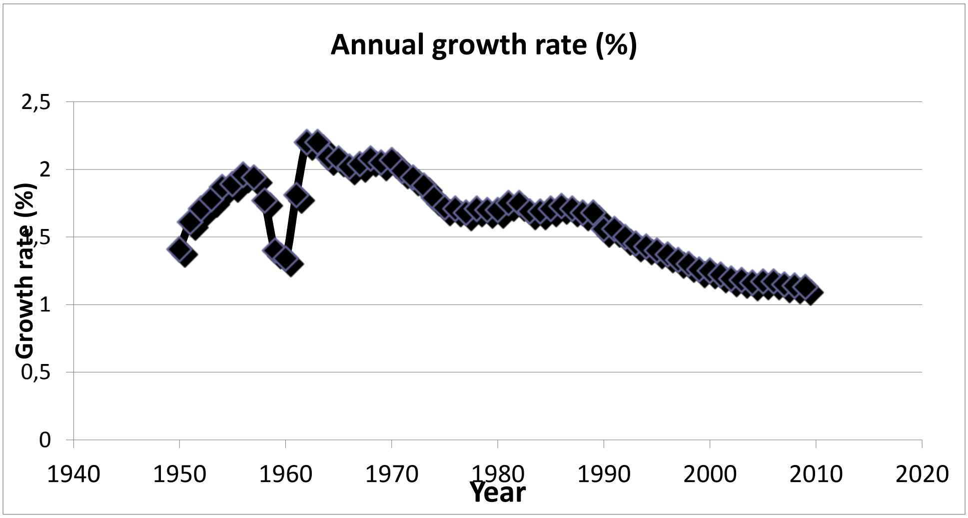

Figure 5 – Annual rate of growth of the world population 1950-2009 (data from US Bureau of Census[14]).

An important point to refer about the figure 5 is the pronounced dip appearing in 1958-1960 that was due to the so called Great Leap Forward that occurred in China in this period, amidst with natural disasters, widespread famine and in the wake of a massive social reorganization that resulted in a toll of tens of millions of deaths. As we will observe in the next section, this dip is also very visible in the historical evolution of the world GDP and warns us about the weight of China and its very important role in economics-related world affairs.

Curiously, and in despite of the data (calculations!) of the US Bureau of Census, the recent evolution (since 1950) of the world population can be finely fitted by a logistic curve, which delivers a slightly different result regarding both – the extrapolation to the year 2050 and the turning point corresponding to the maximum growth rate. This fitting is shown in figures 6a (the logistic curve) and 6b (the same in the form of a Fisher-Pry plot), which were obtained using the IIASA's LSM II program[15]. As can be observed the fitting is absolutely perfect ($R^2 = 1$), what implies that we are amidst a natural growth process, with a characteristic time Δt of about 160 years (1920-2080), with an inflexion point in 2000-2001 (maximum growth rate). The maximum carrying capacity of this process points to a population of about 12 x 10^6 people to be reached by the end of the century, but that can stabilize before this maximum (say by about 10 x 10^9 people, considering that the end of a logistic growth process implies the transition into a new regime). Our curve points to a population in 2050 of about 9.7 x 10^9 people.

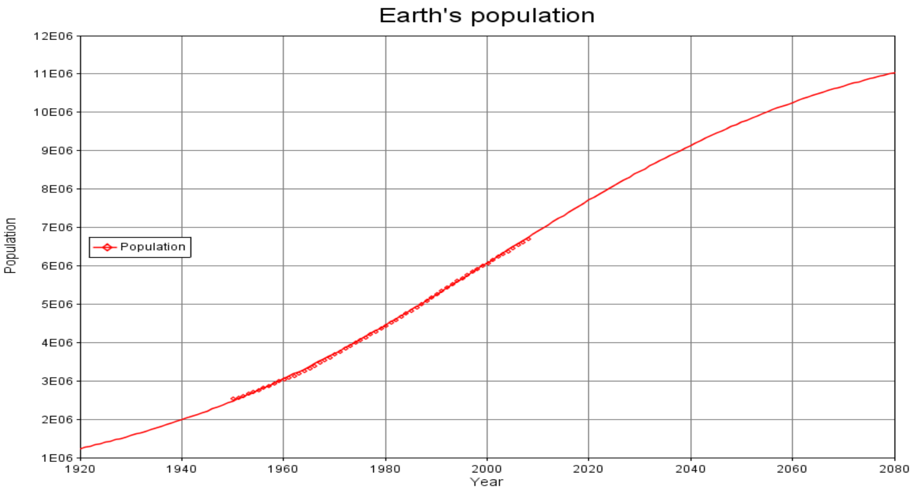

Figure 6a – Logistic growth of the world population 1950-2009 using IIASA's LSM2 software[15].

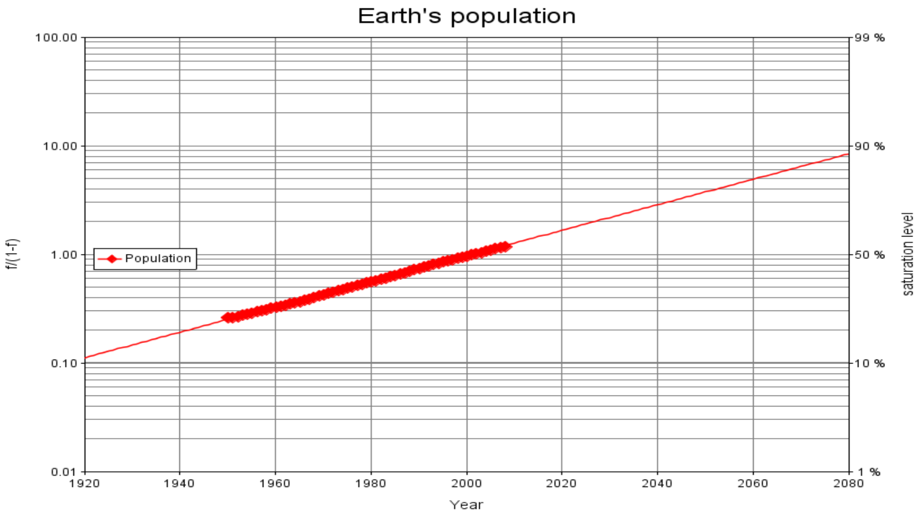

Figure 6b – Fisher-Pry plot of the world population 1950-2009 using IIASA's LSM2 software[15].

In recent paper Boretos[16] performed the same fitting using the US Bureau of Census' data set until 2005 and has found a somehow moderate result, with a characteristic time Δt = 117 years and a turning point in 1995. Accordingly to the set of data used by this author the extrapolation to the year 2050 matches the projection of the US Bureau of Census.

Let's now call our second agent, the aggregate world output, or in other words, the world GDP. Using Maddison's data since 1950 we have also fitted a logistic curve and the result is depicted in figures 7a (logistic curve) and 7b (Fisher-Pry plot). The fitting is not so perfect (R^2 = 0.996) as in the previous case of Earth's population, but works equally well.

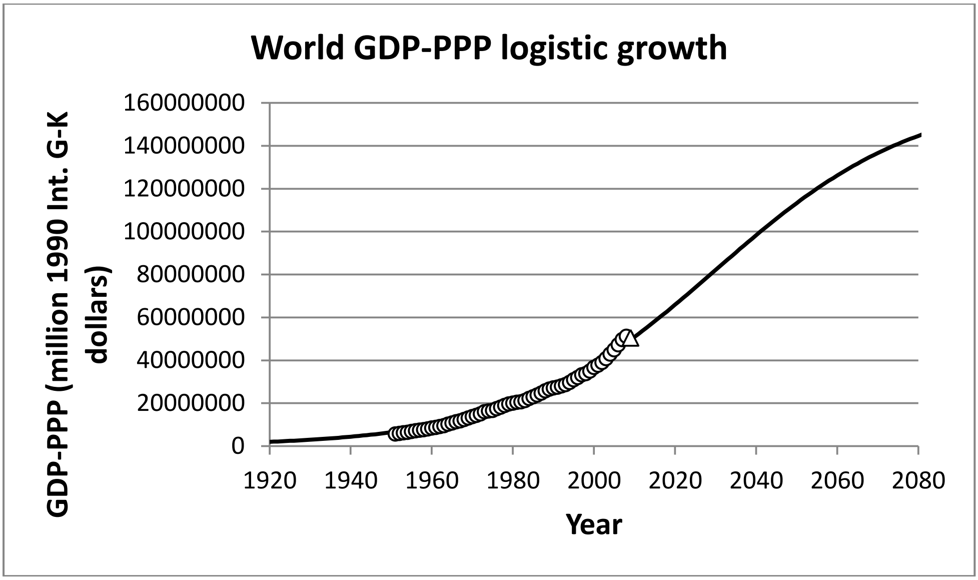

Figure 7a – Logistic growth of the world GDP-PPP 1950-2008. The last point (triangle) is the estimate for 2009 from IMF[9].

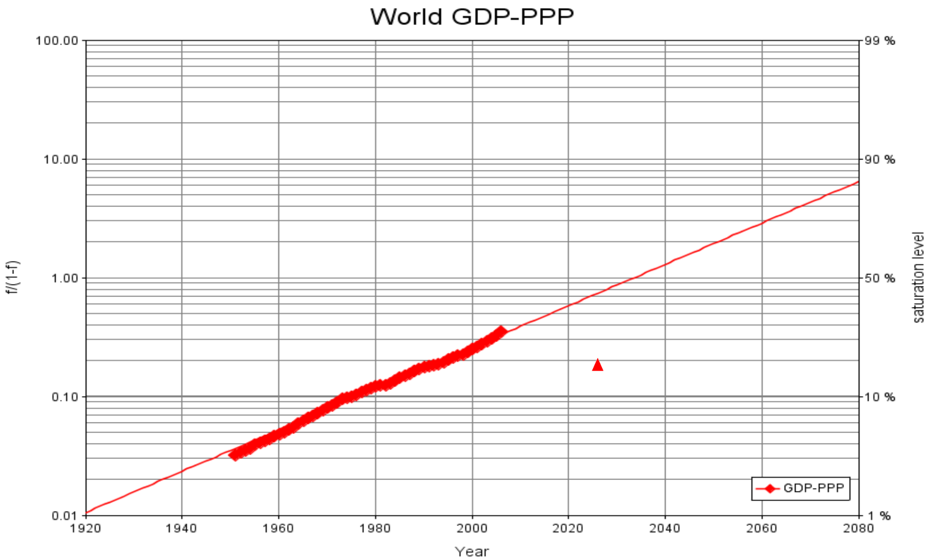

Figure 7b – Fisher-Pry plot of the world GDP PPP 1950-2008. The last point (triangle) is the estimate for 2009 from IMF[9].

The resulting logistic corresponds to a natural growth process with a characteristic time $\Delta t \sim 110$ years that will saturate about 2080 with a turning point (peak of the growth rate) around 2030. Boretos[16] has tried the same fitting using a different dataset and numbers only until 2005 and has found a similar result with a characteristic time of about one century and a turning point in 2015. Unnecessary to stress that these differences are absolutely irrelevant considering that we are using different datasets and in our fittings we have used more recent data (until 2008), which has naturally contributed to a slightly higher carrying capacity and pushed the turning point ahead in time. The main reason for this difference lies in the higher world GDP growth rates observed in the period 2006-2008, which we will further analyze in the next section.

These results require some further thought. What is the meaning of these natural growth processes? Why the GDP has grown faster than population? There are no simple answers to these questions and their in deep analysis deviates from the purpose of this piece. But a few words about their meaning are worth to put.

Human population and its output are growing since the onset of civilization some five millennia ago. But contrary to a widespread impression, the story of world population of the last 5000 years is not one of continuous exponential growth. Rather, it can best be described as a series of three major surges, each more substantial than its predecessor, but both of the first two surges also followed by a long period of population stability[4]. The graph depicted in figure 1 shows only the last stabile period (from year 0 to year 1~000 a.C.) and the last spike-like surge respectively. As already shown by Devezas and Modelski[4], this 2000-year process corresponds to the formation of the *global system*, one of the *global-institutional* processes that monitor the progress of agents, and program their developments. Nested within this longer process there are other shorter *global-institutional* processes like the *global economy* process (~250 years, see table 2 in reference 4) that corresponds to the process being analyzed in this paper.

At this point we wish to make stand out the first important result of the present investigation, which can be easily discerned through the comparison between the actual points and the path of the logistic growth process shown in figures 7a and 7b. In these graphs we have also included the estimated projection for 2009 (the triangle in both graphs, using data from IMF[9]). As can be seen the actual points, mainly between 2005-2008, evidence a slight deviation upwards, and the point corresponding the estimate for 2009 seems to pull the curve downwards in order to match the original path. In order words, *the present crisis seems to work as a kind of self-correction mechanism of the system.*

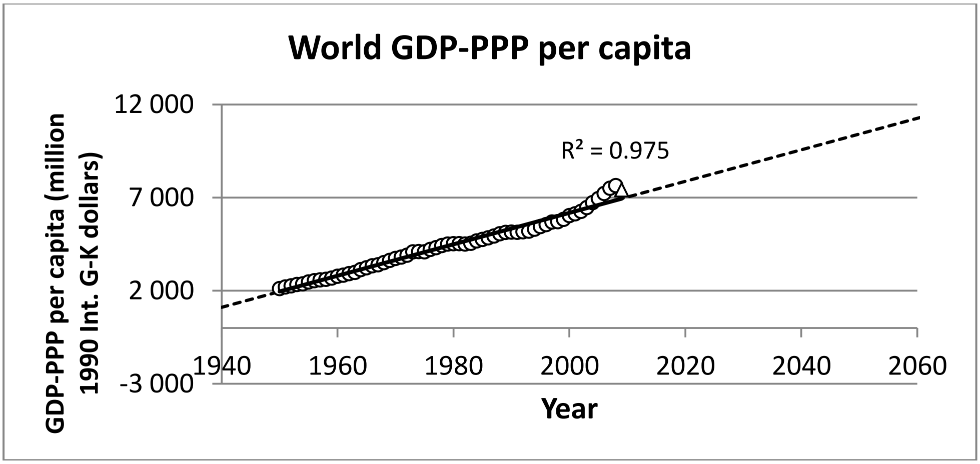

Figure 8 – Linear fitting of the world GDP-PPP per capita 1950-2008. The last point (triangle) is the estimate for 2009 from IMF[9].

The next step was to look at the behavior of the unfolding of the global output per capita. Using the recent data the fitting of a logistic curve does not work well, a result that diverges from those got by Boretos. In figure 4 of his paper this author shows a logistic fit, but the substitution curve is clearly right skewed and the author does not present the error estimates. Boretos states that *'world GDP has increased faster than population at all times'*, but this is not true as alias we can infer from the linear fitting of the GDP/capita exhibited in figure 8 below.

As can be observed the overall linear fitting is not bad (R^2 = 0,975), but most important the linear trend is perfect until 1981, deviating downwards after this date and until at least 2001, what implies a growth rate of GDP below the population's growth during approximately a time span of 20 years. After 2004 and until 2008 the actual data exhibits an inverted behavior, that is, the world GDP has grown faster than population – but this trend stopped abruptly in 2009. Again the extrapolated point that contains the outcome of the actual crisis seems to pull the trend downwards. It is clear that if we use the extrapolation for 2010 the corresponding point will be located still closer to the straight line.

Resuming the results of this section we have:

1 – The present crisis seems to be a kind of self-correction mechanism that brought the global output back to its original logistic growth pattern.

2 – This pattern corresponds to a final phase of the ongoing global economy process, which will saturate before of the end of this century, signaling that we are entering into a new regime (a new learning process) of the socioeconomic world system.

In the next sections we will see how results from other analysis and approaches reinforce these preliminary conclusions.

6 - Comparative analysis of the global output under a larger timeframe

Figure 9 shows the timely evolution of the GDP-PPP for a set of 18 selected countries for which the most complete data are available since 1870. These countries together contribute today for ~ 70 % of the global GDP (74% in 1950, and 73% in 1970). This result is well known; everyone is acquainted with the fact that China is the country exhibiting the most dramatic GDP growth during the last decades, and certainly will surpass the USA in the next decade or so. India and Brazil are also growing at fast paces, but still far below China, while Europe and Japan demonstrate that are losing momentum in this race. It is very evident that the former USSR was hit at the late 1980s by its political-economical transformation and disaggregation, but is also recovering momentum leaded by Russia and some of their former members.

This kind of graphical representation does not allow to discern details and much less to perform reliable forecasts. On the other hand the picture is completely different if we look at annual movements in aggregate activity, or in other words, the annual growth rate of GDP. As will be seen in this section, such visualization allows discerning changes that have appeared systematically across countries, due to catastrophes, political and/or social upheavals, wars, recessions, etc…. Moreover it permits also to distinguish some patterns, as for instance the different phases of K-waves observed since 1870.

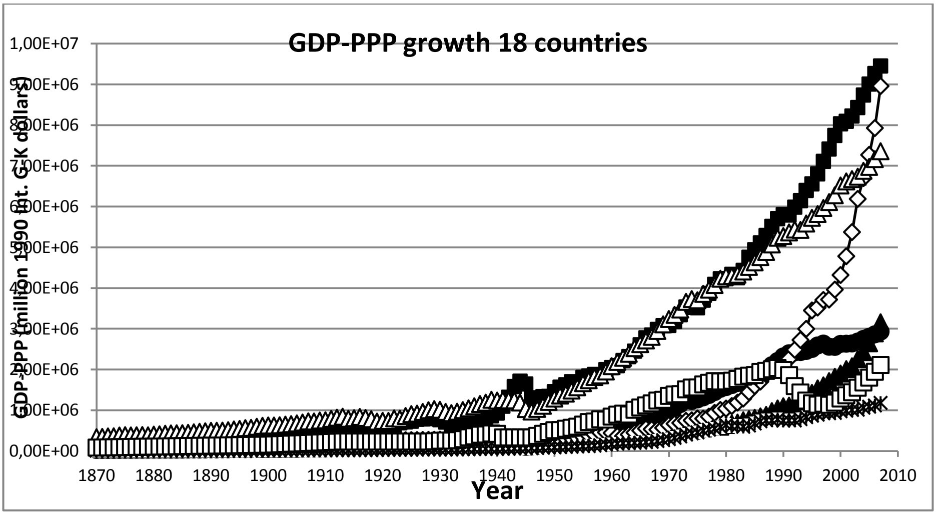

Figure 9 – GDP-PPP growth 1870-2008 for 18 countries – USA (■), China (◇), 12 Western European countries (△), India (.▲.), Japan (.●.), former USSR (.□.), Brazil (x).

In figure 10 that follows it is depicted the historical record since 1870 of the annual growth rate of the world GDP-PPP using Maddison's data. Before advancing commenting on some important details of this picture, it is important to clarify some aspects considered in the construction of this graph.

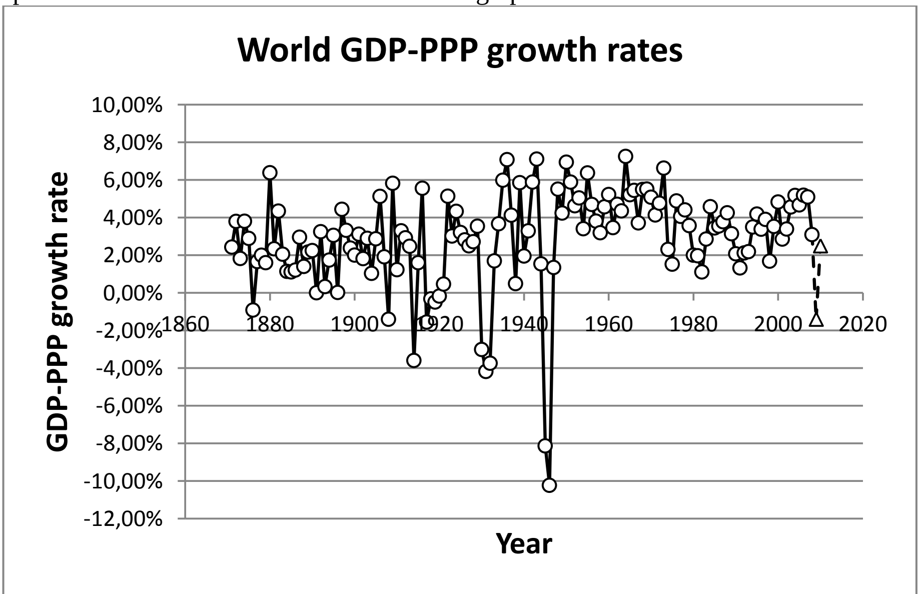

Figure 10 – World GDP-PPP growth rates 1870 – 2008 (data from Maddison [5,6]).
△ – Estimates for 2009 and 2010 from IMF [9].

As already explained in the third section of this work, Maddison's data set is not complete for the entire time span since 1870. For the construction of the graph shown in figure 10 the data corresponding for the interval 1870-1884 are the numbers for USA and 12 WE countries, that where undoubtedly at this time the leading economies in the world (in 1880 corresponding to 55% of the world GDP). Between 1885 and 1927 the numbers include also India, Japan, Indonesia, and Brazil (in 1900 corresponding to ~61% of the world GDP), and between 1928 and 1949 the USSR was added to this group (corresponding in 1940 to ~71% of the world GDP). From 1950 onwards the numbers include all countries.

The validity of this approach can be inferred from the behavior of the two superposed graphs shown in figure 11, showing the unfolding of the GDP growth rates for the world and for the USA plus 12 WE countries. As can be observed, the movements – ups and downs – are perfectly 'in phase', the only clearly observable difference is that the peaks (maximum growth rates) and dips (minimum growth rates) for the world are damped, due to the fact that the performance for some individual countries are not exactly synchronized with the leading countries. This 'damping effect' seems to work well until at least the year 2000, when an opposite effect seems to enter in action. But the general aspect of the graphs suggests clearly that USA plus the 12 WE countries leaded the world economy for the most of the time.

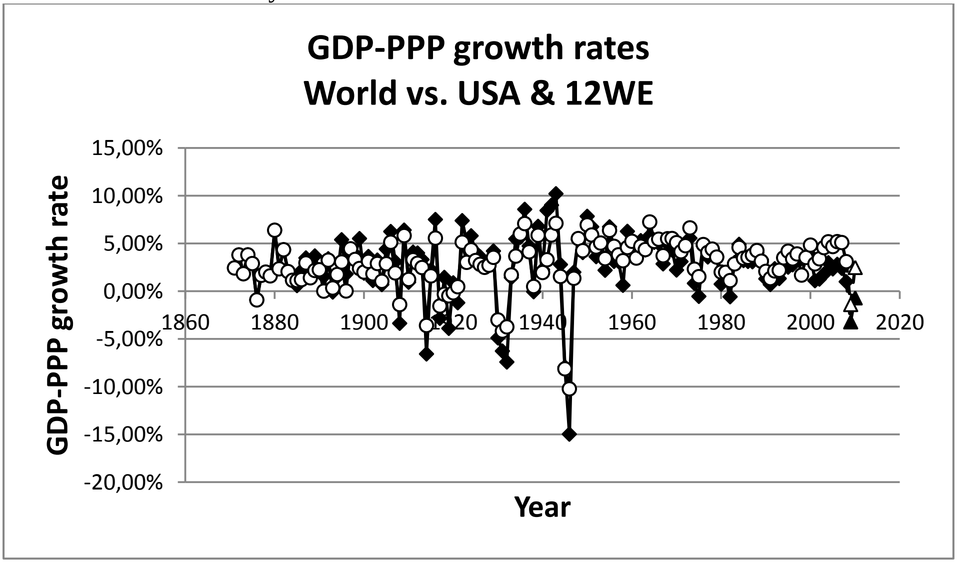

Figure 11 – Comparison between the GDP-PPP growth rates for the world (○) and USA plus 12 WE (◆). The last points △ (world) and ▲ (USA + 12 WE) are the estimates for 2009 and 2010 from IMF[9].

The picture is completely different when we compare the behavior of individual countries, like India and China (data for China are shown only after 1950), both with very troubled history, as shown in figure 12 in comparison with the same world graph. It is very clear that the fluctuations are much more radical for the individual countries and not synchronized with the rest of the world.

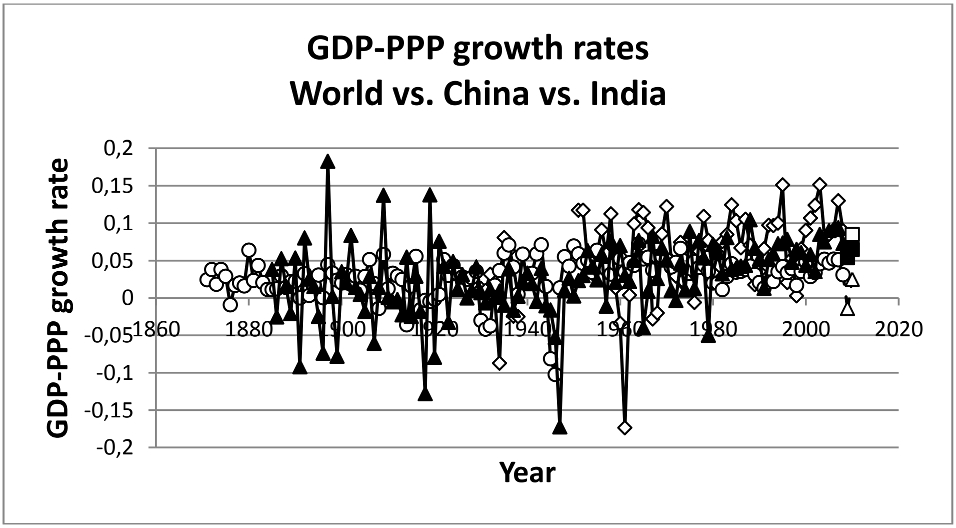

Figure 12 - Comparison between the GDP-PPP growth rates for the world (○), India (▲), and China (◇). The last points △(world), ■ (India) and □ (China) are the estimates for 2009 and 2010 from IMF[9].

Note that estimates for 2009 and 2010 (from IMF) were included in all these previous graphs. It is important also to point out that we have not used weighted averages in these graphs; weighted averages contribute to a biased picture of the whole. What we have in all three graphs represent the very fluctuations of the aggregate output.

Now let's try to present in a resumed form, point by point, the main aspects unveiled when looking at these graphs, or in other words, when observing the secular unfolding of the aggregate world output.

1 – The most striking aspect exhibited by the graph in figure 10 is the very turbulent time during the first half of the 20th century, which carried within with the effect of two world wars and the most painful economic crisis already experienced by the world economy; note that the 'dip' corresponding to this Great Depression is placed exactly in the middle of the 'double dip' corresponding to the two world wars, roughly equidistant by ~15 years.

2 – This turbulent time is confined between two periods of 'peace times', the first one from 1870 until 1913 (then 43 years), and the second one from 1950 until 2008 (then 58 years).

3 – The first period of relative stability and 'peace times' is marked by two pronounced dips with negative growth rates, a first one in 1876 and a second one in 1908. The first dip corresponds to the panic of 1873, which gave place to a strong recession of the world economy, but that was especially severe in USA. The NBER statistics[17] consider it as the longest recorded contraction cycle in the USA (65 months, 1873-1879), and some authors[18] have compared it with the current financial crisis due to many common characteristics. The second dip appears in 1908 and was a consequence of the panic of 1907[19], with also dramatic global consequences, but shorter in duration (in the NBER statistics[17] for the USA a contraction cycle of only 13 months). Despite short it can be considered as a Great Recession comparable in numbers (GDP contraction) to the present crisis.

4 – Still regarding this first ‘peace times’ period, we can distinguish two sub periods: one with a downward trend (decreasing growth rates, considering the mean values) that extended until at least 1896, soon followed by one with an upward trend (increasing growth rates, considering the mean values), that extended until the middle 1920s but was disturbed by the onset of the WWI. We have then two sub periods: ~1870 to 1896 and 1896 to 1922, each with ~26 years, that as suggested by many adepts of Kondratieff long waves correspond respectively to the downwave phase (or phase B) of the 2nd K-wave and to the upwave (or phase A) of the 3th K-wave.

5 – Regarding the second ‘peace times’ period that followed WWII and started after 1948, we can more easily distinguish different sub periods – more exactly three. A first one located between 1948 and 1973, when the global output averaged a growth rate of about 5%, a second one between 1974 and 1992, when the global output averaged circa 3.5%, and a third one after 1993 when an upward trend is observable, reaching and surpassing the mark of 4% (with a brief interruption in 2001 – the dot.com bubble). The reader should note that there is a dip in the world-series corresponding to 1998, but comparing with the graphs shown in figures 11 and 12 we can see that it was not a crisis in the USA or Europe, but the consequence of the famous Asia Crisis[20], which started in July 1997 in Thailand and spread quickly to many other Asian countries, including China and India. Again we have sub periods with time spans averaging two decades – in this case now 25 years and 18 years respectively. K-waves adopters usually associate these sub periods with the up and downwave phases of the 4th K-wave. Following this schema it seems that after 1992 the 5th K-wave might already be started. We will turn to this point in the next section.

6 – Regarding now the actual crisis, translated by the extrapolated points for 2009 and 2010 (small triangles in figure 10), we can’t draw so easily the same conclusion expressed in the previous section of a self-correction mechanism that is pulling the general trend towards its original path. The points for 2009 and 2010 resemble much more a pathological symptom signaling that something is wrong with the existing economic system, or perhaps more exactly expressed, with the existing global financial system. We use here ‘a pathological symptom’ because we are facing neither a world war, nor a worldwide social upheaval. Something else seems to be hidden behind the facts.

7 – A closer look again to our graphs of figures 10, 11 and 12 may help to shed some light upon the facts. A very important detail to stress is that we have historically a very important precedent that happened in 1907, that is, exactly one century ago (or, in other words, two K-waves ago!). The phenomenon, known as the ‘1907 Bankers Panic’[19], was very similar to the actual crisis under at least two important aspects: it occurred during an upward trend of the global economy (i.e., during the A-phase of a K-wave) and was a pure financial crisis involving market liquidity that led to bankruptcy many important agents of the banking system, which quickly spread from New York to Europe and to some Asian countries (see for comparison graph of figure 12). The remedy at that time was the same as nowadays: the injection of large sums of money to shore up the banking system, soon followed by a profound reform of the US financial system, which included the creation of the Federal Reserve System (FED, created in 1913). The reader should observe in the graph of figures 10 and 11 that the dip in 2009 mirrors the one in 1908!

8 – As already referred to in the paragraph preceding figure 11, it is very evident from the graphs comparing the unfolding of the world GDP and the sum of USA plus 12 WE GDPs, that after 2000 a different trend emerged: the growth rates of the world

GDP from this date onwards are higher than for the USA and the European countries together, an inverse behavior of the GDP evolution to this date. This push upwards is clearly motivated by the rocketing GDP growth rates observed for India and China, as can be inferred from figure 12.

9 – Such an inverted trend seems to be a clear signal that we are already witnessing a transition to a new global socioeconomic system, which will carry within with a profound restructuration of world economic affairs. In few words it means that real growth rates of low-income countries have been growing increasingly apart from those of high-income countries. See more details in the conclusions section.

10 – Ajar with the times, the present crisis seems to sum up a mix of self-correction mechanism (or at least the urgent necessity of finding the necessary measures for correction) and signals of an imminent transition to a new world order.

Before closing this section it is worth to bring to the reader's attention the fact that negative fluctuations of the world GDP is not sufficient condition to characterize a great depression. There are more things at stake when we wish to speak of economic recessions with a worldwide impact and severe consequences across countries. In a very recent book (2009) the economists Carmen Reinhart and Kenneth Rogoff[21] shown that in order to characterize a real great depression it is necessary to observe not only a considerable contraction of the GDP, but also a significant retraction of the worldwide commercial exchange. For these authors this phenomenon has only occurred three times in the recorded history: in 1907/1908, 1929/1933 and now in 2007/2009. Many other crises, like those of 1873/1879, 1945/1946, 1987, 1998 or 2000/2001, have not had the same global impact like these three mentioned, because have not hit equally both measures (GDP and commerce) or have had only regional effects (like the 1998 Asian crisis). This aspect is a very important one regarding our previous conclusions and the parallel between the actual crisis and the 1907 Panic.

7 - Scrutinizing the recent record of the global output

Keeping in mind the fact already mentioned in our fifth section (Signals of saturation) that the most reliable data for the global output are those that followed WWII, it is worth to scrutinize further this recent period, which we coined as the second 'peace times' period.

Figure 13a shows the result of applying an 11-year moving average to the data of figure 10 (world GDP-PPP growth rates) in the period 1947 - 2008. As can be seen it is evident a wave-like behavior suggesting the fingerprint of a complete long wave. Figure 13b presents the result of fitting a simple sinus series of the type $P(t) = P_0 + A \sin(2\pi\ t/T) + B \sin^2(2\pi\ t/T) + \ldots$, whose solution is $P(t) = 4 + 1.03 \sin(2\pi\ t/50.14) + 0.03 \sin^2(2\pi\ t/50.14)$, evincing then a periodical movement with a period of about 50 years (the points for 2009 and 2010 were not included in the fitting).

This result comes to reinforce our conclusion resumed in point 5 of the previous section that we can divide this recent period in three sub periods – the first and second corresponding to an entire K-wave and the third corresponding to the upward movement of the following K-wave. The entire K-wave in this curve matches very well the dates that many different authors have presented for the 4^{th} K-wave, which started about 1947/48, reached a maximum about the 1970s, and was completed in the first half of the 1990s.

The extrapolation for the fifth K-wave points to a maximum to be reached shortly before 2020, or in other words, the present expansion movement, although disturbed by the recent crisis, may well continue for more one decade. The much discussed apparent recovery still on course (crisis 2007/2009) seems to hint that the system is indeed resilient.

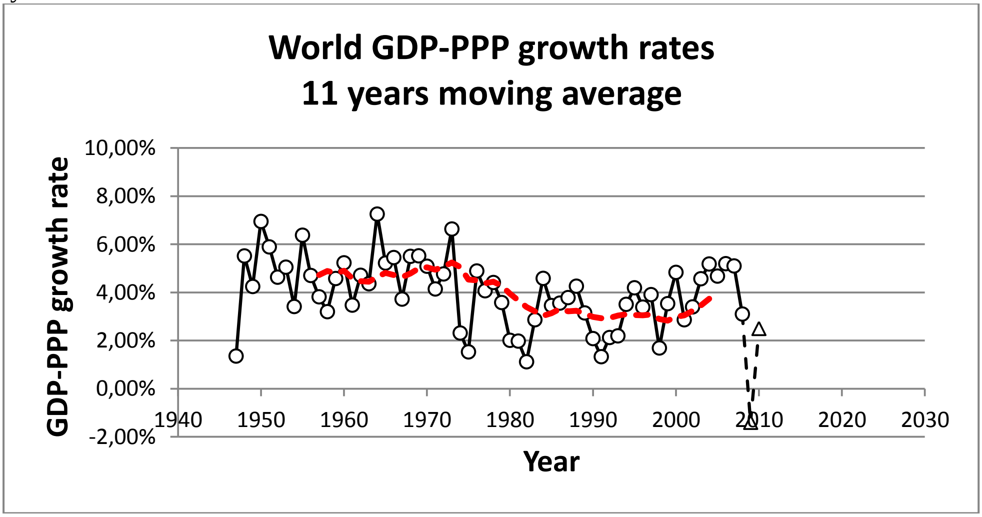

Figure 13a – 11-year moving average applied to the world GDP-PPP growth rates in the period 1947-2008. The estimates for 2009 and 2010 (△) were not included in the MA.

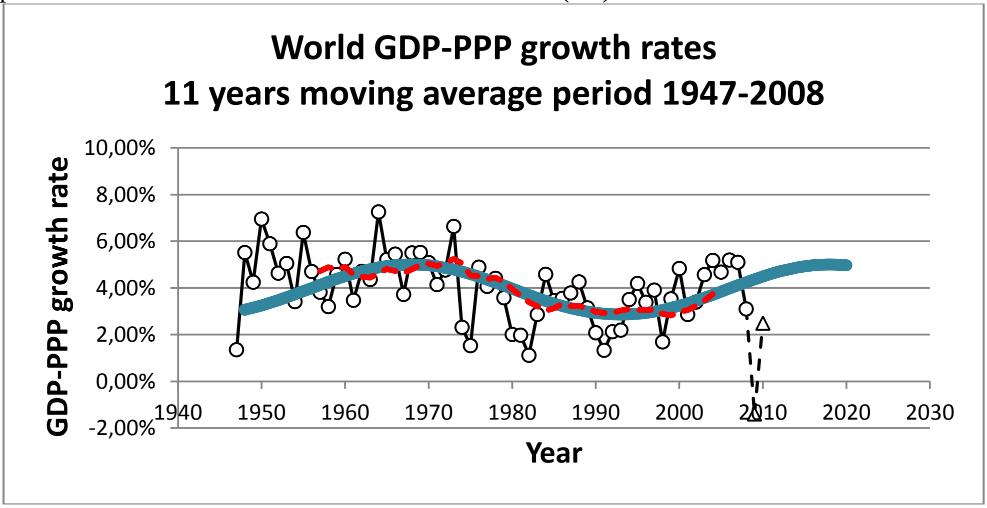

Figure 13b - Result of fitting a simple sinus series $P(t) = 4 + 1.03 \sin(2\pi\, t/50.14) + 0.03 \sin^2(2\pi\, t/50.14)$, evincing a periodical movement with a period of about 50 years (the points for 2009 and 2010 were not included in the fitting).

8 - Shrinking recessions and contractions

In a recent paper the Italian economist Mario Coccia[22] brings to attention the fact that the duration of contraction phases of business cycles are far shorter than the duration of expansion phases. The author observes also that the duration of the recessions corresponding to the downwave phase of a longwave is in average shorter than the upwave phase. In the case of business cycles the author uses statistics from NBER[17] and from the US Bureau of Economic Analysis (BEA[23]), comparing data for

USA, UK and Italy. In the case of longwaves the author uses an extensive comparison of the dates proposed to this phenomenon by many different longwave theorists.

His results point to a mean duration of contractions of business cycles in the USA, between 1854 and 2001, of about 17 months, and a mean duration of expansions of about 39 months, or in other words, an average of 31% of the time experiencing economic contraction and 69% experiencing economic expansion. Regarding the K-waves the author points to an average of about 29 years for upwaves (53% of the total time) and 26 years for downaves (47% of total).

We decided in this research to explore also this phenomenon using the NBER statistics for the USA and were confronted with two very interesting and unexpected results: first, there exists an increasing trend towards shorter contractions and longer expansions, and second, the fingerprint of K-waves is clearly visible also in the history of US Business Cycles.

Figure 14a shows the graph resulting from the distribution in time of the succession of economic expansions and contractions in the history of business cycles in the USA since 1850. In despite of the star field-like aspect of the distribution of the points, one can clearly distinguish the enduring trend towards longer expansions and shorter contractions translated by the straight trend line. The last point in this graph corresponds to the expansion period that lasted from the end of 2001 to the end of 2007 (73 months) and ended with the onset of the actual crisis.

Figure 14b presents the resulting 20-year moving average applied to the same historical statistics. The trend line reveals a wave-like behavior that coincides with the dating schema used by many longwave authors and matches very well our conclusions in the previous sections. In this graph we have added a point to the actual crisis considering it with a supposed duration of 24 months. This point was considered in the moving average in order to observe the path of the trend line. Again we are induced to the same conclusion drawn in point 1 at the end of the fifth section (Signals of saturation) – this last point suggests the action of a self-correction mechanism bringing down a period of excessive growth!

Coocia[22] suggests that these shrinking contraction periods may be due to a learning process during which government(s) have developed functioning methods to undermine the effects of economic recessions. This suggestion comes to reinforce our second conclusion in the fifth section about a secular learning process of the socioeconomic world system.

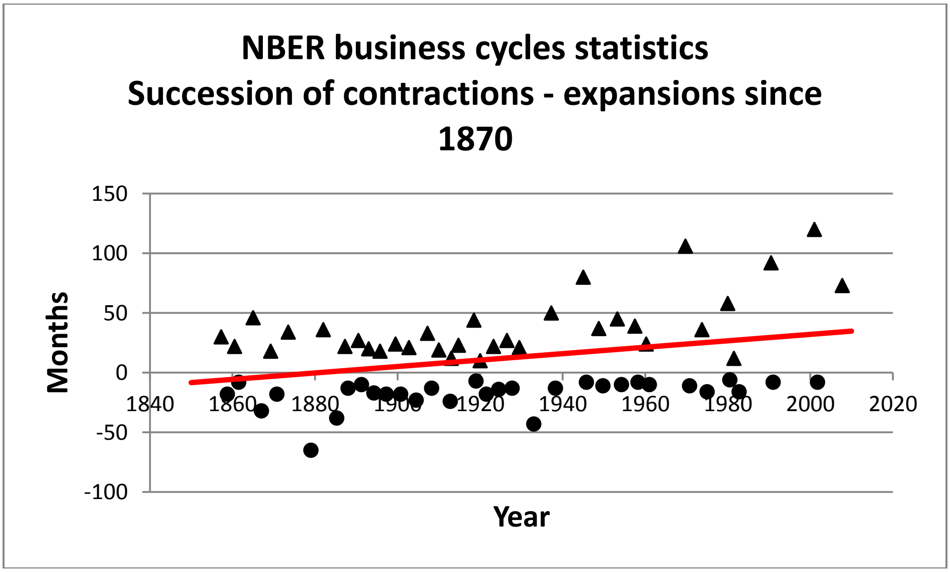

Figure 14a - Star field-like aspect of the distribution of the succession of economic expansions (▲) and contractions (●) in the history of business cycles in the USA since 1870 (data from NBER[17]). The straight trend line translates the trend towards longer expansions and shorter contractions in business cycles.

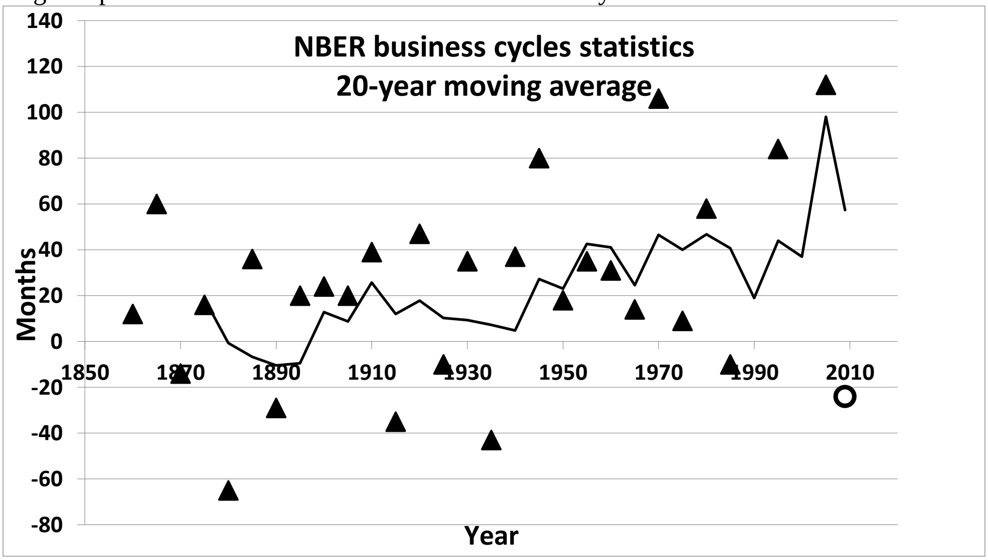

Figure 14b - 20-year moving average applied to the points of figure 14a. The trend line reveals a wave-like behavior that coincides with the dating schema used by many longwave authors corresponding to the 2nd, 3rd, 4th and 5th K-waves. In this graph we have added a point (○) corresponding to the actual crisis considering it with a supposed duration of 24 months.

9 - Maddison's phases of economic growth

In a publication from 2007 Maddison[24] performs a balance of his impressive and massive historical research about the evolution of the world GDP and GDP/capita since the beginnings of the 19th century, as well as a detailed analysis of the works of some longwaves theorists (Kondratieff, Kuznets, Abramovitz, Schumpeter, and longwave revivalists like Rostow, Mandel and Mensch). Maddison concludes that the existence of a regular long-term rhythm in economic activity is not proven and states further that there is no convincing evidence to support the notion of regular or systematic longwaves in economic life.

Based mainly on his own data on aggregate performance Maddison however concedes that there have been major changes in growth momentum of capitalist development since 1820, which he coins as phases of economic growth. He recognizes five phases: 1820-1870 (transition from merchant capitalism to industrial accelerated growth), 1870 – 1913 (liberal phase), 1913-1950 (beggar-your-neighbor phase), 1950-1973 (golden age), and a last one from 1973 onwards (neo-liberal phase). Curiously there is some coincidence between these dates and some very important dates used by longwave adopters either to characterize the duration of a full wave or to mark the transition between phases (up and down) of longwaves.

But there are some oddities to point out in Maddison's whole analysis. In first place his review of authors contributing to bring empirical evidence on the existence of longwaves is far from complete and does not include very import vast research work of authors that have brought robust empirical evidence using most effective mathematical tools. Maddison reviews basically only classical authors that have tried either to advance economic models to explain the longwave phenomenon or to present evidence based only on economic statistics (with the exception of Mensch).

As robust empirical and mathematical evidence one must considers at least two authors that have carried during decades (1980s and 1990s) extensive work on longwaves: the American economist Brian Berry and the Italian physicist Cesare Marchetti, whose works were published in the pages of TF&SC and elsewhere. Berry[25] used convincingly chaos theory and spectral analysis to prove the existence of longwaves and Marchetti[26], leading a research team at the International Institute for Applied Systems Analysis (IIASA), produced some hundred graphical analyses applying the logistic substitution model on physical measures of human aggregate activities. In our point of view there is a touch of nonsense and exaggeration in simply refusing all the massive evidence brought by both authors.

Indeed it is very difficult to prove the existence of longwaves using only economic statistics. There are many variables that must be considered simultaneously and this consists in an almost impossible task. But we must recognize that in despite of this inherent difficulty there is the register of at least two bold forecasts in recorded history: Kondratieff himself, writing between 1922 and 1926, predicted accurately the Great Depression of the 1930s and there is the famous graph published in 1974 by Media General Financial Services that had been widely reproduced by dozens publications on longwaves since then (the graph was also reproduced in one of our previous publications in the pages of TF&SC[27]). This graph, a schematic depiction portraying the cresting unfolding of K-waves since the 1790s, predicted also very accurately the behavior of the world economy in the following decade (1980s), when was observed a global reduction of economic growth and retraction of the world commerce, as alias evinced too through the timely evolution of the world GDP-PPP growth rates shown in figures 13a and 13b.

This kind of schematic depiction of K-waves has been the preferred target of many criticizers of Kondratieff waves, who insist in the fact that such regular long-term oscillations do not exist. It is clear that such monotonic upward movement during about two decades, followed by a subsequent two decades-long downward movement do not exist indeed – what is necessary to comprehend is that such representation is just a schematic portrait of a very complex behavior that include the timely unfolding of several variables and do not tries to translate the evolution of a unique variable. Perhaps a bit more realistic representation should include in the upward and downward movements the within nested shorter business cycles, as we try to express through figure 15. But again it is very important to stress that this is just a schematic depiction of a much complex phenomenon and does not intend under no circumstances to render a real depiction.

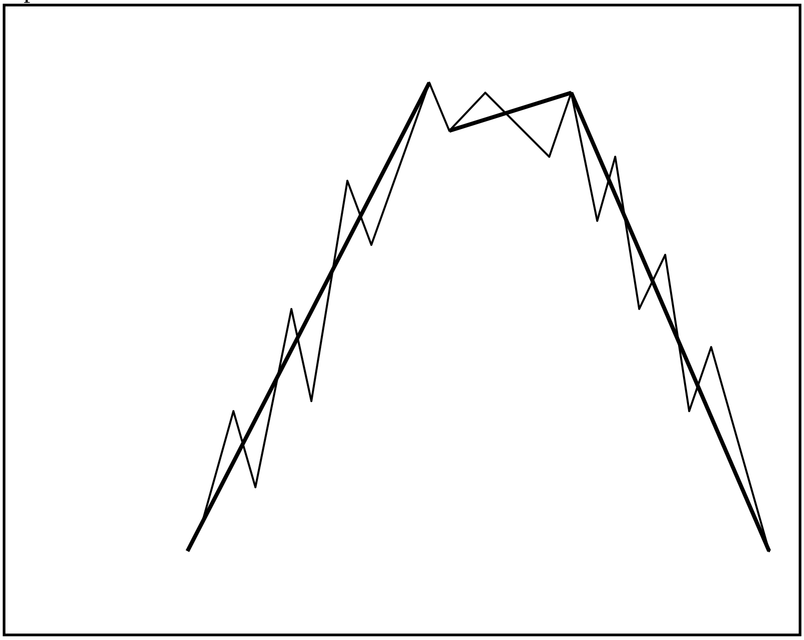

Figure 15 – Schematic depiction of a hypothetic long wave with nested shorter business cycles. As explained in the text this is just a schematic portrait of a very complex phenomenon and does not intend to render a real depiction of a single variable.

As a second oddity in Maddison's whole analysis we wish to point out the lack of graphical analysis. One really wonders why Maddison does not use graphs in his publications. In his famous and very frequently referred 2007 book[6] for instance, among 124 tables, Maddison presents only seven graphs, and just for comparisons of GDP cumulated growth (or comparative levels of GDP/capita) for pairs of countries, like UK/Japan, UK/India, US/China, etc… In his own words[24] he says to use "*inductive analysis and iterative inspection of empirically measured characteristics*", but the most of his analysis and conclusions are drawn only based on tabular constructions, which do not allow perceiving long-term trends and details of an evolutionary process. As can be seen in this work, a simple glance at some graphs allows the perception of fingerprints of K-waves, as well as the observation of details related to the temporal behavior of a given economic-related quantity.

It is hard to understand why Maddison is so adamant in his statements about the lack of evidence on K-waves if he has never applied mathematical analysis on his monumental set of data, as for instance spectral analysis. We have already mentioned above the contribution of Brian Berry. This author, in his 2001 paper[28] has demonstrated the existence of low frequency waves of inflation and economic growth using digital spectra analysis. He and his collaborators have found ~9 and ~18-year oscillations linked to business and building cycles, and additional ~28 and 56-year rhythms linked to inflation alone.

Very recently Korotayev and Tsirel[29] have examined minutely the entire data set of Maddison's GDP-PPP growth rates under the optic of modern spectral analysis and have found very similar results, or in other words, two strong frequency peaks corresponding to the shorter business cycles (in this case ~8 years and ~15 years), and two long-term frequency peaks (~30 and ~52 years) related to long waves – the shorter probably corresponding to upwaves and downwaves movements and the longer corresponding probably to complete K-waves oscillations.

In our research we decided then to verify these results and have applied a simple Fast Fourier Transform using the Sigview software[30]. The result is shown in figure 16 where we can clearly discern the existence of four frequency peaks, in this case 7.5 years, 15 years, 32 years (very weak) and 52 years – again practically the same result as those of Berry and Korotayev-Tsirel. It is important to stress that ours and Korotayev-Tsirels' results were found in the same data set where Maddison says that there is no convincing evidence to support the existence of systematic long waves in economic life.

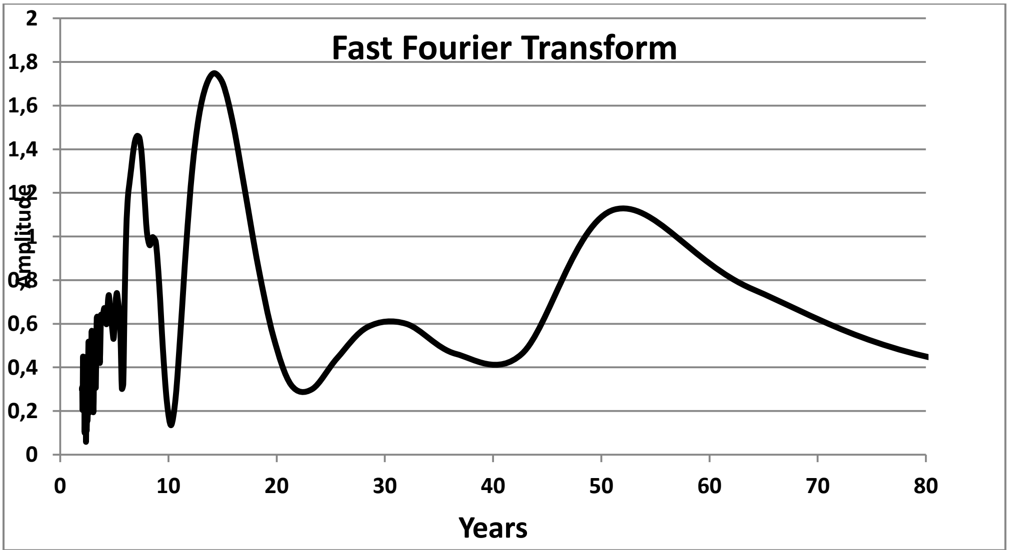

Figure 16 - Fast Fourier Transform using the Sigview software[31] applied to the historical unfolding of the GDP-PPP growth rates presented in figure 10. We can clearly discern the existence of four frequency peaks: 7.5 years, 15 years, 32 years, and 52 years.

Closing this section we wish to briefly discuss a statement of Maddison at page 161 of his paper[24], where he wrote: "*The government regulatory role in the economy has greatly increased. One result of the latter is that the stability of financial institutions has improved. Before the Second World War, depressions were often reinforced by major bank failures, but these are now rarer and their impact is cushioned*".

What is curious in this statement is that it is partially true – in fact, there have been a learning process during which governments have learned a lot how to reduce the impact of economic shocks, as we have already stressed previously, and that explains the phenomenon of shrinking recessions and contractions portrayed in our figure 14a. But on the other hand is completely false regarding the stability of financial institutions. Let's give a discount to Maddison – he has written these lines shortly before the big financial crash of the end of 2007.

10 - Gold – the master of commodities

At the end of the closing chapter of his 1922 book[31] Kondratieff has made a very important observation about the behavior of gold during the unfolding of K-waves, which has been bypassed by most of long waves analysts up to present days. In this chapter, with the suggestive title "*The crisis of 1920-1921 in the system of general movement of conjunctures*" Kondratieff paved the way for his dangerous idea of an incoming (temporary) collapse of the world economy and used gold to reinforce his damned prophecy. The inclusion of the word 'temporary' here is very important, because Kondratieff's dangerous idea was not the forecast of a final collapse (as wished by his Bolshevik opponents), but the anticipation of a new downward wave, which should be followed by another upward wave – or, in other words, a general picture of a wave-like movement of the capitalist system.

Kondratieff wrote: "*Gold output, on the other hand, showed a remarkable movement, too. Since mid-1890s its output was surging to come to a maximum in 1915 and a subsequent continuous decline….The output of gold is quite likely to plunge into a long depression, which is the most remarkable feature of the current epoch*". He follows referring to a study of Joseph Kitchin and presents a table from a publication of this author with data about the annual average growth of gold output, in which can be seen a minimum in 1810, a maximum in 1847, again a minimum in 1868, followed by a new maximum in 1891, and declining again after this date. In the following paragraphs he wrote: "*It can be readily seen that the dates and periods displayed match closely the turnarounds and periods of upward and downward waves of the long cycles. It is also quite obvious that the upward waves are coincident with periods of a high annual growth of gold, and vice versa. In this case, we enter upon the area of relatively low annual growth of gold, which is going to affect the downward conjunctures of the long cycle…..Again this process promises to follow the line of the 1870s*".

Indeed a bold forecast and what happened in the following years is the history everyone knows very well. But after these lines Kondratieff has made too another very important point: "*We can therefore relate to the world economy as being quite likely to enter upon a downward phase of the long cycle. This by no means goes to say that this phase will be clear of its own ups or downs or depressions in terms of minor capitalist cycles. They have always been present in such phases in whatever long cycles of conjunctures of the past. They will surely be present in a downward phase of the long cycle. In a general frame of their variation, however, the conjunctures are most likely to keep downwards. Consequently, elevations in minor cycles of the oncoming period will lack the intensity they would display while on an upward wave of a long cycle. By contrast, crises of this period promise to be sharper, and depressions of minor cycles lengthier*".

Again a bold forecast, and the reader must keep on mind that these lines were written in 1922. What Kondratieff voiced in this last paragraph is exactly what we have tried to express through figure 15 in section eight.

The question now is – what our important actor in world economic affairs, gold, allows us to say about the present trend and what may be forecasted regarding the forthcoming years? As we will see some forecast is indeed possible, but we have first to consider that the behavior of gold has changed dramatically along with the last century, after Kondratieff inspired vision.

The graph depicted in figure 4 (Gold weekly price) cannot tell us much about the future of gold price, except perhaps the fact that we are presently witnessing a strong momentum upwards. Such growth however cannot continue indefinitely, nothing in the universe growths forever. But on the other hand this graph tells us a lot about the gold's past and recent history. As can be seen, since 1900 gold experienced a long period subjected to two levels of constant prices until 1971, when suddenly began to raise, reaching a first modest peak in 1975, soon followed by a strong peak by the end of 1980, outreaching the level of US$ 800. This record was immediately followed by a continuous trend of decreasing prices that endured 20 years, reaching a minimum of about US$ 270 by the end of 2000, when gold entered a new phase of an apparently unstoppable trend towards ever increasing prices.

The long period of constant prices belongs to the old times of the 'gold standard', which started in Britain after the Napoleonic wars. In the second half of the 19th century, a number of nations in Europe and elsewhere followed suit, and the United States adopted the gold standard *de facto* in 1879, by making the "greenbacks" that the Government had issued during the Civil War period convertible into gold; it then formally adopted the gold standard by legislation in 1900, when our graph begins. By 1914, the gold standard had been accepted by a large number of countries, although it was certainly not universal.

During the 1880-1914 period, the "mint parity" between the U.S. dollar and sterling was approximately $4.87, based on a U.S. official gold price of US$20.67 per Troy Ounce (31.1035 grammas) and a U.K. official gold price of £ 4.24 per Troy Ounce. This system worked well during almost forty years when the world economy entered the turbulent phase already referred to when commenting on the graph of figure 10.

We can state that this first period of relative peace corresponded to the real entrenching stage of a successful international capitalist system, when there were no changes in the exchange rates of the United States, UK, Germany, and France (though the same did not hold for a number of other countries). There were few barriers to gold shipments and few capital controls in the major countries. Capital flows generally seem to have played a stabilizing, rather than destabilizing, role. After the outbreak of the First World War, one combatant country after another suspended gold convertibility, and floating exchange rates prevailed. The United States, which entered the war late, maintained gold convertibility, but the dollar effectively floated against the other currencies, which were no longer convertible into dollars. After the war, and in the early and mid-twenties, many exchange rates fluctuated sharply. Most currencies experienced substantial devaluations against the dollar; the U.S. currency had greatly improved its competitive strength over European currencies during the war, in line with the strengthening of the relative position of the U.S. economy.

But in the very beginning of the turbulent phase that followed WWI (and when Kondratieff issued his first publications!), there was a widespread desire in Europe, especially in the UK, to return to the stability of the gold standard, and a worry about the growing attractiveness of the dollar—which was convertible into gold—and of dollar-denominated assets. Following a disastrous five years back on the gold standard, the UK abandoned it in 1931, and others followed over the next few years.

Things began to worse and after the onset of the Great Depression in 1929 Keynesian economics was the evident remedy found to recover the agonizing patient, that so healthful patient of the preceding years. In April 1933, US President Franklin Roosevelt through the Gold Reserve Act imposed a ban on U.S. citizens' buying, selling, or owning gold. While the U.S. Government continued to sell gold to foreign central banks and government institutions, the ban prevented hoarders from profiting after Congress devalued the dollar (in terms of gold) in January 1934. This action raised the official price of gold by more than 65 percent (from $20.67 to $35 per Troy Ounce) and this fact is translated by the first jump to a new level observed in our graph of figure 4.

In 1971, when the Bretton Woods system broke down, President Richard Nixon ended US dollar convertibility to gold and the central role of gold in world currency systems ended, giving birth to a new era of complete liberalization of capital flows. The consequences are very clear in the graph of figure 4: the dollar and gold floated and in January 1980 the gold price hit a record of US$850 per ounce, soon followed by a decrease that endured for almost 20 years. Only after 2000 gold started again to escalate reaching new levels that make look overt the 1980's record. What can be learned from this picture?

The first quite obvious lesson is that the remedy found to fight the system's illness does not hold for a long time. It is as if the doctors (economists) were combating only the symptoms and not really fighting the true intrinsic system's sickness. The relief measures insistently applied until now by mainstream economics consists in failed contra-cyclical policies that systematically overlook some strong forces underlying the global economy. These strong forces are mainly the inexorable human propensity to hoard and the physical-biological imperatives acting upon the complex socioeconomic system. The latter was already analyzed in deep in some of our previous publications[27, 32] and we do not intend to discuss in this paper. It is looking at the former that we can discern some important hints that can help us to correctly reading the historical unfolding of the role played by this important actor – gold – in the whole piece of economic capitalist development.

The reason for our title – the master of commodities – lies in the fact that gold is the most hoardable commodity. Gold does not tarnish or fade; it resists the entropic laws of decay, and its high specific gravity contributes to the fact that the opportunity cost of hoarding gold is far lower than that of hoarding any other commodity. Gold is essentially money of last resort and has been the most effective hedge against turbulent times, be they caused by wars or economic depressions. For all over the recorded history humans have shown an inexorable trend to hoard gold bullions and all the sudden changes observed in the unfolding of the graph depicted in figure 4 were due to governments measures trying to oppose this strong economic force. Unnecessary to point out that such measures have never worked (in the long range) in favor of the health of the socioeconomic realm. The increasing price trend evinced since 2001 is the clearest proof of the action of hoarding per se.

But in order to draw effective conclusions about the future path of the world economic system is necessary to look at gold's history other way around. In 1977 Berkeley's Professor Roy Jastram in his seminal work "The Golden Constant – The English and American Experience, 1560 – 1976"[33] demonstrated, for the first time, how gold's purchasing power had been maintained over the centuries. Dividing the gold price index by the wholesale price index he found that the Purchasing Power of Gold (PPG) has fluctuated around a broadly mean value. However, Jastram's research ended in 1976, and therefore he barely foresaw the impact of the new era of floating gold price, still at its genesis.

Very recently Jastram's original work was updated by Leyland[34] in a research supported by the World Gold Council. The new edition contains two additional chapters (and the relevant statistics) examining the period from 1971 to 2007. The conclusions about the behavior of the Purchasing Power of Gold differ somewhat between the periods before 1971, when the gold price was controlled, and after, when it was free. Nevertheless, one conclusion remains unchanged - that gold maintains its purchasing power over long periods of time even though, over shorter periods, it has fluctuated significantly. But more importantly, this new research demonstrates that now gold moves just the opposite of what it used to do. Before 1971 gold lost value during inflationary spirals, while it appreciated in value during major deflations. The reason was obvious: gold was fixed in price. But after 1971, when the gold was delinked and set free to fluctuate, the price of gold goes up when inflation goes up, and falls when deflation hits.

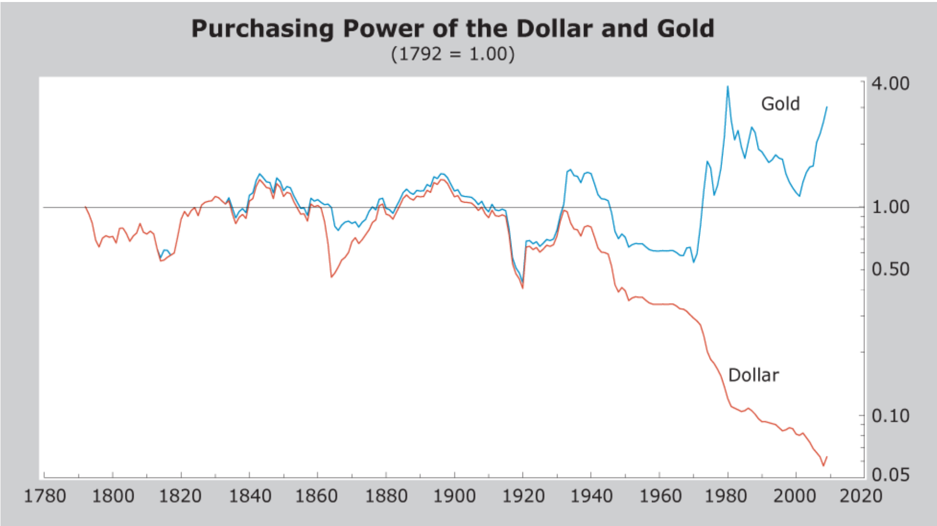

Figure 17 - Purchasing Power of Gold (PPG) compared to the Purchasing Power of US Dollar (PPD) since the 1790s. Data from the American Institute of Economic Research[36].

In figure 17 we present a graph portraying the Purchasing Power of Gold (PPG) and as comparison the Purchasing Power of US Dollar (PPD) since the 1790s recently published in the Web by the American Institute for Economic Research[35]. There are some very important points to infer from this graph that we try to resume below:

1 – Both purchasing powers have unfolded perfectly in phase until at least the early 1930s, when began to diverge and this diversion aggravated substantially after 1971.

2 – There is evidently a wave-like behavior and the maxima and minima of the fluctuations before the early 1920s match closely the dates for the turnarounds of long

waves pointed out by Kondratieff that we referred to at the beginning of this section; the dip in the early twenties also matches Kondratieff's forecast.

3 – The 50-year beat of the maxima of these long fluctuations is absolutely evident – 1840s, 1890s, and late 1930s. Even the peak reached in 1981 falls within the long wave timeframe. It is indeed hard to understand the intestine refuse by mainstream economics in believing in the existence of long waves.

4 – In 1971 for the first time in history PPG jumped suddenly from a value below to a value above its historical average, and no more returned to the field below < 1.00. After a brief hesitation in the mid-1970s, PPG rocketed again in 1980-81, when gold price reached the first maximum shown in figure 4. This was a decade (1970-1980) not just of high inflation but it also included the two oil price "shocks" and what appeared at the time to be the end of the post-war "miracle" growth of the 1950s and 1960s.

5 – After the maximum reached in 1980-1981 PPG entered a 20-year long declining period, during which a self-correction mechanism seemed to act in order to bring it down to its original path along with its historical average. That was the time of the "great moderation" of the decades 1980s and 1990s, a period of disinflation, generally improving economic circumstances, mostly strong stock markets and marked politically by the fall of communism.

6 – In contrast, since 2001 the PPG has risen again due to the well know concern over global imbalances and rising debt, which culminated in the current economic and financial crisis.

7 – Comparing the last decreasing period of PPG (1980-2001 = 21 years) with the preceding ones (1842-1870 = 28 years, 1895-1920 = 25 years, 1940-1971 = 31 years) we can say that it was relatively shorter, but not very far away. Associating this fact with the observation that PPG is presently going away from its historical average we can suspect that we are facing an anomaly, or at least we are experiencing a transition phase as we have already pointed out when analyzing other economic indicators.

8 – Such an anomaly, or if we prefer, the imminence of a transition phase, is evident from the 'bifurcation' (perhaps better, divergence) presented in the graph of figure 17. It is quite possible (in fact it is the case since 1971) that a portion of the increase of PPG is really just the outcome of the decrease of PPD, considering that the change in gold price is simply a mathematical recalculation of an ever-changing US Dollar value.

9 – The history of fiat currencies is that they lose their purchasing power over time. Because a limited amount of gold exists in the world and paper money can be created without limits, gold has been an ultimate protection against the debasement of currencies. If we look at the historical charts of the purchasing power of major currencies as well as the amount of these currencies in circulation (see for instance the graphs presented by Financial Sense University[36]) what we see is that all major currencies have lost steadily purchasing power since 1971 – US Dollar is now at 20% of its level in 1971, GB Pound at 18%, Canadian Dollar at 18%, Australian Dollar at 10%, Japanese Yen at 70% and Swiss Franc at 70%. Opposed to this decrease the amount of circulating paper money of the same currencies grew by a factor 8 (USD), 5 (GBP), 10 (CAD), 20 (AUD), 10 (JPY), and 3 (CHF) respectively. On the other hand the amount of mined gold has grown slowly and almost linearly, from about 95 x 10^3 metric tonnes in 1971 to about 160 x 10^3 metric tonnes in 2008 (a factor of only 1.6 in more than three decades) [36]. Resuming this point, the amount of available gold (or gold

output) is not the cause of the movements of PPG after 1971; the subjacent cause lies in the combination of two other linked factors – an ever-increasing debasement of currencies and declining (1971-1981) or improving (1981-2001) economic circumstances.

10 – But supposing that in despite of the changed circumstances the system is resilient and that the PPG will not deviate very much from its historical average (considering also that hoarding has its natural limits), we might conjecture that the actual increasing trend of PPG (and naturally also of gold price) can continue until 2010-2011 (a decade after 2001), but will return to its historical mean value, a process that may involve one or two decades of economic growth that will coincide with the upward phase of the 5th K-wave peaking about 2020. This forecast matches well our previous considerations when discussing the world GDP.

There is also another way to look at the historical unfolding of gold price calling to playing other of our important agents – the Dow Jones Industrial Average (DJIA). We can calculate a ratio dividing the DJIA weekly index by the weekly price of gold, or in other words, to determine the historical record of the answer to the question: how many ounces of gold buy the Dow Jones Industrials index. The Canadian financial analyst Ian Gordon[37] originally developed this method, which he uses as economic forecasting tool. The resulting graph is shown in figure 18, and as we can see there is also a clear regular wave-like pattern.

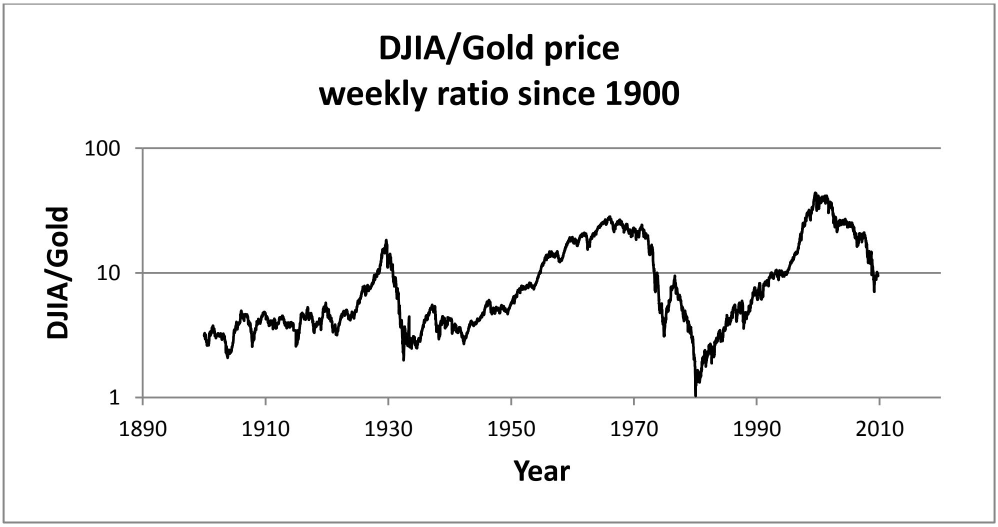

Figure 18 – Ratio DJIA/Gold price considered weekly since 1900. The data used are the same as in figures 3 and 4.

The pattern however is quite different from that presented in figure 17 – it seems inverted with relation to PPG, some of the PPG peaks are now pronounced dips and the waves have now a skewed aspect, evidencing two or three decades of growth followed by sudden falls. The first quick movement downward was soon after the stock market crash of 1929, and lasted only until 1933, recovering after Roosevelt's Gold Reserve Act. It followed an upward movement during almost three decades, which stopped around 1965-66 in consequence of a hesitating stock market. In 1971 again a sudden drop after the end of the US dollar convertibility, which extended until 1974 and was followed by the profound dip in 1981 that was due to the combination of a bearish stock market and an accentuated gold rally in prices.

The last wave begins then in 1981 and it is readable in this curve the timid stock market crash of 1987, which was followed by a rapid increase of the ratio DJIA/Gold (mainly after 1992), not only due to a worldwide bullish stock market, but also due to the healthy economic growth (and consequently to the cheaper gold) of the 1990s, which the Nobelist Joseph Stiglitz[38] coined as "The Roaring Nineties". A peak in the ratio happened in 2000, and after the dot-com bubble burst it has followed a steadily downward trend.

The actual situation is one of a hesitating stock market, mainly due to fears of an imminent inflation, and of a gold rally that many financial analysts[39] want to believe that will continue for a while with gold prices escalating until over US$ 2900! May be such a so high price level will never be reached, but a simple extrapolation of our curve of figure 18 induce us to hope that a minimum of the ratio might be reached very soon, which may be soon followed by an upward movement, implying in a recovering economy. Considering also the regular beat of the peaks – 1929 – 1965 – 2000, or in other words, a period of about 30-35 years (or a half K-wave), we can speculate that the next peak might be reached by about 2030 or earlier.

Concluding this section we can state that the historical evolution of gold allow us to foreseen that the present circumstances of a weakening dollar, a bearish stock-market, and increasing gold prices will reach an end very soon and a renewed economic upsurge may well take place lasting at least until the decade 2020-2030.

Conclusions

In this paper we have investigated the global secular evolution of four important economic-related actors, whose interplay when scrutinized with the suitable analytical tools evince some historical patterns that shed some light on what is going on with the world economic system. These actors are: the world population, the world aggregate output known as Gross Domestic Product (GDP), the historical leader of all commodities – Gold, and the still most important financial index, the DJIA (Dow Jones Industrial Average). Also the succession of economic depressions and expansion periods in the US was examined.

The main conclusions of this research are resumed below:

1 – Fingerprints of Kondratieff longwaves are ubiquitous in all observed time-series used in this research – world GDP growth rates, succession of economic expansions-contractions in the US, purchasing power of gold and the historical ratio DJIA/gold price.

2 – Regarding the present crisis we can state that it has some unique characteristics, which distinguish it from all previous economic depressions. But in despite of its unique characteristics a parallel with the panic of 1907 may be drawn – both have occurred amidst a strong international growth period and are perfectly symmetric in the observed space-time pattern.

3 – The most important conclusion concerning this crisis is that it seems to sum up a mix of a self-correction mechanism that brought the global output back to its original logistic growth pattern, and signals an imminent transition to a new world economic order.

4 – The next decade will be probably one of worldwide economic growth, corresponding to the second half of the expansion phase of the fifth K-wave, but that will saturate soon after the 2020's.

5 – There are strong signals that we are already witnessing a transition to a new global socioeconomic system, which will carry within it a profound restructuration of world economic affairs, with a multipolar world leadership and a new world currency. The in this research applied trend analysis using logistic curves, spectral analysis and the singularity approach converge to the same general result of an evolutionary trajectory leading the world system toward a true age of transition.

Regarding this last conclusion it is important to make stand out the fact translated by our results shown in figure 11 (and commented on in point 9 of section 6) that real growth rates of low-income countries have been growing increasingly apart from those of high-income countries. Since the onset of the Industrial Age high-income countries have contributed with at least about 70% for the global output measured as world GDP growth rate. Recent numbers presented by Marone[40] of the United Nations Development Programme show that this historical trend was maintained up to the mid 1990s, with the contribution of all income categories being roughly constant. But after this point and up to 2007 growth contribution from low-income countries surged by more than threefold, from around 10% (mid 1990s) to almost 35% (2007). In the mid 1990s high-income countries contributed with 77% for the global output growth, and low/middle-income countries contributed with 23%. Presently these numbers have radically changed to 95% from low/middle-income and only 5% from high-income countries. *Indeed we are amidst a great transformation.*

In this work we have applied a broad perspective approach with the main goal of exploring past events encompassing the action of the four actors/variables/agents together in order to find patterns of behavior that can concede us to comprehend what is going on. We just tried to construct a '*timescape*' using these variables that allow us to discern for instance that an incoming transition seems to be on marsh and that the present crisis exhibit symptoms of a saturating world economic system. We avoided bold forecasts and have speculated only about the very near future, within a time horizon of about two decades, a future that somehow is already determined by today's actions (and non-actions) and circumstances.

But as we all very well know, contingency exists and there are much more variables that must be considered in order to construct the most probable scenarios. We hope that our present results may contribute for more embracing studies that applying the multiple perspectives approach may lead to the enhancement of our ability to think constructively about the future of economics on a global scale.

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TIME SCALES AND MECHANISMS OF ECONOMIC CYCLES: THE CONTRIBUTIONS OF KONDRATIEFF, KUZNETS, SCHUMPETER, KALECKI, GOODWIN, KALDOR, AND MINSKY[1]

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Abstract

The current work highlights the empirical and epistemological contributions made by economists regarding the cyclical nature of economic and social development. We examine the main mechanisms of economic cycles involving different time scales, with a particular focus on long wave theory. Long wave theories include Kondratieff's theory of cycles in production and relative prices; Kuznets' theory of cycles arising from infrastructure investments; Schumpeter`s theory of cycles due to waves of technological innovation; Keynes – Kaldor – Kalecki demand and investment oriented theories of cycles; Goodwin`s theory of cyclical growth based on employment and wage share dynamics; and Minsky's financial instability hypothesis whereby capitalist economies show a genetic propensity to boom-bust cycles. The paper also discusses the methodological and empirical challenges involved in detecting long duration cycles.

JEL classification codes: C61; C63; G21; D83; D92

Keywords: production cycles, infrastructure cycles, accelerator - multiplier mechanism, innovation cycles, Goodwin cycles, Keynes-Kaldor Cycles, Samuelson accelerator-multiplier cycles, Kalecki cycles, Minsky asset price-leveraging cycles, spectral analysis, wavelet analysis

[1] We would like to thank Arkady Gevorkyan and Aleksandra Kotlyar for excellent research assistance in writing this paper. We also want to thank the organizers and participants of the Kuznets conference in Kiev, May 2011, for helpful discussions and suggestions. Discussions with various colleagues from the New School for Social Research and the University of Bielefeld, as well as with Peter Flaschel, James Ramsey, and Mauro and Marco Gallegati are gratefully acknowledged.

All things come in seasons - Herakleitos

One can never step into the same river twice – Herakleitos

1. Introduction

After a thirty year period of relative tranquility in the world economy -- the so-called period of “great moderation” -- the US economy suffered a financial meltdown in 2008 that triggered the “great recession”. These events have motivated new interest in theories that can explain long periods of expansion that end suddenly with deep recessions. One approach, which has been intellectually unfashionable for many years, is the theory of long economic waves.

This paper examines the empirical and epistemological contributions made by economists regarding the cyclical nature of economic and social development. The paper discusses the main mechanisms of economic cycles involving different time scales, with a particular focus on long wave theory. As part of this survey, the paper shows the continuing relevance of the theoretical constructs developed by Nikolai Kondratieff (also, Kondratiev, Кондратьев) and Simon Kuznets (Кузнец), both for modern macroeconomics and for assessing possible future scenarios. The paper also shows the difficulty of modeling long wave analysis as it poses significant challenges to the equilibrium method which dominates shorter period economic analysis.

Empirical economists and economic historians have voiced diverse views on economic cycles. On one hand, there seems to be good evidence for business cycles based on a shorter time scale, and the endogenous dynamics of shorter cycles appear to be clear and distinct. On the other hand, long wave cycles are more controversial, involve different theoretical mechanisms, and are harder to verify empirically – in part because data is inevitably more limited owing to the reduced frequency of such cycles. Several different theories of the long wave exist. These include Kondratieff’s theory of cycles in production and relative prices; Kuznets’ theory of cycles arising from infrastructure investments; Schumpeter`s theory of cycles due to waves of technological innovation; Keynes – Kaldor – Kalecki demand and investment oriented theories of cycles; Goodwin`s theory of cyclical growth based on employment and wage share dynamics; and Minsky’s financial instability hypothesis whereby capitalist economies show a genetic propensity to boom-bust cycles.

Business cycles of shorter duration can be explained by inherent mechanisms that generate cyclical fluctuations in economic activity. However, the mechanical view of long waves is more problematic and challenging. We discuss both those challenges and recently “discovered” evidence regarding components of long duration cycles. The notion of a financially based long wave Minsky super-cycle, which has been largely overlooked by contemporary economist, appears to have become more relevant in the wake of the financial crisis and the end of the “Great Moderation”.

The paper is organized as follows. Section 2 examines the long wave theories of Kondratieff and Kuznets. Section 3 builds on the preceding discussion and analyzes varying time scales and mechanisms of economic cycles prevalent in economic theory. Section 4 examines a Minsky-type of long-period cycles. Section 5 discusses the methodological and empirical challenges involved in detecting economic cycles, particularly those of long duration. Section 6 concludes the paper.

2. The Legacy of Kondratieff and Kuznets

2.1 Kondratieff and Theory of Long Waves

Writing in the early 1920s Nikolai Kondratieff advanced the idea of the probable existence of long wave cycles in capitalist economies lasting roughly between 48 and 60 years. Within that, there is a period of accumulation of material wealth as productive forces move to a newer, higher, level of development. But at a certain point there commences a decline in economic activity, only to re-start growing again later (Kondratieff, 1922 [2004]). This mechanism has been dubbed, in economic literature, as Kondratieff cycles.

It should be noted that prior to Kondratieff, some empirical efforts on systematizing the cyclicality of economic crises was carried out by van Gelderen (1913), Buniatian (1915), and de Wolff (1924), which Kondratieff admits to in his publications (see end note in Kondratieff, 1935). Though Kondratieff's ideas were not well accepted by the official Soviet economics he insisted on his main argument and in short time followed up with more rigorous publications. Only few English language translations were available at the time (most notably, Kondratieff, 1925; and Kondratieff, 1935). Nevertheless, the potency of his ideas was recognized quickly entering the work of subsequent economists (e.g. Schumpeter, 1934; Kuznets, 1971; Rostow, 1975; and others) as we review in the next section.

The gist of Kondratieff's argument came from his empirical analysis of the macroeconomic performance of the USA, England, France, and Germany between 1790 and 1920. The economist looked at the wholesale price levels, interest rate, production and consumption of coal and pig iron, production of lead for each economy and price movements (Kondratieff, 1935). Using a peculiar statistical method-- de-trending the data first and then using an averaging technique of nine years to eliminate the trend as well as shorter waves of Kitchin (Kitchin, 1923) type-- Kondratieff suggested a regularity of ups and downs in the data on a long time scale. Within that there were intermediate waves along with long waves. As a result Kondratieff stated that economic process was a process of continuous development. Among possible explanations to the long wave cycles Kondratieff mentions a) changes in technology; b) wars and revolutions; c) appearance of new countries on the world map; and d) fluctuations in production of gold (Kondratieff, 1935; and Kondratieff et al. 2002).

All four appear as valid external shocks in pushing any particular economy or the world economy into a downward or upward cycle path. However, after careful analysis it became evident that external factors could not be the sole determinants of shocks in economic transformation. The missing part is the accumulation of preceding events, and the development of economic -- but also social, and political -- relationships over long cycles that may help to endogenize the external factors.

Subsequently, with popularization of Kondratieff's views, extensions to the original analysis, roughly following the 40-60 years rule, began to appear. Figure 1a illustrates an approximation of Kondratieff's original timeline of long wave cycles. Kondratieff’s original estimation was based on a commodity prices index for the U.S., England, and France in Kondratieff (1935). Subsequently, with popularization of Kondratieff's views, extensions to the original analysis, roughly following the 40-60 years rule, began to appear.

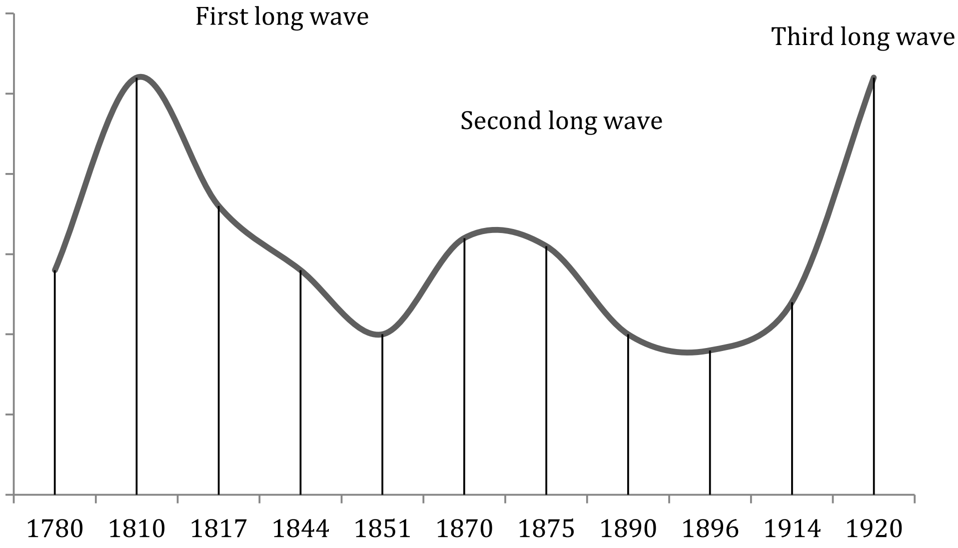

Figure 1a: Long Waves - Cycles Illustration[2]

One of the first to catch on the logic was Schumpeter (1939) who pointed out the distinction between short (3-4 years or Kitchin), medium (9-10 years Juglar)[3], and long (54-60 years Kondratieff) cycles in his analysis of economic development. We discuss some of this below.

As to mechanisms, Kondratieff already pointed to a large-scale accumulation of innovative activity, i.e. inventions and processes modifications that required fifty or more years before complete insertion, absorption in the production method. The role of innovation, implied in Kondratieff's work and the workings of those internal dynamic tendencies are described in detail in Schumpeter's Economic Development (1934). In turn, Garvy (1943) subjects Kondratieff's proposition to sharp criticism from positions of Soviet economists and from the point of view of western economics. Paradoxically, in either case the conclusion appears to be that Kondratieff was too hasty in assigning the term "cycle" to his propositions, as those do not correspond to the internal evolutionary dynamics following some mechanism of cycles.

There was a difference however in the Western economists' views and their contemporary Soviet counterparts. From the western economist point of view, articulated by Garvy (1943), there was no sufficient statistical evidence to warrant any regularity, i.e. cyclicality, to Kondratieff's analysis. The Soviet economists writing around the time of Kondratieff's original publications and shortly after (e.g. Studensky, Oparin, Pervushin, Bogdanov, Sukhanov and others, see Garvy, 1943 for concise discussion and references) rejected the term "cycle" in reference to the capitalist production mode since that implied some type of capitalist system's perpetuity. At the time that was in direct opposition with the socialist beliefs of gradual phasing out of the capitalist economy into its next logical stage of socialism, as was implied by then dominant interpretation of Marx's Capital (1867). These beliefs in rapid phased

[2] Source: authors' approximation based on Kondratieff (1935)
[3] See Juglar, 1862

successions picked up from simplistic interpretations would feed into initial enthusiasm around shock therapy reforms in post-socialist economies in the early 1990s (Gevorkyan, 2011).

Recently, researchers working within Kondratieff's original methodological scope have attempted to extend their analysis across the twentieth century with focus on predictive capabilities of such work into the nearest future. Some find the ongoing economic deterioration in the world economy fitting calculations of the Fifth Long Wave of the Kondratieff cycle (e.g. Korotaev and Tsirel, 2010; Kondratieff et al. 2002; Akaev, 2009; and others), some of them using spectral analysis. A re-validation of the very four exogenous shocks (technology, wars, shifts in boundaries, and value of gold) so carefully documented and refuted by Kondratieff himself took place in some of those papers. Exogenous shocks are surely important “occurrences”, yet, the internal dynamics in the evolution of economic relationships over a long time period and staging economic development must be considered as well. We address this in further detail below, using more modern empirical methods.

2.2. Kuznets' Novel Analysis of Development

Simon Kuznets received the Nobel Prize in Economics in 1971 for his empirical analysis of economic growth, where he identified a new era of “modern economic growth”. Like Kondratieff, Kuznets relied on empirical analysis and statistical data in his pioneering research. Absorbing his findings on historical development of the industrial nations with initially abstract categories of the national income decomposition, Kuznets developed a concept of long swings, though disputed, now referred to in literature as Kuznets cycles or Kuznets swings (e.g. Korotaev and Tsirlin, 2010).

The Kuznets swings' period is ranged between 15-25 years and initially connected by Kuznets with demographic cycles. In that analysis, the economist observed and quantified the cyclicality of production and prices, linking with immigrant population flows and construction cycles. Researchers have attempted to connect these cycles with investments in fixed capital or infrastructure investments (Korotaev and Tsirel, 2010 for literature review). Focusing on developed economies of North America and Western Europe, Kuznets computed national income from late 1860 forward with structural breakdowns by industry and final products. He also provided measures of income distribution between rich and poor population groups.

Kuznets unveiled the deficiency of constrained theoretical work built on simplified assumptions. He was critical of capital and labor as the sole factors sufficient for economic growth. Instead analysis must encompass information on technology, population and labor force skills, trade, markets, and government structure. Kuznets carried his analysis further in developing measures of national income through categories of consumption, savings, and investment (e.g. Kuznets, 1949, 1937, 1934, etc.), eventually leading to a system of national income accounting.

It should also be noted that working on the problems of income inequality, Kuznets was one of the first to look at economic growth measurements in the developing world and (e.g. Kuznets, 1971, 1966, 1955). His well-known inverted U-shaped curve measuring inequality on the y-axis and economic development, expressed as change in GNP on the x-axis was an intellectual breakthrough of the time (see Figure 1b). The conclusion is that while the economy remains in agricultural stage income inequality among different groups within the economy is low. As the national economy embarks on the process of industrialization inequality rises over time, then it falls again.

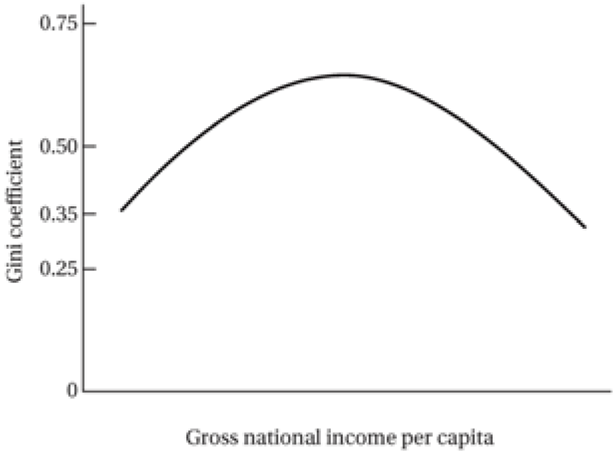

Figure 1b. Kuznets curve

This describes the experience of developed economies in Western Europe and North America, i.e. the initial phases of industrialization cause sharp rises in inequality. Upon reaching a critical saturation point, inequality subsides while economic growth continues. This happens through the emergence of a “middle class,” improved education facilities, health care, and governance. It is interesting to note that further structural change and the shifting of resources to services and the financial sector, may increase inequality again, as, for example, is seen in the US economy since the 1980s. It may be argued that this somewhat correlates to a popular analysis in development economics on the transition mechanisms from traditional to modern industrial sectors.

A variant of the Kuznets curve is also utilized in the analysis of environmental problems. This application suggests an immediate deterioration in air quality and intensification of environmental problems at the initial industrializing stages until spreading affluence and emergence of middle class introduces legislative and other controls on hazardous production (WB, 1992; Grossman and Krueger, 1995 and more recently Stern, 2004). Elsewhere, these implied predictions of fading inequality offered a strong intellectual foundation for the mentioned reforms of the early 1990s in Eastern Europe and former Soviet Union (Gevorkyan, 2011). There omitted in studies of sequencing of market liberalization reforms and limitations of the state in the economy were the negative externalities of shock therapy policies. Yet, in the early 1990s, the promise of immediate market reforms and potential access to greater income opportunities did not materialize at the height of the reforms. In fact, income inequality problems still remain relevant and critical on policymakers' agendas two decades since the “transition”. The absence of the universal tendency of declining income inequality raises a question of how one measures economic development and what time-frame to consider is "sufficient" to measure the rise of “welfare” over time.

Common between the work of both Kondratieff and Kuznets was the motivation to define the mechanisms of economic growth and development, and systematize core tendencies driving the transformational momentum. That in turn connects directly to the earlier discussion on cyclicality in development. And so, finally, Kuznets (1973) brings up six key characteristics of modern economic growth, based on methodology consistent with national income accounting and historical analysis of economic development: 1) increase in per capita growth and population in developed economies; 2) increasing productivity rates; 3) increasing rate of structural transformation; 4) rising urbanization and secularization; 5) spread of technology and infrastructure improvements (communications); 6) limits to wide-scale spread of economic growth and benefits. Therefore despite seeming improvements, Kuznets noted persistence of disproportionate economic growth worldwide and apparently some broader measures of welfare.

3. Time Scales and Mechanisms of Economic Cycles

As mentioned, the work of Kondratieff and Kuznets fostered a systematic approach to modern understanding of long economic swings. Numerous authors have further proposed not only different mechanisms underlying cycles but also cycles on different time scales. An early theory of cycles was put forward by Robert Owen in 1817, who stressed wealth inequality and poverty, originating in industrialization, yielding under-consumption as a reason for economic crises. Sismondi, in the middle of the 19th century took a similar view and developed a theory of periodic crises due to under-consumption. This led to the discussion of the "general glut" theory of the 19th century, which Marx and other classical economists also extensively contributed to.

More specifically, a mechanism of cycles on a shorter times scale, of 8-10 years duration, was developed by Juglar (Juglar cycles), resulting, as he saw it, from the waves in fixed investment. Later, Kitchin, in the 1920s, introduced an inventory cycle of 3-5 years. Later an important contribution was made by Schumpeter (1939), who referred to the "bunching" of innovations and their diffusion as a cause for long waves in economic activity.

Roughly at the same time, Samuelson (1939), influenced by the Spiethof accelerator and the Keynesian multiplier principle, developed the first mathematically-oriented cycle theory using difference equations[4]. Others, such as Rostow (1975), had proposed the theory of stages of growth. Simultaneous with Samuelson, Kalecki (1937) developed his theory of investment implementation cycles where he saw significant delays between investment decisions and investment implementations, formally introducing differential delay systems as tool for studying cycles.

Kaldor (1940), based on Keynesian theory, developed his famous nonlinear investment-saving cycles, which took into account aggregate demand. Later, Goodwin (1967) proposed a model of growth cycles, which took into account classical growth theory, but is actually based on an unemployment-wage share dynamics, since the growth rate as well as productivity growth is kept constant in the long run. We will first discuss cycle theories on a longer time scale and then move to the Goodwin and Keynes-Kaldor cycles. We also briefly include a discussion of Kalecki`s cycle (1971) theory and how it might relate to Kondratieff.

3.1 The Kondratieff Long Swings

The above review raises a few critical questions that need proper evaluation. For example, it is difficult to detect clear mechanisms in the Kondratieff cycles (e.g. as sketched in Figure 1a above). If anything is working here as a mechanism, it must be

[4] A review of the mechanisms of cycles on a shorter time scale is given in Semmler (1986).

some exhaustion of endogenous and exogenous factors: in the long upswing prices are rising, interest rates rise and wages rise, raw materials and non-renewable resources may be exhausted, causing to drive up prices and wages. New technologies are discovered in periods of long down swings which come to be used in a new upswing. New resources are discovered, such as iron ore, coal, gold and other metals, which Kondratieff argues to be endogenously expanded through new discoveries but both technology and resources will finally be exhausted too: resource and product prices rise, deposits at saving banks rise, but also interest rates and wages rise and a downturn begins. There is a struggle for markets and resources. New countries are opened up. There are market limits, such export limits, which restrict further expansions, as Kondratieff data on French exports show. Then, in the long downswing, prices fall, wages fall, interest rates fall, plenty of resources and unused production capacity push prices down, and unemployment reduces wages. Overall, there are some mechanisms indicated in Kondratieff, but not specifically modeled.

3.2. The Kuznets Long Swings

Further, Kuznets theory of development and fluctuations can be seen as an interesting intersection of two traditions in the economics of his time. On the one hand, he was interested in cyclical movements in numerous time series data, such as volume of all types of production and prices, seasonal and secular movements in industry income and national income and its components, and long swings in economic activities, and business cycle analysis. On the other hand, he saw development as a time irreversible process of industry and national income development, which evolves in stages of economic growth, with plenty of structural changes. Each stage may have its particular saving rate, consumption patterns, unevenness and disequilibrium as well as income inequality. As described above, inequality first rises with industrialization and later declines. Kuznets conceptual framework can be seen as a mixture of cycle theories, referring to the accelerator principle for infrastructure investments, and a theory of stages of economic growth that were similar to those pursued by Rostow (1975). A similar view on stages of growth, that taken by Kuznets and Rostow, is also pursued by Greiner, Semmler and Gong (2005). Overall, Kuznets was ambiguous whether there are regular mechanisms generating cycles. He conjectured that cycles may be in the economic data solely as a result of certain historical “occurrences”.

3.3. The Schumpeter Innovation Cycles

Schumpeter's concept of competition deviates from the neoclassical conception in some essential aspects: First, competition is not limited to price or quantity adjustments. It is described as an evolutionary process, as a process of «creative destruction». The engines of this development are capitalist enterprises: «Capitalism, then, is by nature a form or method of economic change and not only never is but never can be stationary .... The fundamental impulse that sets and keeps the capitalist engine in motion comes from the new consumer's goods, the new methods of production or transportation, the new markets, the new forms of industrial organization that capitalist enterprise creates» (Schumpeter, 1970, p. 83). The incentives for developing these types of technical change originate in transient surplus profits. What is taken as given in neoclassical general equilibrium analysis as parametric data, when the price and quantity adjustments occur is the explicandum in Schumpeter: process innovation, product innovation, new forms of organization of the firm and new forms of financial control.

Second, Schumpeter stresses that competition is not necessarily an equilibrating force. When referring to the existence of entrepreneurial firms and their rivalry, Schumpeter maintains that «there is in fact no determinate equilibrium at all and the possibility presents itself that there may be an endless sequence of moves and counter-moves, an indefinite state of warfare between firms» (Schumpeter, 1970, p. .79). Moreover, competition as an evolutionary process takes place through time, in discrete steps. For example, he writes: «Now the first thing we discover in working out the propositions that thus relate quantities belonging to different points in time is the fact that, once equilibrium has been destroyed by some disturbances, the process of establishing a new one is not so sure and prompt and economical as the old theory of perfect competition made it out to be, and the possibility that the very struggle for adjustment might lead such system farther away instead of nearer to a new equilibrium. This will happen in most cases unless the disturbance is small» (Schumpeter, 1970, p. 103). Indeed, in Schumpeter it is the product and process innovation, undertaken by the entrepreneur, which brings the economic system out of equilibrium, resulting in long waves and business cycles. Moreover, he even does not seem to be very interested in a theory of centers of gravitation for market forces as developed by the classical economists.

Third, in Schumpeter, competition is an evolutionary process, one of rivalry between firms motivated by the search for surplus profit. He calls this surplus profit the transient "monopoly profit" of new processes and new products: "Thus, it is true that there is or may be an element of genuine monopoly gain in those entrepreneurial profits which are the prizes offered by capitalist society to the successful innovator. But the quantitative importance of that element, its volatile nature and its function in the process in which it emerges put it in a class by itself" (Schumpeter, 1970, p. 102). "The transient surplus profit does not appear as deviation from the perfectly competitive state of the economy and as a waste in the allocation of resources, but as a reward for the innovator and a gain for the capitalist society. On the contrary, the perfectly competitive economy, where every market agent behaves in the same way under the condition of parametrically given external conditions seems to imply a waste of resources ... working in the conditions of capitalist evolution, the perfect competitive arrangement displays wastes of its own. The firm of the type that is compatible with perfect competition is in many cases inferior in internal, especially technological, efficiency. If it is, then it wastes opportunities" (Schumpeter, 1970, p. 106). Thus, in Schumpeter's view, the entrepreneurial firms are powerful engines of progress and "in particular of the long-run expansion of total output" (p. 106).

Following Schumpeter`s footsteps, the literature after Schumpeter has distinguished between radical and incremental innovation. The major waves of radical innovations, which where followed by the diffusion of this new technology and incremental innovations where[5]:

- The water-powered mechanization of the industry of the 18th and early 19th century.
- The steam-powered mechanization of the industry and transport of the middle of the 19th century (rail ways, steam engines, machine tools)
- The electrification of industry, transport and homes at the end of the 19th century

[5] For details see Reati and Toporowski (2004)

- Motorization of industrial production, transport, civil economy and the war machinery (from ~1914 onward)
- Computerization and information technology from the 1960s and 1970s onward.

According to Schumpeter oriented long wave theories, each of those radical innovations did not only create long waves in economic development, but each of those long waves were driven by different technology, originated in different countries and then diffused world wide.

3.4. The Samuelson Accelerator-Multiplier Cycles

A model of the medium time scale is the one by Samuelson (1939). It is a model of the interaction of the accelerator - multiplier model. The basic construction is as follows. Sales accelerates investment and output change results in income changes, through the multiplier, which again stimulates sales.

The multiplier-accelerator model of Samuelson (1939) can produce cycles. Take C_t= consumption, I_t= investment, Y_t= income, C_0= autonomous consumption, I_0= autonomous investment, and $= C_0 + I_0$; $I = S = sY$, therefore the multiplier is: $Y = \frac{1}{s}$. Use:

$$C_t = C_0 + \alpha Y_{t-1} \tag{1}$$
$$I_t = I_0 + \beta(Y_{t-1} - Y_{t-1}) \tag{2}$$
$$Y_t = C_t + I_t \tag{3}$$

substituting (1) and (2) into (3) gives

$$Y_t = C_0 + \alpha Y_{t-1} + I_0 + \beta Y_{t-1} - \beta Y_{t-2}$$
$$G = C_0 + I_0$$
$$Y_t = G + (\alpha + \beta)Y_{t-1} - \beta Y_{t-2}$$

The standard form of a second order linear difference equation is:

$$Y_t - (\alpha - \beta)Y_{t-1} + \beta Y_{t-2} = G \tag{4}$$

which is stable or unstable depending on the size of β . Moreover, one can have contracting or expanding cycles depending on whether there imaginary parts of the eigenvalues, see figures 2a-2d.

When we replace income by profit flows R_t one can turn the above into a kind of Kalecki model such as:

$I_{t+1} = A + \alpha R_t + b(R_t - R_{t-1})$; If one writes for $s_p R_t = I_t$, $R_t = \frac{I_t}{s_p}$ we get a similar second order difference equation:

$$I_{t+1} = A + \frac{(a+b)}{s_p} I_t + \frac{b}{s_p} I_{t-1} \tag{5}$$

which again can be stable or unstable and it can produce unidirectional change or oscillations. The Kalecki model is further studied in sub-section 3.7.

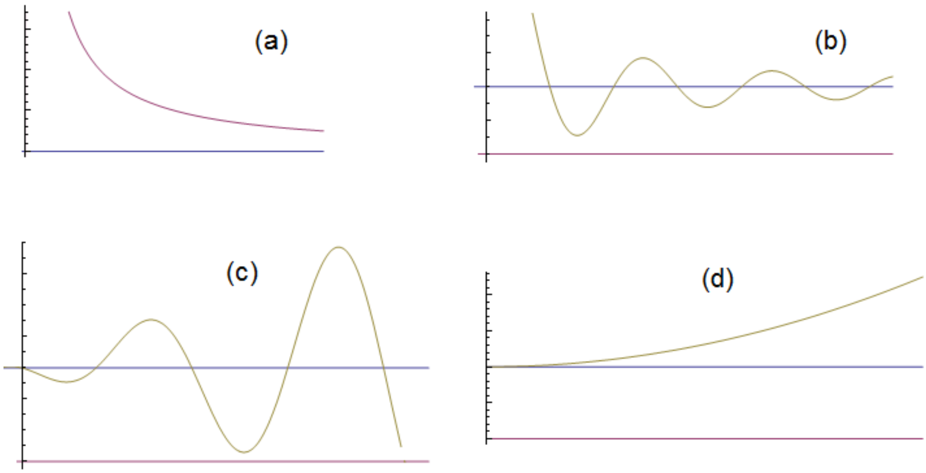

Figure 2a-2d: Stable and unstable development and oscillations

3.5. The Goodwin Growth and Income Distribution Cycles

Other types of cycles that have been discussed, particularly in the Post War II period, where Goodwin`s growth cycle theory that postulates an interaction of employment and wage share. It looked like a business cycle model when it was first proposed but, in fact, empirically it seems to operate also on a medium time scale[6].

Goodwin (1967) postulates cycles driven by growth and income distribution. Low growth, generated by low profits and investment, generates unemployment, which in turn limits wage growth as compared to productivity. This gives rise to lowering the wage share: low wage share means high profit share and slowly rising investment, which reaches a turning point as employment and wage growth make the wage share rising and the profit share falling. By utilizing nonlinear differential equations, originally developed by Lotka and Volterra for models of interacting populations, we can rewrite the Goodwin model of wage-employment dynamics as follows.

$$\dot{x} = P(x, y) = (a - by)x$$
$$\dot{y} = Q(x, y) = (cx - d)y$$

or as

$$\frac{\dot{x}}{x} = a - by$$

$$\frac{\dot{y}}{y} = cx - d$$

where $\dot{x}$ represents the time rate of change of the ratio of the employed to the total labor force and $\dot{y}$ is the change of the wage share. Both variables depend on the level of x and the constants $a, b, c, d > 0$. The coefficient a denotes the trend of

[6] For details of the subsequent dynamic modeling, see Semmler (1986)

employment if all income is reinvested $(y = 0)$ and *d* is the fall in real wage if $(x = 0)$. The symbol *by* denotes the influence of the wage share on the employment ratio, and *cx* the positive influence of employment on the wage share. Due to this interaction of the variables the employment ratio is prevented from rising and the wage share from falling without limits.

For a growth model with trends as represented by Goodwin, the coefficients can be interpreted as follows: $a = b - (m + n)$ where b is the output/capital ratio *(Y/K)*, *m* is the growth rate of productivity and *n* is the growth rate of the labor force. All of those are taken as constants. Assuming a linearized wage function (for instance, $\frac{\dot{w}}{w} = -e + cx$) and with *m* the growth rate of productivity as before, we obtain for the growth rate of the wage share the term $\frac{\dot{y}}{y} = \frac{\dot{w}}{w} - m$, with *d*= *e*+*m*. Thus the second pair of differential equations can be written as

$$\frac{\dot{x}}{x} = b(1 - y) - (m + n)$$

$$\frac{\dot{y}}{y} = cx - (e + m)$$

which is indeed equivalent to the first equation of the (above) system, except that it is written in terms of growth rates. The core of the last system shows that the change of the employment ratio depends on the profit share *(1-y)* and that the change of the wage share depends on the employment ratio. This form has been used to explain the fluctuation of the employment ratio and the fluctuation of the industrial reserve army in Marx (Marx, 1867, vol. I, ch. 23; see Goodwin, 1967). The basic structure of this model represents the interacting variables of the employment ratio and wage share as dynamically connected.

The last system has two equilibria: *(0,0)* and $(\frac{d}{c}, \frac{a}{b})$. The linear approximation of the system is with ξ_1, ξ_2 as small deviations from the equilibrium values

$$\begin{pmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{pmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix}$$

The calculation of the Jacobian for the first linear approximation gives for the equilibrium $(\frac{d}{c}, \frac{a}{b})$

$$J = \begin{bmatrix} 0 & -bd/c \\ ca/b & 0 \end{bmatrix}$$

The real parts of the eigenvalues are zero and the linear approximation of the equilibrium point represents the dynamical structure of a center (Hirsch and Smale, 1974, p. 258). With real parts of the eigenvalues zero, the linear approximation of the system through the Jacobian does not allow conclusions regarding the behavior of the dynamical system in the neighborhood of the equilibrium. Yet, as can be shown, by constructing a Liapunov function for the above system, which is constant in motion and hence has time derivatives $\dot{V} = 0$, the wage share-employment dynamics results in closed solution curves (Hirsch and Smale, 1974, p. 258 and Flaschel and Semmler, 1987).

The closed trajectories of the system are, however, only closed curves and the wage share-employment dynamics does not allow for persistent cycles, such as limit cycles (Hirsch and Smale, 1974, p. 262; Flaschel, 1984). In addition (see Flaschel and

Semmler 1987), the dynamical system is structurally unstable, since small perturbations can lead to additional interaction of the variables J_{11} or J_{22}can become nonzero). This leads to a qualitatively different dynamical behavior of the system, hence it can become totally stable or unstable. Under certain conditions the above system can also become globally asymptotically stable. This can occur if the conditions for Olech's theorem are fulfilled (see Flaschel, 1984).

Equivalent results are obtained when in place of a linear wage function a nonlinear wage function is substituted in the system (see Velupillai, 1979). The wage share-employment dynamics worked out originally by Goodwin for a model of cyclical growth and then applied by him to explain an endogenously created unemployment of labor depict a growing economy, whereas often models of nonlinear oscillations refer only to a stationary economy.

Since the change of the wage share and the change of labor market institutions such as bargaining and other protective legislature are slow, this model of economic cycles, however, does not really model business cycles but rather medium run cycles. On the other hand for a theory of longer cycles the dynamical interaction of other important variables over time (such as waves of innovations, changes of capital/output ratio, productivity, relative prices and interest rates) as well as demand factors are neglected.

3.6. The Keynes-Kaldor Demand Driven Cycles

The demand factors are considered in the next section presented here. The Keynes - Kaldor model seems to operate on a shorter time scale. It essentially refers to the role of demand, defined by the relation of investment and savings. In his 1940 article, Kaldor proposed such a shorter scale cycle model, a nonlinear model of business cycles, which after that has been reformulated in the light of mathematical advances in the theory of nonlinear oscillations which take into account demand changes (Kaldor, 1940, 1971; Chang and Smyth, 1971; Semmler, 1986).

Kaldor relies on a geometric presentation of a business cycle model which depends on a nonlinear relation between income changes and capital stock changes and which seems to generate self-sustained cycles without rigid specifications for the coefficients, time lags and initial shocks. The geometric presentation of his model of persistent business cycles due to the dynamic interaction between income changes and accumulation and dissolution of capital, indeed also includes the possibility of limit cycles, i.e. asymptotically stable cycles regardless of the initial shocks and time lags.

His ideas are also formulated for a stationary economic system and can be represented by nonlinear differential equations in the following way (Chang and Smyth, 1971):

$$\dot{Y} = \alpha(I(Y, K) - S(Y, K)$$

$$\dot{K} = I(Y, K)$$

where α is a reaction coefficient, $\dot{Y}$ the rate of change of income, $\dot{K}$ the rate of change of the capital stock, I =investment and S =saving as functions of the level of income and capital stock. According to the assumptions underlying the model, there is a unique singular point (Chang and Smyth, 1971, p. 40). This type of Keynesian-Kaldorian model can give rise to persistent cycles, see Semmler (1986), it does not model the specific role of growth and income distribution, as Goodwin has stressed. Yet it stresses the role of endogenously changing demand. The linear approximation is:

$$\begin{pmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{pmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix}$$

where the Jacobian is

$$\frac{\partial(\dot{Y}, \dot{K})}{\partial(Y, K)} = \begin{bmatrix} \alpha(I_Y - S_Y) & \alpha(I_K - S_k) \\ I_Y & I_K \end{bmatrix}$$

where $(I_K - S_k) > 0$, since $I_K < S_K < 0$ and $I_Y > 0$ (Chang and Smyth, 1971, p. 41). The determinant is $(I_Y S_k - S_Y I_K)$, which is positive because for the existence of a unique singular point it is assumed that $(I_Y S_k < S_Y I_K)$. The element, $J_{22} = I_K$, is always negative. The linear approximation with the Jacobian represents at its core the investment-income dynamics according to which the change of income depends negatively on the level of the capital stock J_{12} and the change of capital stock depends positively on the level of income J_{21} , but there is a negative feedback effect from the level of capital stock to the change of capital stock and an ambiguous feedback effect from the level of income to the change of income J_{11}. This will be explained subsequently.

Analyzing the singular point one can conclude that the equilibrium is a focus or a node and that the equilibrium is stable or unstable accordingly as $\alpha(I_Y - S_Y) + I_K > (<) 0$. This singular point also allows for a limit cycle, since the necessary condition for a limit cycle is that the dynamic system has an index of a closed orbit which is 1 (Minorsky, 1962, p. 79). This excludes a saddle point as a singular point (see Minorsky, 1962, p. 77). Moreover, the most interesting point in this dynamic system is the ambiguous element J_{11}. According to Kaldor's graphical presentation, it is assumed (see Kaldor, 1940, p. 184) that

(1) $I_Y > S_Y$ for a normal level of income;
(2) $I_Y < S_Y$ for abnormal high or abnormal low levels of income; and
(3) the stationary state equilibrium has a normal level of income.

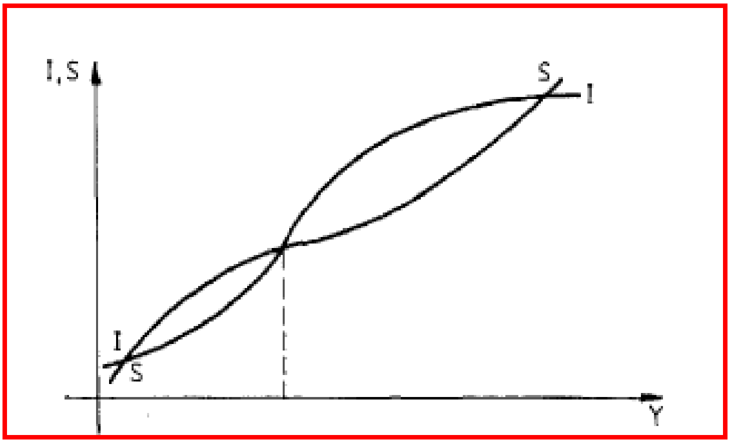

Figure 3: Kaldor graph on nonlinear investment and saving functions.

This might be illustrated by Figure 3 with Y the level of output which shows that the normal level of Y is unstable and the extreme values of Y are stable. Mathematically this means that the trace $J_{11} + J_{22}$ changes signs during cycles. This is the negative criterion of Bendixson (Minorsky, 1962, p. 82) for limit cycles, i.e. if the trace $J_{11} + J_{22}$ does not change signs, persistent cycles -- limit cycles -- cannot exist (see also Guckenheimer and Holms, 1983, p. 44). As proven by Chang and Smyth (1971, section V) there indeed exists the possibility of stable cycles, limit cycles, under the assumption proposed by Kaldor.

However, the three conditions as formulated above and originally formulated by Kaldor (1940, p. 1984) are not necessary for the existence of cycles. What is actually necessary for cycles is only that $I_Y > S_Y$ (i.e. that J_{11} switches signs) at some level of Y . Moreover, the singular point at the normal level of Y does not have to be unstable as a necessary condition for a limit cycle. The critical point can be stable (see Minorsky, 1962, p. 75). In addition there also is the possibility that the system is globally asymptotically stable. This is the case if:

$\alpha(I_Y - S_Y) + I_K < 0$ and (2) $(S_Y I_K < I_Y S_k)$ everywhere.

The global asymptotic stability under these conditions follows from Olech's theorem (see Ito, 1978, p. 312).

Evaluating Keynes - Kaldor's model of a demand driven business cycles one can say that Kaldor's formulation of an income-investment dynamics brought some advances regarding a theory of endogenously produced business cycles, especially formulations of the theory of cycles in terms of a theory of nonlinear oscillations (see also Kaldor, 1971) one can extend this to include a formulation concerning the dynamics in employment and wage share which was originally more visible in classical models that referred to the profit-investment dynamics.

3.7 The Kalecki Profit and Investment Cycles

To draw some similarities to the Kondratieff long wave theory, we can follow Kalecki (1971) and replace the income, Y , by profit flows Π[7] and allow for $J_{11} = \alpha(I_\Pi - S_\Pi)$ to change signs during the cycle. In some sense the role of profit, wages and income distribution -- as in the Goodwin model-- can be allowed to come in here.

In general it could be assumed that:

(1) $\frac{\partial I}{\partial \Pi} > \frac{\partial S}{\partial \Pi}$, for profit income in an interval such as $\Pi_1 < \Pi < \Pi_2$(see Figure 4). This may be due to a previous decrease in capital stock, production and employment which entail low construction cost for plants, low material and resource cost and low wage costs (relative to productivity), high profits and low interest rates and easy access to credit. These factors then may give rise to an expectation of rising profits on investments.

On the other hand in other regions we can have:

(2) $\frac{\partial I}{\partial \Pi} < \frac{\partial S}{\partial \Pi}$, with two clarifying conditions:

(a) for $\Pi > \Pi_2$ due to capacity limits, rising construction cost for plants and rising material and wage cost (relative to productivity), exhaustion of exhaustible

[7] This conversion seems permissible as long as there are no savings out of workers income and thus workers income is completely spend for consumption. This is what Kalecki assumes.

resources, rising interest rates and but falling actual and expected profits. Profits and expected profits may fall due to the rise of those costs and wages -- that cannot be passed on-- in the long upswing. This looks similar to a mechanism that Kondratieff has indicated to eventually occur in his long upswing (see sections 2.1 and 3.1).

(b) for $\Pi < \Pi_1$ in a recessionary or slow recovery period, where firms invest in financial funds instead of in real capital (Minsky, 1983) but due to the economic conditions in a recessionary period, the rate of change of saving in response to falling profits tend to drop faster than the rate of change of investment. Wage share may have been rising previously, and profit share falling but here investment is still not dropping completely to zero. This resembles the Kondratieff scenario of a long downswing and recessionary or stagnation period.

Though the economic intuition appears the same in our above stylized business cycle dynamics and the Kondratieff long waves phases, the time scales are are probably different ones: one is a shorter one and the other a longer one, but the mechanisms may be the same. Yet, for a longer time scale much of the economic structure and relationships are likely to change.

In the history of economic thought the change of sign for J_{11} during the economic cycle was verbally anticipated by many writers on the study of capitalist dynamics (Kalecki, 1971, p. 123; Kaldor, 1940, p. 184) and can be regarded as an essential for a theory of fluctuations in economic development. Mathematically $J_{11} + J_{22}$ must change signs in order to generate self-sustained cycles. If J_{11} and J_{22} were zero, J_{12} and J_{21} alone would determine the profit-investment dynamics. There would only be structurally unstable harmonic oscillations. The negative signs of J_{12} and J_{22} exert a retarding influence on accumulation, and J_{21} represents an accelerating force on capital accumulation, whereas J_{11} exerts a retarding influence in the boom period and an accelerating impact on profit and accumulation in the later phase of the recession.

Intuitively, the existence of self-sustained cycles can be seen in figure 4 from the fact that the trajectories of $\Pi(t)$ and $K(t)$ are bounded in absolute values and the profit-investment dynamics follow certain directions in the plane. Roughly speaking, for large enough $\Pi(t)$, $\dot{\Pi}$ turns negative and for large enough $K(t)$, $\dot{K}$ turns negative and vice versa. Geometrically, this is illustrated by figure 4.

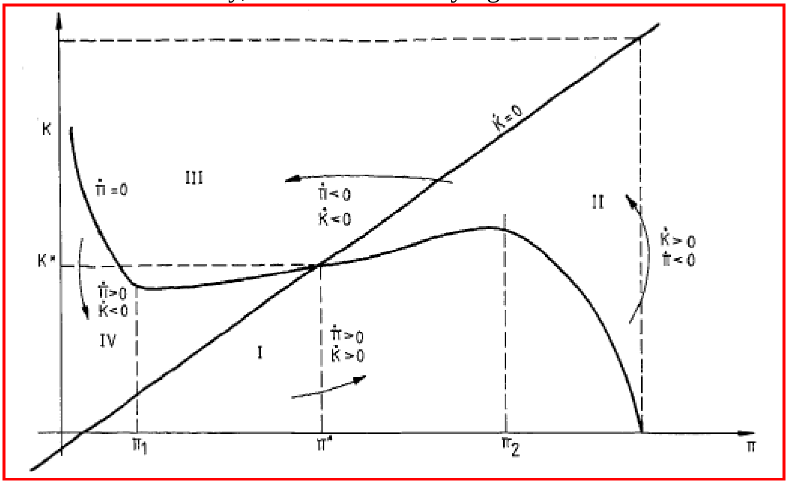

Figure 4: Phase Diagram

For $\dot{\Pi} = 0$ we get the slope

$$\frac{dK}{d\Pi} = \frac{S_\Pi - I_\Pi}{I_K - S_K} \lesseqgtr 0$$

and for $\dot{K} = 0$ the slope is .

$$\frac{dK}{d\Pi} = -\frac{I_\Pi}{I_K} > 0$$

Thus in the plane of the Figure 4 there are four quadrants. For reasons of simplicity we have assumed a linear investment function in Figure 4. The system has a unique solution at $\Pi*$ and $K*$ since the curve $\dot{K} = 0$ has a steeper slope than $\dot{\Pi} = 0$ when the latter is upward sloping in a certain region. This follows from the assumption in the model[8]. The determinant of the Jacobian of the dynamical system above is $\alpha(S_K I_\Pi - S_\Pi I_K) > 0$. The singular point is a focus or a node and is stable or unstable accordingly as $\alpha(I_\Pi - S_\Pi) + I_K \lesseqgtr 0$. A saddle is excluded, and the singular point has index 1 as necessary condition for a self-sustained cycle (Minorsky, 1962, p. 176). (The singular point does not have to be unstable as Kaldor originally assumed, Kaldor, 1940, p. 182.) The existence of a self-sustained cycle follows intuitively from the analysis of the vector fields in the different regions which correspond roughly to stages of economic cycles.[9]

For region I, which expresses the dynamics of a recovery period, $K(t)$ is below the $\dot{K} = 0$ curve and $\Pi(t)$ is below the $\dot{\Pi} = 0$ curve; the decline in capital stock and its effect on profit (i.e. the effects of cases (1) and (2) as well as other changes in economic conditions in a recessionary period will generate a positive rate of change of profit (since $I_\Pi > S_\Pi$ in region I, see also condition 1). Therefore, in region I we will find $\dot{\Pi} > 0$ and $\dot{K} > 0$.

The increase of profits and investments after a recessionary period will lead to rising $K(t)$, but through the effect of cases (1) and (2) (i.e. the negative effect of growth of capital stock on profits) the growth rate of II will become negative. Thus in region II, indicating a boom period, we have $\dot{\Pi} < 0$ and $\dot{K} > 0$. Hence the arrows in Figure 4, indicating the direction of the vector field of Π and K , will start bending inward (see condition (2)(a) which leads to $I_\Pi < S_\Pi$). With capital stock rising and $\dot{\Pi} < 0$ due to a magnitude of capital stock greater than its stationary value K^*, the capital stock must eventually decline (i.e. through the effect of case (2). We also have $\dot{\Pi} < 0$ due to $I_\Pi < S_\Pi$ at the beginning of a downswing period (capital may be accumulated more as money capital than as real capital).

In region III, indicating a downswing period, through the influence of $\dot{\Pi} < 0$ on *K(t)* , *K(t)* also starts declining; thus$\dot{\Pi} < 0$ and $\dot{K} < 0$. Hence for $\Pi(t) < \Pi^*$ and $K(t) < K^*$ the vector field is pointing inward. A decline of capital stock below K^* in region IV the recessionary period, however, causes profits eventually to rise. The recessionary period may slowly then turn into a recovery period, indicated by region I. This, of course, assumes again that eventually $\dot{\Pi} > 0$. The investment of financial

[8] The curve $\dot{\Pi} = 0$ is downward or upward sloping when $S_\Pi > I_\Pi$ (or $S_\Pi < I_\Pi$). By assuming that for a certain region $\Pi_1 < \Pi^* < \Pi_2$, $\dot{\Pi} = 0$ is upward sloping and $\dot{K} = 0$ also has a positive but steeper slope, it follows that there is only one unique equilibrium point. For similar assumptions concerning an income/ investment model, see Chang and Smyth (1971, p. 40).
[9] A proof using the Poincare-Bendixson theorem is given in Semmler (1986)

funds turns into investment in real capital, thus investment out of profit tends to become greater than savings out of profit. The recessionary period (with wage increase below productivity, low material and capital cost, low interest rates and easy access to credit as well as a decline in capital stock and thus rising profit expectation[10] must have its impact on $\dot{\Pi}$, for otherwise the recessionary period will endure.

Therefore under the economic conditions stated in conditions (1), (2)(a) and (2)(b) the profit-investment dynamics creates its own cycles by which profit, investment and thus output and employment cannot exceed certain boundaries. The dynamic system is self-correcting and fluctuates within limits: for large enough $K(t)$ is $\dot{K} < 0$ and for large enough $\Pi(t)$ is $\dot{\Pi} < 0$. A similar argument holds for small enough $K(t)$ and $\Pi(t)$. Thus, whereas the system with cases (1) and (2) becomes stable at its outer boundaries (indicated by the negative sign of $J_{11} + J_{22}$), it cannot converge towards equilibrium, since the equilibrium is unstable (indicated by the positive sign of $J_{11} + J_{22}$). Therefore, the dynamics of the system will result in cycles, see Semmler (1986). These self-sustained cycles, resulting from the profit-investment dynamics, can be regarded as close to classical dynamics and conceptions and the original Kalecki model and reflects to a certain extent also the dynamics of output, income, resource cost, price level, wage and bank deposit and interest rate dynamics of the Kondratieff long wave theory. Though for such a cycle on long time scale many structural changes may occur that could significantly change the mechanisms and economic relationship over the cycle.

4. The Minsky financially driven basic cycle and super-cycle

Next we discuss a Minsky long cycle: a financially-based approach to long wave theory. Long cycles have historically been interpreted as an interaction of real forces with cost and prices. Kondratieff cycles emphasize secular changes in production and prices; Kuznets cycles are associated with economic development and infrastructure accumulation; Schumpeterian cycles are the result of waves of technological innovation; while Goodwin cycles are based on changes in the functional distribution of income arising from changed bargaining power conditions in period of high growth rates and Keynesian theories express demand factors.

The work of Hyman Minsky provides an explicitly financially driven theory of business cycles. Minsky’s own writings were largely devoted to exposition of a short-run cycle and a very long-run analysis of stages of development of capitalism. The short-run analysis is illustrated in two articles (Minsky, 1959a, 1959b) that present a financially driven model of the business cycle based on the multiplier-accelerator mechanism with floors and ceilings. A later formalization is Delli Gatti et al. (1994) in which the underlying dynamic mechanism is increasing leveraging of profit flows, which roughly captures Minsky’s (1992a) hedge-speculative-Ponzi finance transition dynamic that is at the heart of his famous financial instability hypothesis. The very long-run analysis of stages of development of capitalism is illustrated in Minsky’s (1992b) essay on “Schumpeter and Finance“. That stages of development perspective has been further elaborated by Whalen (1999) and Wray (2009).

[10] A very important factor for the change of signs in J_{11} for a monetary economy seems to be the financial condition of firms and the banking system (see Minsky, 1983)

Recently, Palley (2010, 2011) has argued Minsky's (1992a) financial instability hypothesis also involves a theory of long cycles. This long cycle explains why financial capitalism is prone to periodic crises and it provides a financially grounded approach to understanding long wave economics.

A long cycles perspective provides a middle ground between short cycle analysis and stages of development analysis. Such a perspective was substantially developed by Minsky in a paper co-authored with Piero Ferri (Ferri and Minsky, 1992). However, unfortunately, Minsky entirely omitted it in his essay (Minsky, 1992a) summarizing his financial instability hypothesis, leaving the relation between the short and long cycle undeveloped.

Minsky's financial instability hypothesis maintains that capitalist financial systems have an inbuilt proclivity to financial instability that tends to emerge in periods of economic tranquility. The dynamic behind this proclivity can be summarized in the aphorism "success breeds excess breeds failure". Minsky's framework is one of evolutionary instability and it can be thought of as resting on two different cyclical processes (Palley, 2010, 2011). The first is a short cycle and can be labeled the "Minsky basic cycle". The second is a long cycle can be labeled the "Minsky super cycle".

The Minsky basic cycle has been the dominant focus of interest among those (mostly Post Keynesians) who have sought to incorporate Minsky's ideas into macroeconomics and it provides an explanation of the standard business cycle. The basic cycle is driven by evolving patterns of financing arrangements and it captures the phenomenon of emerging financial fragility in business and household balance sheets. The cycle (see Table 1) begins with "hedge finance" when borrowers' expected revenues are sufficient to repay interest and loan principal. It then passes on to "speculative finance" when revenues only cover interest, and the cycle ends with "Ponzi finance" when borrowers' revenues are insufficient to cover interest payments and they rely on capital gains to meet their obligations.

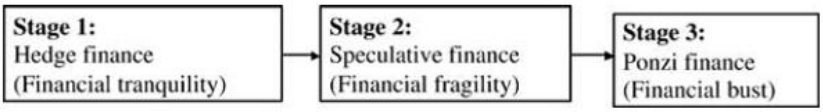

Table 1: Minsky financing practices

The Minsky basic cycle embodies a psychologically based theory of the business cycle. Agents become progressively more optimistic in tranquil periods, which manifests itself in increasingly optimistic valuations of assets and associated actual and expected revenue streams, and willingness to take on increasing risk in belief that the good times are here forever. This optimistic psychology affects credit volume via the behavior of both borrowers and lenders - not just one side of the market. That is critical because it means market discipline becomes progressively diminished. Leveraging is increased but the usual text book scenario of corporate finance, whereby higher leverage results in higher risk premia, is not visible in the cost of credit. Instead, credit remains cheap and plentiful because of these psychological developments.

Below, in our empirical analysis in section 5.4, this credit dynamic is illustrated for the recent long financial cycle starting in the 1990s. Initially, it was a real cycle driven by information technology (IT). This IT bubble burst around 2000/2001. However, expansion resumed owing to Minsky's financial cycle of overoptimism, high leverage, underestimation of risk, and expansion of new financial practices. The data show a high degree of leveraging during this period, an optimistic view of profit expectations, low risk premia, low credit spreads, and few credit constraints. Thus, contrary to corporate finance textbooks, the market generated high leveraging with low risk premia.

This process of increasing optimism, rising credit expansion and low risk perception is evident in the tendency of business cycle expansions to foster talk about the "death of the business cycle". In the U.S. the 1990s saw talk of a "new economy" which was supposed to have killed the business cycle by inaugurating a period of permanently accelerated productivity growth. That was followed in the 2000s by talk of the "Great Moderation" which claimed central banks had tamed the business cycle through improved monetary policy based on improved theoretical understanding of the economy. Such talk provides prima facie evidence of the operation of the basic Minsky cycle.

Moreover, not only does the increasing optimism driving the basic cycle afflict borrowers and lenders, it also afflicts regulators and policymakers. That means market discipline is weakened both internally (weakened lender discipline) and externally (weakened regulator and policymaker discipline). For instance, Federal Reserve Chairman Ben Bernanke (2004) openly declared himself a believer in the Great Moderation hypothesis.

The Minsky basic cycle is present every business cycle and explains the observed tendency toward increased leverage and increased balance sheet fragility over the course of standard business cycles. However, it is complemented by the Minsky super cycle, that works over a longer time scale of several business cycles. This long-cycle rests on a process that transforms business institutions, decision-making conventions, and the structures of market governance including regulation. Minsky (Ferri and Minsky, 1992) labeled these structures "thwarting institutions" because they are critical to holding at bay the intrinsic instability of capitalist economies. The process of erosion and transformation of thwarting institutions takes several basic cycles, creating a long phase cycle relative to the basic cycle.

The basic cycle and long-cycle operate simultaneously so that the process of institutional erosion and transformation continues during each basic cycle. However, the economy only undergoes a full-blown financial crisis that threatens its survivability when the long-cycle has had time to erode the economy's thwarting institutions. This explains why full scale financial crises are relatively rare. In between these crises the economy experiences more limited financial boom - bust cycles. Once the economy has a full scale crisis it enters a period of renewal of thwarting institutions during when new laws, regulations, and governing institutions are established. That happened in the Great Depression of the 1930s and it is happening again following the financial crisis of 2008.

Analytically, the Minsky long-cycle, can be thought of as allowing more and more financial risk into the system via the twin developments of "regulatory relaxation" and "increased risk taking". These developments increase both the supply of and demand for risk.

The process of regulatory relaxation has three dimensions. One dimension is regulatory capture whereby the institutions intended to regulate and reduce excessive risk-taking are captured and weakened. Over the past twenty-five years, this process has been evident in Wall Street's stepped up lobbying efforts and the establishment of a revolving door between Wall Street and regulatory agencies such as the Securities and Exchange Commission, the Federal Reserve, and the Treasury Department. A second dimension is regulatory relapse. Regulators are members of and participants in society, and like investors they are also subject to memory loss and reinterpretation of history. Consequently, they too forget the lessons of the past and buy into rhetoric regarding the death of the business cycle. The result is willingness to weaken regulation on grounds that things are changed and regulation is no longer needed. These actions are supported by ideological developments that justify such actions. That is where economists have been influential through their theories about the "Great Moderation" and the viability of self-regulation. A third dimension is regulatory escape whereby the supply of risk is increased through financial innovation. Thus, innovation generates new financial products and practices that escape the regulatory net because they did not exist when current regulations were written and are therefore not covered.

The processes of regulatory capture, regulatory relaxation, and regulatory escape are accompanied by increased risk taking by borrowers. First, financial innovation provides new products that allow investors to take more risky financial positions and borrowers to borrow more. Recent examples of this include home equity loans and mortgages that are structured with initial low "teaser" interest rates that later jump to a higher rate. Second, market participants are also subject to gradual memory loss that increases their willingness to take on risk. Thus, the passage of time contributes to forgetting of earlier financial crisis, which fosters new willingness to take on risk, The 1930s generation were cautious about buying stock in light of the experiences of the financial crash of 1929 and the Great Depression, but baby boomers became keen stock investors. The Depression generation's reluctance to buy stock explains the emergence of the equity premium, while the baby boomer's love affair with stocks explains its gradual disappearance.

Changing taste for risk is also evident in cultural developments. One example of this is the development of the "greed is good" culture epitomized by the fictional character Gordon Gecko in the movie Wall Street. Other examples are the emergence of investing as a form of entertainment and changed attitudes toward home ownership. Thus, home ownership became seen as an investment opportunity as much as providing a place to live.

Importantly, these developments concerning attitudes to risk and memory loss also affect all sides of the market so that market discipline becomes an ineffective protection against excessive risk-taking. Borrowers, lenders, and regulators go into the crisis arm-in-arm. Lastly, there can also be an international dimension to the Minsky long cycle. That is because ideas and attitudes easily travel across borders. For instance, the period 1980 – 2008 was a period that was dominated intellectually by market fundamentalism which promoted deregulation on a global basis. University economics departments and business schools pedaled a common economic philosophy that was adopted by business participants and regulators worldwide. Organizations like the International Monetary Fund and World Bank also pushed these ideas. As a result, developments associated with the Minsky long cycle operated on a global basis giving rise to common financial trends across countries that multiplied the overall effect.

The twin cycle explanation of Minsky's financial instability hypothesis incorporates institutional change, evolutionary dynamics, and the forces of human self-interest and fallibility. Empirically, it appears to comport well with developments between 1981 and 2008. During this period there were three basic cycles (1981 – 1990, 1991 – 2001, and 2002 – 2008). Each of those cycles was marked by developments that had borrowers and lenders taking on increasingly more financial risk in a manner consistent with Minsky's "hedge to speculative to Ponzi" finance dynamic. The period as a whole was marked by erosion of thwarting institutions via continuous financial innovation, financial deregulation, regulatory capture, and changed investor attitudes to risk, all of which is consistent with the idea of a Minsky long cycle.

The Minsky long cycle enriches long wave theory. In addition to adding financial factors, the Minsky cycle has different implications for the pattern of long waves compared to conventional long wave theory. Conventional theories see a separate long wave on top of which are imposed shorter waves. In contrast, the Minsky long cycle operates over a long time scale to gradually and persistently change the character of the short cycle (i.e the Minsky basic cycle) until a crisis is generated.

This pattern of evolution is illustrated in Figure 5, which shows a series of basic cycles characterized by evolving greater amplitude. This evolution is driven by symmetric weakening of the thwarting institutions which is represented by the widening and thinning of the bands that determine the system's floors and ceilings. Eventually the thwarting institutions become sufficiently weakened and financial excess becomes sufficiently deep that the economy experiences a cyclical downturn that is uncontainable and becomes a crisis.

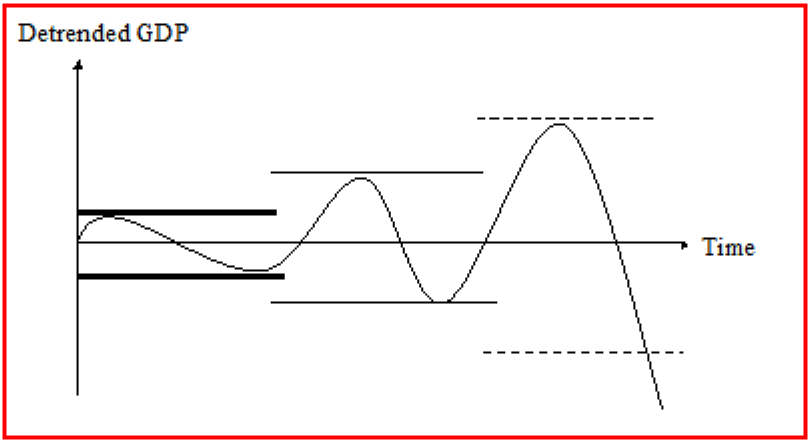

Figure 5: De-trended GDP - Symmetric Fluctuations

Figure 5 shows the case where economy undergoes basic cycles of symmetrically widening amplitude prior to the crisis. However, there is no requirement for this. Another possibility is that cycles have asymmetrically changing amplitude. This alternative case is shown in Figure 6 which represents Minsky's endogenous financial instability hypothesis as having an upward bias. In this case thwarting institution ceilings are less durable than the floors, giving rise to stronger and longer booms before crisis eventually hits. A third possibility is a long-cycle of constant amplitude and symmetric gradual weakening of thwarting institutions that eventually ends with a financial crisis. This richness of dynamic possibilities speaks to both the theoretical generality and historical specificity of Minsky's analytical perspective. The dynamics of the process are general but how the process actually plays out is historically and institutionally specific.

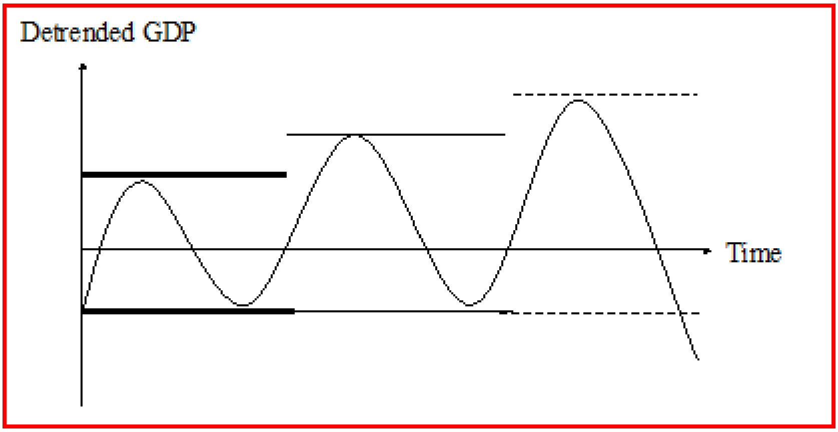

Figure 6: De-trended GDP - Asymmetric Fluctuations

Analytically, the full Minsky system can be thought of as a combination of three different approaches to the business cycle. The dynamic behind the Minsky basic cycle is a finance-driven version of Samuelson's (1939) multiplier – accelerator formulation of the business cycle (see Section 3.4). This dynamic is essentially the same as that contained in new Keynesian financial accelerator business cycle models (Bernanke et al., 1996, 1999; Kiyotaki and Moore, 1997). Thwarting institution floors and ceilings link Minsky's thinking to Hicks' (1950) construction of the trade cycle. The thwarting institutions are explicitly present in the floors and ceilings, but they can also be present in the coefficients of the multiplier - accelerator model which determine the responsiveness of economic activity to changes in such variables as expectations and asset prices. The long-cycle aspect is then captured by shifting and weakening of floors and ceilings and changing of behavioral coefficients. This connects Minsky to long wave theory, with the role of financial innovation linking to Schumpeter's (1939) construction of an innovation cycle.

Despite these commonalities with existing cycle theory, formally modeling Minsky's financial instability hypothesis is difficult and can be potentially misleading. Though models can add to understanding, they can also mislead and subtract.

One problem is formal modeling imposes too deterministic a phase length on what is in reality a historically idiosyncratic process. Adding stochastic disturbances jostles the process but does not adequately capture its idiosyncratic character, which Heraleitos described as "One never steps in the same stream twice". A second modelling problem is that the timing of real world financial disruptions can appear almost accidental. This makes it seem as if the crisis is accidental when it is, in fact, rooted systematically in prior structural developments which had generated vulnerabilities.

A third problem is the financial instability hypothesis is a quintessentially non-equilibrium phenomenon in which the economic process is characterized by the gradual inevitable evolution of instability that agents are blind too even though it is inherent in the structure and patterns of behavior – and agents may even know this intellectually.

This problematic of non-equilibrium is explicitly raised by Minsky (1992b, p.104) in his "Schumpeter and Finance" essay:

"No doctrine, no vision that reduces economics to the study of equilibrium seeking and sustaining systems can have long-lasting relevance. The message of Schumpeter is that history does not lead to an end of history."

5. Empirical Evaluation of Cycle Theories of Different Time Scales

Next we discuss some methodology used in the extraction of cycles from data. In the literature there are three typical methods to empirically study cycles. These are first spectral analysis (Fourier`s theorem), second filtering methods (HP - filter, BP - filter and penalized splines), and third wavelet theory.[11] Since the advantages and disadvantages of the second one have been discussed widely we will here more extensively focus on the first and the third methods.

5.1 A General Approach of Extracting Cycles from Data: Fourier's Theorem

Generally speaking, a function is termed periodic if it exhibits the following properties:

$$f(x) = f(x + T)$$

In this case, T is known as the "period" and, if x is time, then $\frac{1}{T}$ is the frequency. In the physical world there are many phenomena that exhibit periodic behavior, e.g., pendulums, springs, and waves, to name just a few. Mathematical examples also abound.

It is interesting to consider what happens when periodic functions are added together. For example, consider the following:

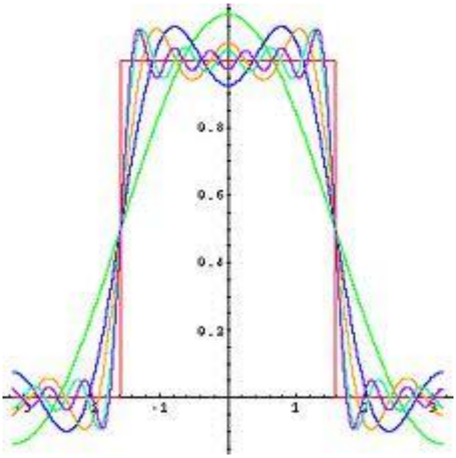

Figure 7: The reinforcing/complementing effects of multiple periodic functions.

[11] On the usefulness of wavelets to study cycles at different time scales, see Gallegati, Ramsey and Semmler (2011).

We can see that when several periodic functions are added together, some parts reinforce each other (when both are positive) and other parts cancel each other (when the functions are of opposite sign). But the interactions may be more or less complex and form surprising shapes, a square wave, as is shown in Figure 7.

From the physical world, we can readily observe certain properties of periodic phenomenon, e.g., cancellation, reinforcement, damping, etc. When one moves away from two sound sources emitting tones of different frequencies, one hears, alternately, louder and softer tones.

It was observations of this kind that motivated Joseph Fourier, in the early 1800s to speculate that virtually any function could be formed by adding together the correct combination of periodic functions. In his famous analysis, Fourier defined a sequence of trigonometric values as follows:

for any function, f, which is integrable from $-\pi \ to \ \pi$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx$$

using these terms, then the function, *f*, may be approximated by

$$f(x) = \frac{a_0}{2}\sum_{n=1}^{\infty}[a_n\cos(nx) + b_n\sin(nx)]$$

Thus, any function may be approximated by a sum of trigonometric functions. This is a powerful result. For example (Tolstov, 1962), we may write that the trivial function, y=x, thus:

$$f(x) = x \sim 2(\sin(x) - \frac{\sin(2x)}{2} + \frac{\sin(3x)}{3} \ldots)$$

This is illustrated below:

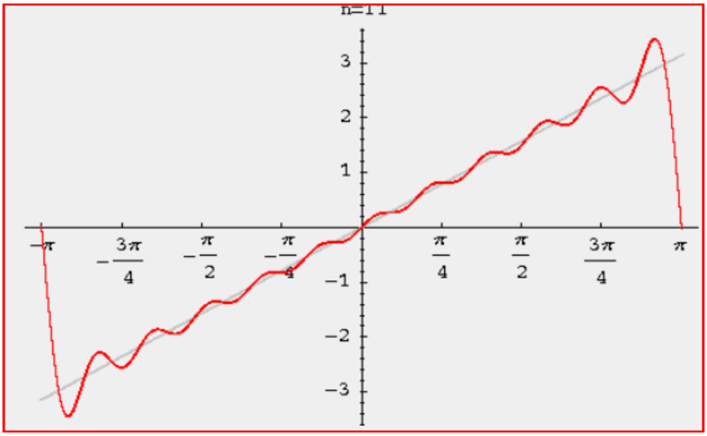

Figure 8: The function $y = x$ expressed as a sum of periodic functions

5.2 Spectral Analysis and Kuznets, Kondratieff, and other Waves

The mathematical implication is that for any time series, a sequence of periodic functions may always be found that add up to approximate the original time series. The above mathematical fact does not, in and of itself, imply that there is any actual or other interpretation of this equivalence. In other words, the fact that there is a mathematical equivalence does not imply that there are real phenomena that exhibit the same characteristics. Nonetheless, it does not imply the reverse either, i.e., that there may be periodic behavior lurking behind some phenomena. In this case, Fourier analysis could be useful in teasing out the details.

Mankiw (2008) states flat out that there are no regularities in economic phenomena. Garrison (1989) states that Kondratieff waves are a product of "creative empiricism" and equivalent to the fanciful shapes, e.g., head-and-shoulders, of technical stock traders - and have "no basis whatever in theory." However, he later modifies this position to allow for wave-like phenomena that have some structural basis.

In fact, as we have shown (above), economists have much reason to suspect that the latter is the case. They have long recognized periodic phenomena of both long and short periods. Business cycles are but one example, which are easily detected and found in data. Thus, it is not unreasonable to replicate the stylized facts of an economic phenomena by suggesting that it is, in fact, the combination of a number of periodic phenomena. This has the advantage of reducing observable phenomena to other phenomena already explained.

Kondratieff waves, described in the previous sections, are cycles that alternate between periods of high growth, with rapid price rises, and periods of relatively slow growth, with falling prices. Regardless of the existence of the illustrated sequence of historical events, it remains controversial if there is, in fact, any fundamental periodic phenomenon of which these fact are manifest. However, recently, as we have outlined in the above sections, a number of researchers have found evidence for such waves.

A number of arguments against this include: (1) the fact that even though certain types of human events tend to recur, people learn from their mistakes and some expectations of cycles may smooth them out. Also, (2) the types of production and investment change over time, (3) long waves are hard to verify empirically, (3) we have shown (see section 3) that there may be different mechanisms working for cyclical behavior at different time scales, and, lastly, (4) Fourier's theorem shows one can always find waves in any data set (even in a data set generated by random numbers).

Although Korotayev and Tsirel (2010) find evidence not only for Kondratieff waves, but also for Kuznets swings, Juglar cycles, and Kitchin cycles. Without going into too much detail, suffice to say that each of these periodic phenomena are characterized by different frequencies and amplitudes. Thus, it is no surprise, see (4) above, that analysis of data will show, with suitable adjustments/calibration, that the data series can be replicated by a sum of periodic functions.

Korotayev and Tsirel (2010) use spectral analysis in their research. They study world GDP growth rates and prices going back over 100 years. The particular form of spectral analysis they use is adapted to time-series. In this technique, the time-series is analyzed "based on the assumption that a broad class of aperiodic natural, technical, and social processes may be represented as sums of random process with stationary increments of different orders." Now although this seems natural enough, and, in fact, given what we know about Fourier Series, must be mathematically true, the problem with the reasoning is this: we are assuming, in some sense, that what we want to find is already there; and then we go and find it. On the other hand, in any sort of modeling, one generally assumes some sort of structural relationship and then considers empirical data to see if there is evidence that supports it. Some might criticize the method of Korotayev and Tsirel because the period of the Kondratieff waves in their research has a period of around 50 years. Thus, no more than three complete cycles could exist in their database.

However, their approach is statistical, not a simple Fourier decomposition, and it has been shown that, even with such a small sample, the test statistic follows a χ^2 distribution. Thus, they obtain low p-values for those components with periods of approximately 50 years (Kondratieff waves; p=0.04), periods of around 8 years (Juglar cycles, p=0.025), and periods of close to 3.5 years (Kitchin waves, p=0.025). With such p-values, most statisticians would accept the presence of these cycles. The key arguments regard the interpretation of historical economic and political events. Note that Korotayev and Tsirel prefer to regard Kuznets swings as harmonic elements of Kondratieff waves, rather than as a separate cycle.

5.3 Other Methods of Cycle Detection

Another issue that comes up in Korotaev and Tsirel is the pre-processing of data. For example, in addition to eliminating the years of the two world wars, (1914–1919, 1939–1946), they also have “replaced all the values for the period between 1914 and 1946 with geometric means (1.5% per year).” This seem a rather extreme and arbitrary replacement. If cycles are to explain economic behavior, only limited adjustment of the data should be permitted. Further, in a second more radical departure from the actual data, “the values for years between 1914 and 1946 were replaced by the mean value (3.2%) for the whole period under study (1871–2007), that is, those values were actually excluded from the spectral analysis.” Thus, it seems to bring into question as to what, in fact was being analyzed.

Additionally, we believe that a wiser course would have been to follow a more robust method of analysis - one that does not require such a large degree of pre-processing. For example, in Gallegati et. al. (2011) a wavelet approach is used to determine the factors that effect output with considerations of size, scale, and time.

The key issue in the empirical analysis is the fact that there may be cycles of different times scales. This leaves open the possibility that they may amplify or counteract each other. For example, Kondratieff cycles in output and prices are estimated to have periods of around 45-60 years; Kuznets infrastructure cycles have periods of around 25 years; Schumpeter's 'innovation,' 50 years, the Goodwin cycle of maybe 20 to 30 years, and Keynes-Kaldor-Kalecki cycles of demand: 7-9 years. Thus the empirical analysis needs to be able to verify these cycles.

Wavelet analysis is similar to and sometimes more accurate than traditional spectral analysis because it uses short 'wavelets' instead of infinite periodic functions. In contrast to the Fourier analysis, wavelet analysis analyzes the signal at varied frequencies with varied resolutions. Instead of the fixed time-frequency results of the Fourier analysis, the wavelet method provides excellent frequency resolution at low frequencies and good time resolution at high frequencies. Thus, this methodology allows both time and frequency to vary in time-frequency plane, but also the mechanisms driving those cycles .

In Gallegati et. al. (2011), a wavelet approach was used to analyze the time series data underlying the Phillips-curve:[12]

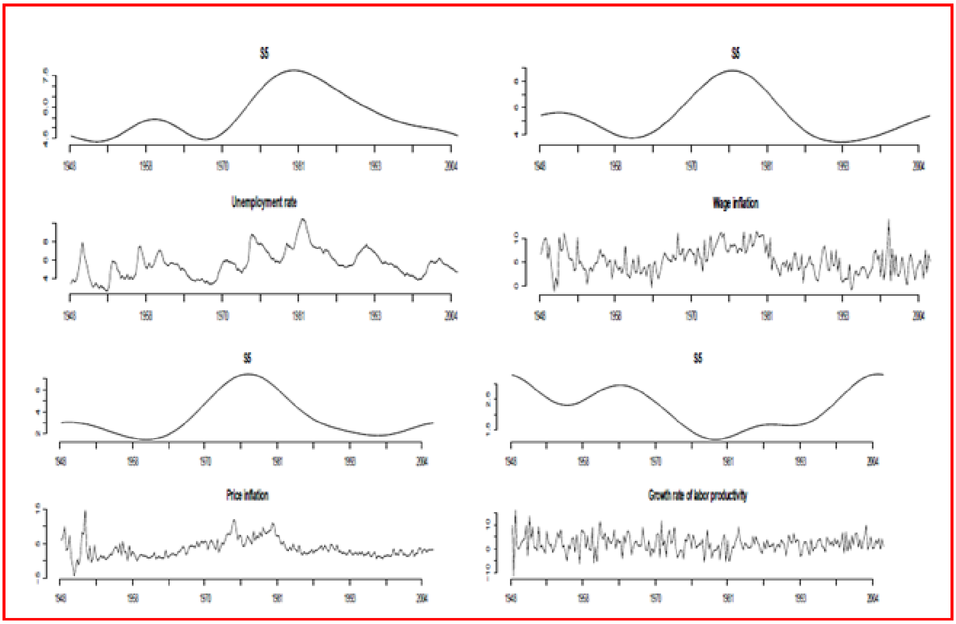

Figure 9: Wavelet analysis; longest times scale and actual time series for unemployment, wage inflation, price inflation and growth rate of labor productivity[13]

Thus, wavelets provide a good method to see relationships on different times scales and allow one to disentangle what drives output at different time horizons. Wavelet variance and cross-correlation methods can be used to determine leads and lags in time series and how different time scales effect them. This is likely to be better approach to cyclical analysis of macroeconomic time series. Figure 9 provides an example for the composition of the time scale for US Phillips curves time series variables. One does not observe not much co-movement between of unemployment, wage and price inflation, or productivity; yet, they are very visible when using a long time scale.

Yet another methodology for (see Kauermann, et. al., 2011) the decomposition and filtering of time series is the technique of penalized splines. Here, a time-series is decomposed into a smooth path and a series of residuals, which are assumed to be stationary around the trend. This technique is robust with regard to correlation of residuals. The residuals exhibit business cycle features.

Splines are basically a type of smoothing, using basis functions, in which piece-wise polynomial functions are joined together to form a "smooth" shape. The "smoothed" shapes can then be studied or periodicity and other features more easily

[12] Note that Goodwin uses such a Phillips-curve but assumes a constant productivity growth rate and real variables.
[13] US data 1948.1-2006.4

than the original data-stream. In their paper, Kauermann, et. al. discuss several sub-methodologies including the Hodrick-Prescott (HP) Filter and the Bandpass (BP) Filter; these are contrasted with the method of penalized spline. They study GDP and its components from 1953 to 1996. The data and the resulting filters are shown below.

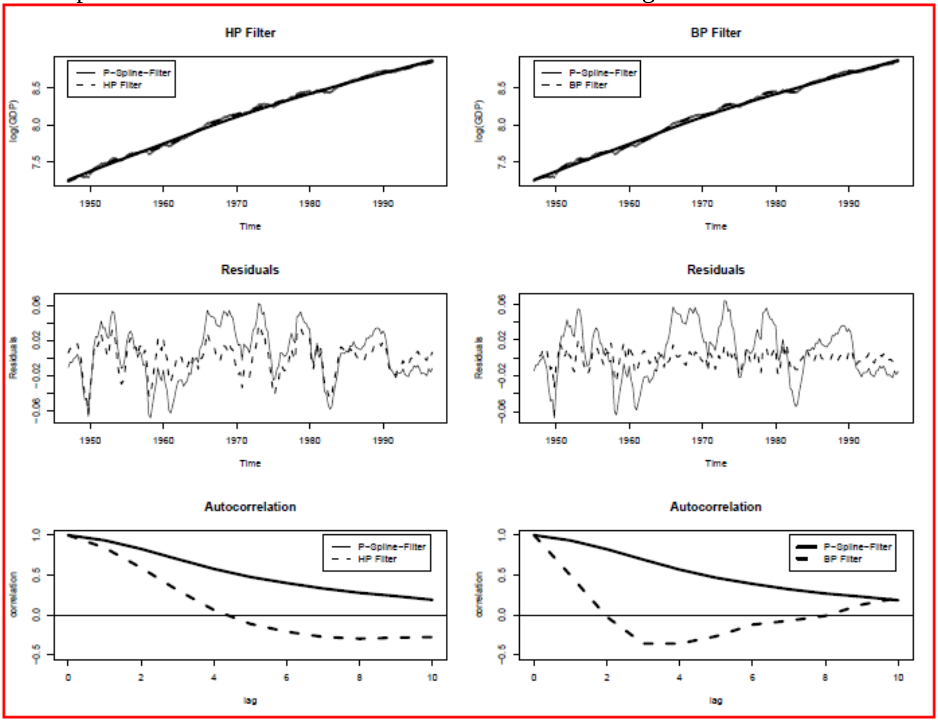

Figure 10: HP and BP filters and penalized splines; US GDP, quarterly data, 1953.1-1996.4

The top line illustrates the penalized spline filter as dark line in contrast with the HP filter (left) and with the BP filter (right). The first one shows almost a linear trend and business cycle components come out more clearly as compared to the HP and BP filters as shown in the middle row. The penalized spline filter can allow for distinct residuals with serial correlation. This is also seen in the auto-correlations of the residuals, which are illustrated at the bottom.

We see that there are a variety of approaches to the identification of cycles within time series methods. Each of them have some advantages and disadvantages.

5.4 Some Empirics on the Goodwin Cycle

Other analysis, for example, Flaschel, et. al. (2008) show how cyclical behavior can appear as Goodwin cycles, based upon predator-prey dynamics as discussed in Section 3.5. In their case, they show how, with suitable assumptions about the wage-price spiral and certain other variables, a Lotka-Voltera type of model gives rise to periodic phenomena, as explained in section 3.5 above. In this case, the ambiguities are only pushed into the background, i.e., into the parameters and structure of the pair of differential equations that give rise to the dynamical system. In other words, it is not in question if the system they develop gives rise to periodic behavior, it does. The question is whether the system is well-grounded in the empirics of the variables being used.

At this point, we do not seek to advocate for or against the existence of wave-like phenomena in economic behavior. Instead, we only wish to point out two things: (1) Fourier's theorem guarantees that one can find a set of waves which fully simulate any curve; (2) there is a fundamental ambiguity about the nature of the economic behavior being explained, with respect to frequency, amplitude, etc. as there is sufficient freedom for interpretation of virtually any periodic phenomena as “economic” phenomena.

An empirical test of the cyclicality of synthesis of the Goodwin and Keynes-Kaldor models are given in Flaschel et al (2008). Often the Goodwin model has been interpreted as business cycle dynamics, but as Flaschel et al (2008) show, the employment and wage-share dynamics seems to hold more for a longer time scale, where the wage-share movement can be found to be related to a large time scale with a delay. Employment seems to lead the change of the wage-share in the context of long waves; see figure 11.

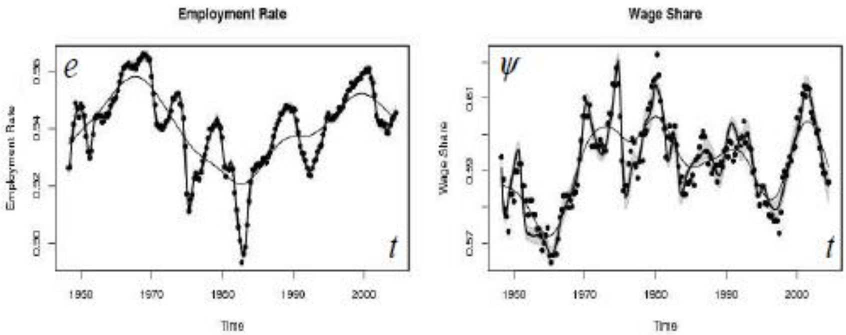

Figure 11: US Goodwin Cycles

Yet, business cycle frequency there is some negative correlation between employment and wage share. This interaction appears to come less from real wage movements, but rather from pro-cyclical productivity movements. As to the longer time scale -- here captured by the thin solid trend line -- as it is observable from the Figure 11 there seems to be strongly a delayed reaction: With employment rising wage share seems to rise with a delay, and as wage share is rising, employment seems to fall with a significant delay. Most of our current cycle models --- on a short or long scale -- have not properly build in such delays, since those models are difficult to solve.

5.5 Some Empirics on the Minsky Cycle

The period 1981 – 2007 provides evidence from the U.S. economy that is strongly supportive of the idea of a long Minsky cycle. The Minsky basic cycle embodies a sentiment-based theory of the business cycle; see also Semmler and

Bernard (2012). The tranquil period generates increasingly greater risk taking as agents become progressively more optimistic. That optimism manifests itself in increasingly optimistic valuations of assets and associated revenue streams. It also manifests itself in credit markets where both borrowers and lenders become more optimistic. That is critical because it means lender-imposed market discipline becomes progressively weaker. Leveraging increases but the usual text-book scenario of corporate finance, whereby higher leverage implies a higher risk premium, is not visible in the cost of credit. Instead, credit remains cheap and plentiful.

Figure 12, shows the ratio of home prices to rents during the period of the 1960s - 2006. Beginning in 1999, the ratio suddenly starts to increase. Such a development would be consistent with a dramatic drop in interest rates, thereby generating a large increase in the present value of anticipated rents. However, that did not happen. Instead, the increase in home prices relative to rents was driven by speculative anticipations of higher resale values. This corresponds exactly with the Ponzi phase of the basic Minsky cycle in which agents borrow to finance asset purchases in anticipation of higher resale values.

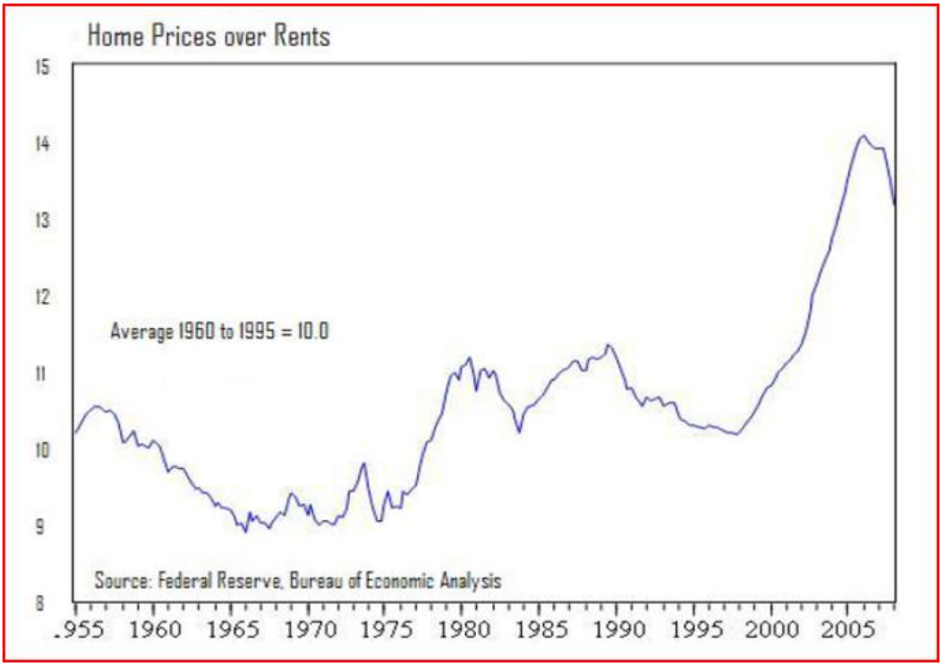

Figure 12: Ratio of Home Prices : Rents

Figure 13 shows nominal mortgage rates over the period 1971 - 2008. The figure shows that nominal mortgage rates rose sharply in the 1970s through to 1981, and then fell steadily through to 2002. Under "normal" circumstances, it would be surprising to see a simultaneous increase of mortgage interest rates and the home price to rent ratio. However, the 1970s and early 1980s were a period of stagflation - rapid inflation plus relatively high unemployment. The rise in the home price to rent ratio can therefore be explained by the combination of increased demand for hard assets that are more protected against inflation and lowered rents attributable to a weak economy.

With regard to the Minsky cycle, the main feature of interest in Figure 13 is that mortgage interest rates remained roughly constant over the period 2002 – 2007. This was a period when the bubble in house prices had already set in (as shown in Figure 12) and buying homes therefore involved larger mortgages. Yet, despite this, there was no increase in risk premiums, reflecting the increased optimism and complacency of lenders.

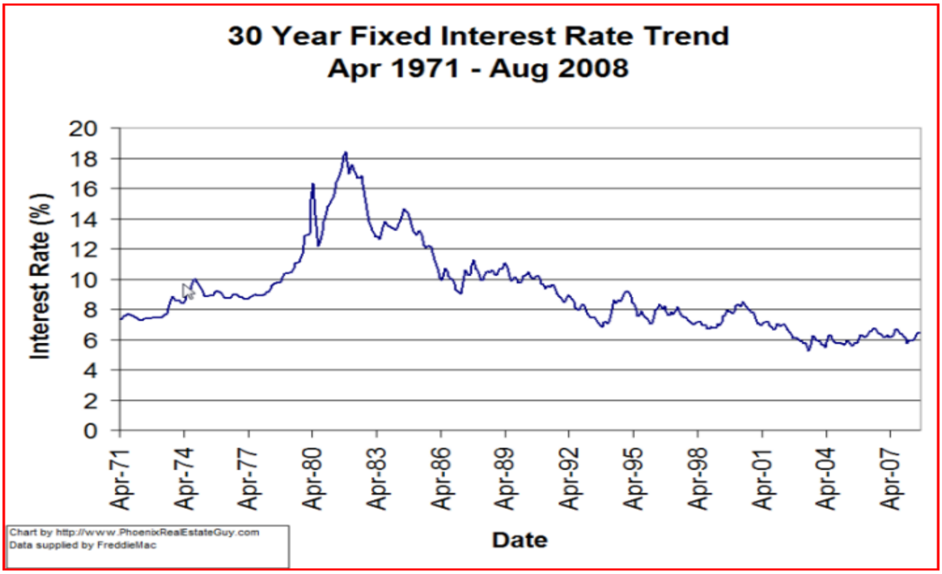

Figure 13: Historical Mortgage Rates

Two further pieces of evidence consistent with the Minsky basic and super cycles are provided in Figures 14 and 15. Figure 14 illustrates the percentage of disposable income devoted to servicing debt for the period 1980 - 2007. This is a fairly good proxy of risk since the lower the percentage of disposable income a borrower needs to pay (i.e the lower the debt service burden), the less risky is the loan. Figure 14 shows a cyclical pattern, with the debt service burden rising in the expansion of the 1980s and then falling when the economy went into recession. It rose again with the expansion of the 1990s, briefly flattened at the end of that expansion, and then continued increasing in the 2000s. This pattern is consistent with the interaction between the basic Minsky cycle and the Minsky super-cycle discussed earlier.

The basic cycle is evident in the expansions of the 1980s and 1990s, but by the 2000s the old thwarting institutions had been rendered obsolete and the economy enters a period of unsustainable boom that ends with a financial crisis. During this last period, leverage increases massively but there is no increase in interest rate risk premiums because lender discipline was in tatters owing to the spread of optimism amongst lenders that weakened market discipline.

Figure 15 shows the volume of funds in Collateralized Debt Obligations over the period 1992 – 2002. Collateralized debt obligations (CDO) are financial assets constructed by bundling smaller loans. The interest and principal on these loans are paid to a trust entity, which then divides those payments among CDO owners.

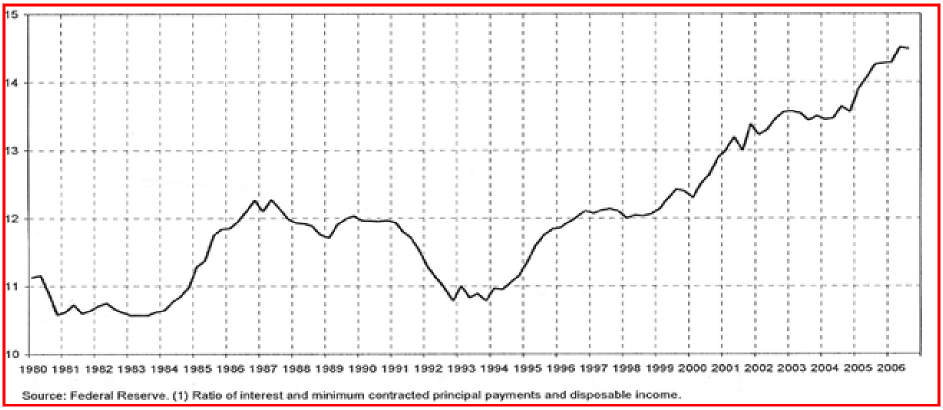

Figure 14: Debt Service to Disposable Income Ratio

Distributions from the trust entity may also be tranched, with less risky CDOs receiving payment first and more risky CDOs receiving the remaining income. Mortgage-backed Securities (MBS) represent a specific type of CDO backed by home mortgages. As discussed earlier, an important ingredient of the Minsky super-cycle is financial innovation that escapes the regulatory net, permits increased risk-taking, and encourages financial complacency. CDOs represent such an innovation. The bundling of loans in CDOs enabled banks and other lenders to sell their loan portfolios and thereby off-load risk. This created the "originate and distribute" model whereby banks and other lenders shifted to selling their loans rather than holding them. That in turn changed patterns of incentives, giving banks an incentive to push loans rather than engage in sound lending. That is because banks increasingly made money by taking the fees, commissions, and profits associated with creating CDOs and did not bear the ultimate risk associated with loan performance. If the loan subsequently went bad it was no longer on the bank's book. The new CDO "originate and distribute" lending model therefore relaxed lender discipline since lenders felt they were not ultimately on the line. This helps explain why debt service burdens were allowed to increase so much and why interest rates did not rise to reflect increased risk premiums.

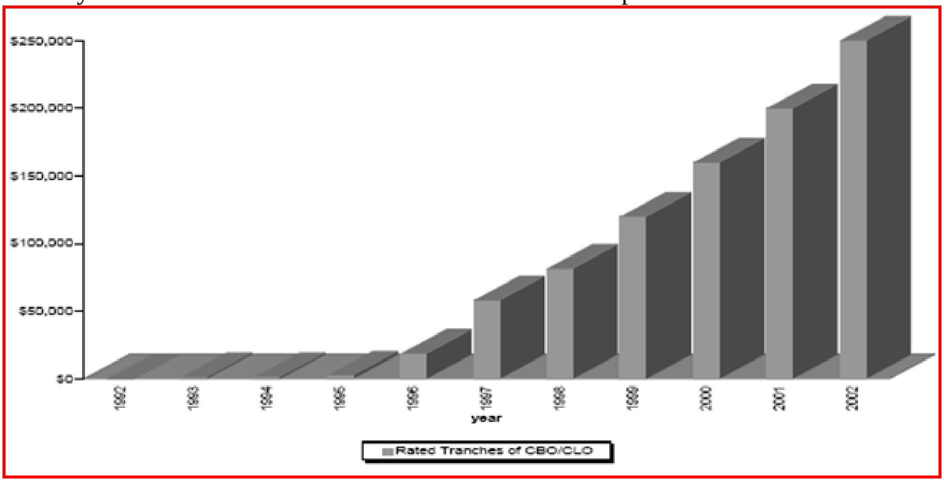

Figure 15: Securitization of debt: Complex securities

6. Conclusion

Empirically detecting the mechanisms of long cycles is difficult. First, there are technical challenges associated with filtering and spectral methods. Second, and more important, economies are characterized by continuous change that becomes ever more significant as the period of analysis lengthens. For instance, Kuznets and Kondratieff waves of 25 to fifty year duration will inevitably take place in a context of significant structural change. Over the last two hundred years, a repeated sequence of structural change has been the transformation of economic activity from agricultural dominance, to manufacturing dominance, then on to service sector dominance. Economies are also characterized by institutional change concerning labor markets, regulatory arrangements, and the organization of firms. These institutional changes alter the processes of decision making, introduce new decision actors and interest groups, and change the balance between markets and government. Technological change has promoted a trend towards economic activity that involves less physical production and resource use, and is more intensive regarding knowledge-based production activity. As a result, the character and forces of growth are likely to change.[14] From that perspective, one can never step in the same stream twice.

The existence of so much historically idiosyncratic matter makes it empirically difficult to detect cycles of fixed periodicity and amplitude based on time invariant cycle generating mechanisms. Wavelet methods appear to be the most suitable means of empirically identifying economic relationships over cycles of different duration. Technical difficulties notwithstanding, the data for some macroeconomic variables (particularly profits) exhibit the dynamics of ups and downs. It also appears possible to talk about stages, or phases, regarding the economic dynamics of developed and emerging market economies. If long cycle theory holds, that raises the question of where we stand today. Are we in the middle of a cycle or at the end of one and awaiting the beginning of another? Those are questions that the application of long cycle theory and methods may help to answer.

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DO INTERNATIONAL OIL PRICE AND THE MACROECONOMIC VARIABLES LIKE THE GDP DANCE THE SAME MUSIC? A HP APPROACH TO ESTIMATE ECONOMIC CYCLES

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Issue: Cyclical dynamics trends of economy in the 20-21st centuries;

Abstract

Since the seminar studies of the economical crises and cycles undertaken by several scientists, that the researchers and the governments of the countries paid more attention to them. In a world where non-renewable energies are becoming more and more scarce, theirs prices tend to increase as their scarcity increases and the demand of big eastern players inflate the market. Our hypothesis is that the growth of the world's economy needs more and more energy and this forces oil price and related energy prices to become higher and higher. Does the historical data confirm or deny our hypothesis? Is there a parallelism among economic and oil price cycles? In this research we are going to adopt the Hodrick-Prescott filter in order to identify the long term trend and the economical cycles present in the very long period oil price evolution. After identifying all the cycles, we are going to study their characteristics and to compare them with the corresponding characteristics of NBR cycles in order to study their parallelism. With this research we expect to confirm or to deny the parallelism of the two cycles, the number of cycles is not very different in the two situations, the troughs and peaks appear in similar moments and the duration of both economic cycles are similar. The results are important in political economic terms since from them depend the different actions that the governments should take in order to increase the well being of the populations.

Keywords: economic and business cycles, oil price cycle, Hodrick-Prescott Filter, energy economics

1. Introduction

Robert Owen (1817) expressed the first systematic thoughts about the economic activity in his Report to the Committee of the Association for the Relief of the Manufacturing Poor, 1817, concluding that the causes of economic cycles are overproduction and underconsumption, which, in turn, are caused by wealth inequality. Sismondi (1819) also wrote about periodic economic crises; this reasoning is in opposition to the existing theory of economic equilibrium in the *Nouveaux Principes d'économie politique* (Sismondi, 1819). Till then the classical economists denied the existence of business cycles, saying that oscillations were due to external factors like wars, and only studied the long term. Sismondi found evidence in favor of his point of view, in the panic of 1825, the first internal economic crisis that occurred in peacetime. Both Owen and Sismondi advocated solutions for these crises. This work did not generate interest among classical economists, though underconsumption theory was later developed as a new branch in economics, systematized by J. M. Keynes in the 1930s. The Sismondi's point of view influenced C. Dunoyer and J. K. Rodbertus that developed the theory of alternating *cycles* and similar theories. The periodic crises in capitalism formed the basis of the theory of Karl Marx, author that said that these crises were inevitable and more and more severe. In 1860, Clement Juglar, a French economist, identified the presence of economic cycles 8 to 11 years long, without claiming any rigid regularity. Another expert is J. Kitchin (1861-1932), a British businessman and statistician that studied the American and English systems of interest rates and found evidence for short business cycles of about 40 months in average. His contribution is considered fundamental for later economists such as N. Kondratieff, S. Kuznets, and J. A. Schumpeter. The last one of these 3 names, J. A. Schumpeter (1883-1950), a famous Austrian economist said that a Juglar cycle has four stages (expansion, crises, recession and recovery) the recovery and prosperity being associated to increases in productivity, consumer confidence, aggregate demand, and prices. Later, in the mid-20th century, Schumpeter and others proposed a typology of business cycles according to their periodicity: the Kitchin inventory cycle of 3–5 years (after J. Kitchin), the Juglar fixed investment cycle of 7–11 years (often identified as 'the' business cycle), the Kuznets infrastructural investment cycle of 15–25 years (after S. Kuznets), and the Kondratiev wave or long technological cycle of 45–60 years (after Nikolai Kondratiev). N. D. Kondratiev, (1892-1938), the Russian economist, that proposed the New Economic Policy (NEP) in the Soviet Union, argued that western market economies have long term of 50 to 60 years cycles of boom followed by depression, that began to be known as "Kondratiev waves" in his honor. S. Kuznet (1901-1985), the Russian-American 1971 Nobel Memorial Prize in Economic Sciences, discovered the "Kuznets swings" when working on business cycles and disequilibrium aspects of economic growth; his studies promoted the development economics new subject. He is also known due to his research on inequality over time, whose results formed the well-known Kuznets Curve. E. R. Dewey (1895-1978), another Harvard economist, identified a number of cycles in the U.S. economy after being assigned the task of discovering the cause and underlying dynamics of the Great Depression in 1931 by the Department of Commerce. Dewey's work led to the lifelong calling in cycles. He combined his enormous research in business cycles with research from leading biologists on cycles in nature and in wildlife, discovering “that cycles of identical length were found in both disciplines and similar cycles from different areas reached their peaks and troughs at the same time”. A. Burns and W. Mitchell (1946), devoted their book, “Measuring Business Cycles”, to the business cycles, one of their main insights is that many economic and social indicators move together, increasing and decreasing during expansion and contraction phases, respectively.

There are some recent studies that tie the oil price and the economic cycles. Among these we have Kilian et al. (2006) that studied the effects of demand and supply shocks in the global crude oil market on several measures of countries’ external balance, including the oil and non-oil trade balances, the current account, and changes in net foreign assets (NFA) during 1975–2004, taking a global perspective, using the Lane–Milesi-Ferretti NFA data set, they document the presence of large and systematic (if not always statistically significant) valuation effects in response to oil shocks, not only for the U.S., but also for other oil-importing economies and for oil exporters. The estimates suggest that increased international financial integration will tend to cushion the effect of oil shocks on NFA positions for major oil exporters and the U.S., but may amplify it for other oil importers. Another interesting paper is the one of T. Naccache (2010) that says that "Many papers have been documenting and analyzing the asymmetry and the weakening of the oil price-macroeconomic relationship as off the early eighties. While there seems to be a consensus about the factors causing the asymmetry, namely adjustment costs which offset the benefits of low energy prices, the debate about the weakening of the relationship is not over yet. Moreover, the alternative oil price specifications which have been proposed by Mork (1989), Lee et al. (1995), and Hamilton (1996) to restore the stability of the relationship fail to Granger cause output or unemployment in post-1980 data. By using the concept of accelerations of the oil price, the authors show that the weakening of this relationship corresponds to the appearance of slow oil price increases, which have less impact on the economy. When filtering out these slow oil price variations from the sample, we manage to rehabilitate the causality running from the oil price to the macroeconomy and show that far from weakening, the oil price accelerations-GDP relationship has even been growing stronger since the early eighties" (Naccache, 2010).

2. Methodology

In this research we are going to adopt the Hodrick-Prescott filter in order to identify the long term trend and the economical cycles present in the evolution of real the oil price over a very long period. After identifying all the cycles, we are going to study their characteristics and to compare them with the corresponding characteristics of NBR cycles in order to study their parallelism. The trend-cycle decomposition is very important in macroeconomics; the idea of this process is to isolate the secular trend and the transitory deviation, the cycle, of the log of an economical series like the GDP. To isolate the economic cycle first we need to estimate the trend, to detrend the series and after this we have the cycle. The Hodrick-Prescott (1997) filter (HP) is the most common method used to extract the trend (g_t) and to d$_e$trend the time series (y_t). The tre$_n$d can be extracted solving the following minimization problem:

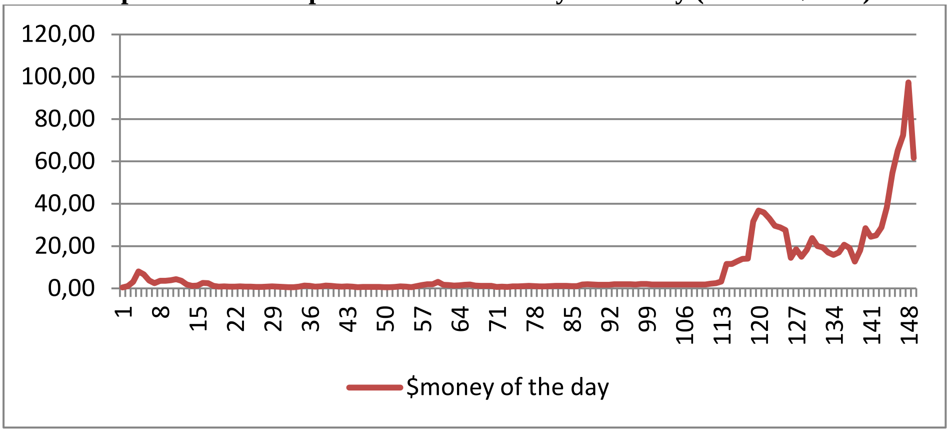
$$\min_{g_t} \sum_{t=1}^{T}(y_t - g_t)^2 + \lambda \sum_{t=2}^{T-1}[(g_{t+1} - g_t) - (g_t - g_{t-1})]^2 \qquad (1)$$

The smoothing parameter, λ, controls the smoothness of the adjusted trend series, g_t (as λ tends to 0 the trend approximates y_t, the actual series, and as λ tends to ∞ the trend becomes linear).HP suggest a value of λ=100 for yearly data. Marcet and Ravn (2003) estimated the parameter exogenously and concluded that for annual data the value should be between 6 and 7. Relatively to the power HP(1997) suggest a power of 2 while Ravn and Uhlig (2002) suggest a power of 4.Related to this see also Maravall (2004).The first part of (1) gives the goodness of fit, the second the penalty for roughness.

3. Results:

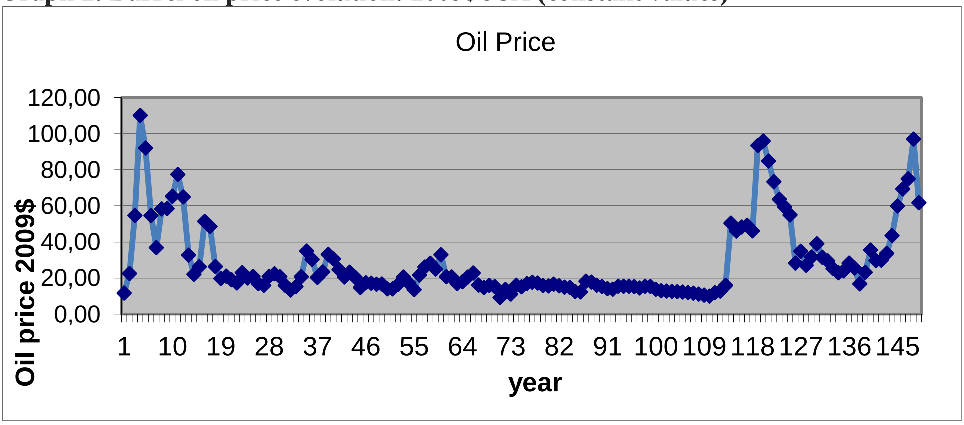
The evolution of the barrel oil price expressed in current US$ or money of the day can be seen in the graph n. 1. In this graph the year number 1 is 1861, the last one is 149 and corresponds to 2009. This price is the cost of a barrel of Arabian light and Brent oil that were sold in the USA (from 1861 on).

Graph. 1: Barrel oil price evolution: money of the day (current $USA)



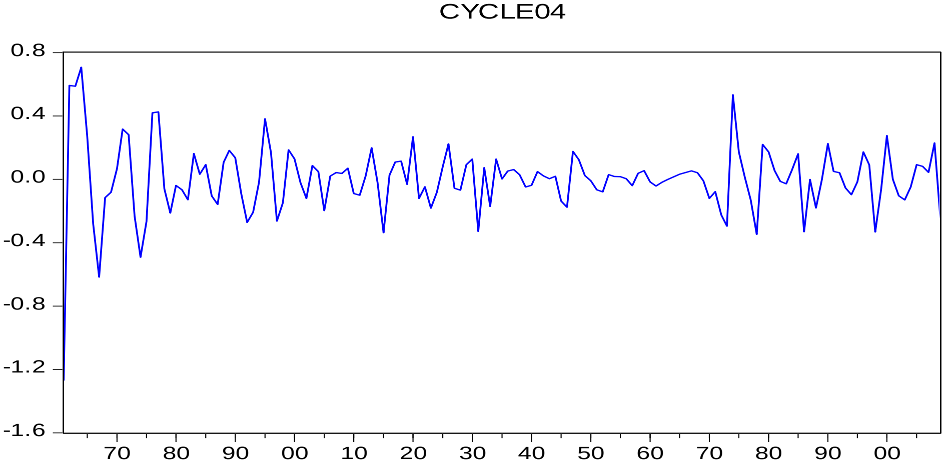
As interesting comparison with this trajectory of the current prices of the barrel of oil is the one that can be seen in graph n. 2 where this price is presented in constant values (2009$USA).

Graph 2: Barrel oil price evolution: 2009$USA (constant values)



Our analysis shows, using the Hodrick-Prescott filter with the logs of the yearly real oil prices (2009US$), showed that between 1861 and 2009 (149 years) there are 28 cycles with an average duration of 5.28 years that varies from a minimum of 2 years (in one cycle, 1931 to 1933) to a maximum of 12 years (1961 to 1973) if we measure the cycle from trough to trough. Other cycles have the following durations: 3 cycles of 3 years ,6 cycles of 4 years, 7 of 5 years, 5 of 6 years, 2 of 7 and 8 years each one and 1 of 12 years. The mode of the duration is 5 years. If we count the cycle from peak to peak we have a maximum of 27 cycles whose durations vary from 2 years in 3 cycles to 8 years in 1 cycle; other cycles have durations of 4 years in 5 cycles, 5 years in 3 cycles, 6 years in 12 cycles and 7 years in 3 cycles. The mode, i. e., the dominant duration (mode) and the average are both 5.48 years. The contraction periods of the cycle, i.e., the periods that go from the peak to the trough vary from 1 to 6 years, the average contraction being 2.96 years. In turn the expansion period of the cycles, i.e., the number of years that go from the previous trough to the next peak of the cycles vary from 1 to 6 years, also, the average being 2.32 years. An interesting conclusion is that, in the average, the contraction periods (2.96 years) are longer than the expansions periods (2.32 years), more 7.2 months. What has been written can be seen in the graph n. 3 and especially in the graph n. 3.

Graph 3: Oil price cycles' sequence since 1861



Using the NBER (2010) data for the USA we can say that between the same years, 1861 and 2009, there are 32 complete cycles with an average duration of 4.70 years that varies from 2.3 years (from 1980/07 to 1982/11) to 10.7 years (1991/3 to 2001/11) if we measure the cycle from trough to trough. If we measure the cycle from peak to peak we have also 32 cycles whose durations vary from 1.4 years (1919/3 to 1921/7) to 10.7 years (1991/3 to 2001/11). The mode is 3 years if we measure the cycle from trough to trough and 3.3 years if we measure it from peak to peak, the averages are both 4.7 years. The contraction periods of the cycle, i.e., the periods that go from peak to trough vary from 6 months (1980/1 to 1980/7) to 65 months or 5.4 years (1883/10 to 1879/3), the average contraction being 1.45 years and the mode is 8 months. In turn the expansion period of the cycles, i.e., the number of years that go from the previous troughs to the next peaks of the cycles vary from 10 months to 10 years, the average being 3.25 years, the most dominant is 1.8 years. An interesting conclusion is that, in the average, the contraction period (1.45 years) is smaller than the expansions period (3.25 years), less 21.6 months. The biggest cycle had duration of 21 years, between 1952 and 1973, and the shortest had duration of 3 years between 1928 and 1931.

Comparison of the two types of cycles

Comparing the oil price cycles to the NBER economic cycles (NBER, 2010) we can conclude that they are not coincident or parallel, as should be expected, the number of the economic cycles from 1861 to 2009 being 32 (compares to 28 of the oil price cycles), there are 4 more economic cycles than oil price cycles. In terms of the average duration of the economic cycles identified by NBER is 4.70 years (that compares with 5.28 years of the oil price duration), the average contraction period of the economic cycles being 1.45 years (that compares with 2.96 years of the oil price cycles), the average expansion period of the economic cycles (from previous trough to peak) is 3.25 years (that compares with 2.32 years of the oil price cycles). These last values mean that the average contraction period of the oil price cycles (2.96 years)is bigger than the average contraction period of the economy (1.45 years), more18.1 months. The average expansion period of the oil price cycles (2.32 years) is smaller than the average expansion period of the economy (3.25 years), so, less 11.2 months.

Is there parallelism or antagonism between oil price cycles and economic cycles?

To answer this question we superposed the two graphs with the two cycles and the conclusions are the following: there is no parallelism between the two types of cycles; neither we can speak of a total antagonism, especially in the cycles that are contemporaneous to periods of low oil prices; there is clearly a causal effect from high rising prices and low economic activity; this causality is especially visible if we take into account time lags, as the economy reacts with some delay to the stimulus representing the rising price of oil; however, there are quite visible periods of antagonism, especially in months that follow sudden oil price hikes; during these periods it is common to assist to the oil price expansion and simultaneously, but possibly with some lag time delay, to a reduction of the economic activity and eventually to a recession; in most cases there is some conflict or antagonism between the two cycles, with rising oil prices entailing the economic crises and decreasing prices pushing or carrying the resumption of the economic activity. These considerations apply solely to countries that are mostly oil importers; in the oil exporters there are certainly more parallelism between the two types of cycles with increases in prices followed by economical increases. Besides the antagonism up referred in countries that are highly dependent from oil imports we found empirical evidence, at least in Portugal in the post 1970s, to say that high oil prices means, almost surely, government change since energy and crude oil in particular play important roles in the occidental oil importing world as they affect all the economic sectors specially on account of the transportations' costs, and this means firms that close and unemployment rates growth, the ingredients that often condemn governments to crash.

4. Conclusion/Discussion

Our analysis carried out with the HP filter shows that between the oil price cycles and the economic cycles identified by NBER there are some similarities but also some differences, depending from being oil producer or oil importer. The number of cycles is different in both situations, and their durations differ. There is a kind of parallelism for the 1st group of countries and a clear antagonism for the 2nd one, this antagonism happens with some time lag, this meaning that if a high oil price increase is verified then, after a certain period of time, the economies of the oil importer countries enter in deep crises with firms closing, and unemployment rates rising and, very often, with government overthrows. We documented a causal relationship from oil price variations to economic oscillations with lag time delay an important result as it can help governments to implement adequate measures to fight against high level oil prices and especially against their consequences in economical and social terms.

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THE ECONOMIC SPACE-TIME PARADIGM AND A NEW ECONOMIC CYCLE MODEL[1] - FOLLOWING KUZNETS' SUGGESTION ON THE SECONDARY CYCLE

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Cornel Ionescu, Ph.D[2]

Abstract:

The paper aims to revisit the fundamental coordinates inside which the economic cycle is conceptualized and assessed. All the considerations on the economic cycle were based on the clock time and physical space, generating exogenous models of it. The authors claim a new approach of the issue. They believe the economic cycle is moving inside a new concept of time: economic time depending on the economic process described, respectively inside a new concept of space: economic space also depending on the economic process described. This means the economic cycle can truly be described, analysed, anticipated (predicted) only on the bases of the continuum economic space-time (CEST) paradigm. The paper introduces the two new concepts (economic time, and economic space) and tries to offer a logical model of the economic cycle based on them. The CEST model of economic cycle seems to deliver better perspectives to understand, describe, and predict the economic phenomenology from both a theoretical and a methodological point of view, because the auto-poiesis phenomenology can rise only in this model (that is, in a logically vivid system model, like the CEST model). As exemplification, the authors include into the presentation some considerations on critical thresholds, bifurcations, attractors, and self-fuelling, introducing them into the proposed logical model of the economic cycle.

Keywords: economic space-time, economic cycle, critical thresholds, logically vivid system

JEL Classification: E30, O11.

[1] This paper is a specific development from the research project supported by CNCSIS – UEFISCSU, project number 821/2008
PNII – IDEI 600 / 2008 – „Mix adjustment policies in light of achieving sustainable economic convergence with the European Union of Romania".

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1. NON-TECHNICAL INTRODUCTION

The economic cycle is a macroscopic manner (paradigmatically observable) in which the individual economic processes take place. This is a mechanism of aggregation and also a mechanism of self- fuelling. This study will only approach some basic aspects regarding the self-fuelling mechanism of the economic cycle.

Generally, by economic cycle we will understand a complex process, economic by its nature, conditioned by three factors: natural, institutional and psychological. Each of these types of conditioning has its own pattern of development, particularly in terms of the two essential categories of the processuality: a) the time; b) the space. Thus, the time specific to the natural conditionality is the clock time, the mechanic time, which doesn't depend on the measured/quantified process. This is a Laplacean time, absolutely independent of the process. The time specific to the institutional conditionality is as autonomous, in relation with the process, as the time specific to the natural conditionality; the difference, however, is that the first one is a discretionary time (or rather temporal interval), while the latter is a non-discretionary time. The time specific to the psychological conditionality is a time assigned to the process, therefore it is a time whose pace depends on the measured/quantified process. We suggest that when we deal with an economic process[3], we will call economic time the resultant of overlapping the three categories of time associated to the economic process. Analogous considerations can be done regarding the space, reaching the concept of economic space.

The problem here is the following: why do we need to introduce a time specific to the economic process? The answer is simple, yet fundamental: because the rate of the economic process depends on the very economic process. In other words, within an economic process we will have accelerations and decelerations of the economic rate in relation with the rate equal with self, absolute and autonomous, of the clock time. These accelerations or decelerations of the economic rate in relation with the clock time explain, in our opinion, in their deepest substances, both the temporal geometry of the economic cycle and its real causalities. Probably, the cause for which the economic discipline or science is so little apt to predict the economic process is the very use of the clock time instead of its own economic time. The proper time of the economic process should have the same "right" to existence as so many other types of proper time used both in nature sciences (for instance in the quantum mechanics or in the general theory of relativity) and in human sciences (for instance in psychology, sociology and history).

The methodological solution to introduce the economic time to measure/quantify the economic process is to reintegrate the economic subject within the economic process by giving up the principle of objectivity specific to the nature sciences, by keeping the second principle of the scientific knowledge, the

[3] The economic is that attribute of an act which involves the intervention of the human being in nature with the purpose to accomplish the material exchange between the actional subject and the nature, via an artefactual membrane.

principle of intelligibility (comprehensibility). The reintegration of the economic subject within the economic process (ultimately, accepting the logical subject-object indiscernibility within the economic process) implies, however, a new paradigm of the economic process: the paradigm of the logically vivid system. The economic system will thus have to be regarded as a logically vivid system. The logically vivid system presumes that the considered economic system contains two additional predicates of sufficiency: a) dissipativity, which is the capacity to preserve or decrease the intra-system entropy; and b) functional redundancy (which involves the structural redundancy). These two predicates of sufficiency required to regard an economic system as a logically vivid system, are the bases for the next epistemological level: the sustainable system. Therefore, ultimately, the introduction of the economic time and space to describe the economic processuality entails the description of the economic system as a sustainable system. We will see that a logically vivid system becomes a sustainable system if another two predicates of sufficiency are added: a) the microscopic hyper-cycles; and b) the dominance of the negative feed-back.

Within this conceptual context we will now develop some considerations of the necessity to review the epistemological and methodological bases of understanding, explaining, describing and modelling the economic cycle.

CONCEPTUAL FRAMEWORK OF THE ORTHODOX MODELS OF ECONOMIC CYCLES

The orthodox theories (models) which currently describe the economic cycle are characterized by three conceptual-methodological features:

a. They take the dominance of one of the classical components of the economic "metabolism": the demand and offer[4] as identification criterion for the onset of a new economic cycle. Figure 1 shows an abstract picture of this feature:

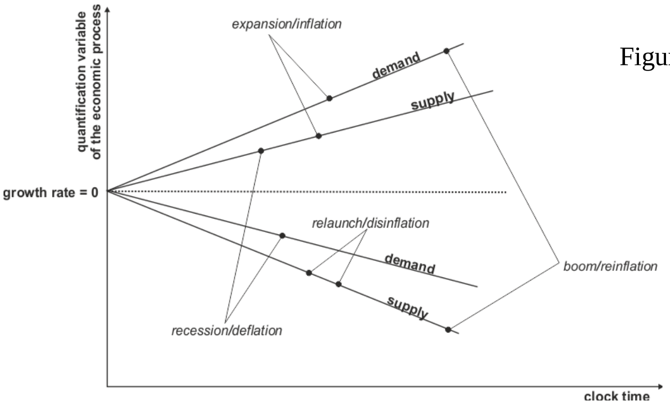

Figure 1. Abstract design showing the formation of the cyclic elements within the orthodox economic process

Thus, for instance, if for a specific reason, the demand for a particular good or service increases in relation with the offer of that particular good or service, there will be a direct pressure to increase the price of that good, which encourages the

[4] We have in view here the three basic economic markets, each one having it own offer-demand pair: 1) the market for goods and services; 2) the monetary market; 3) the labour market.

producers (according to the curve of the demand on the coordinates: price-demanded amount) to expand their business, by investments if necessary. Since the investments can exceed the necessary increase of the demanded supply, a gap in the opposite direction will form between the demand and the offer, and so on. This design can become rather complicate if we take into consideration more complex aspects such as the dynamics of the population, the technological innovation, the life cycle of the product/service, the financial innovation etc; however, essentially this is a dynamic gap between the demand and offer on a specific economic market;

b. They take the clock time (calendar time), which is not influenced by the measured process, as "counter" of the economic process. There are "typologies" of the economic cycle established according to the clock time: annual or sub-annual cycles, decades, centuries etc.

c. The take the market of the goods and services as basic (primary) economic market, while the monetary and labour markets are considered to be derived markets. This presumes, implicitly, that both the monetary-financial flows and the workforce flows follows closely, maybe with a lag or lead, the real flows. In the best case they accept that the monetary-financial flows, at least, might become autonomous in relation with the real flows, leaving thus room for imbalances, but these imbalances are still are taken to be at the level of the demand-offer gap on the market of goods and services. Although the economic crises of the past eight decades showed a completely different thing about the causality of the economic cycle, the orthodox theory has yet failed to go beyond this pattern of analysis and description. Such an example is the determination of the internal economic equilibrium with the IS-LM (investment-saving – liquidity-money) model, relying on the direct „communication" between the real economy market and the nominal economy market.

Figure 2 shows a traditional (orthodox) representation of the economic cyclicity:

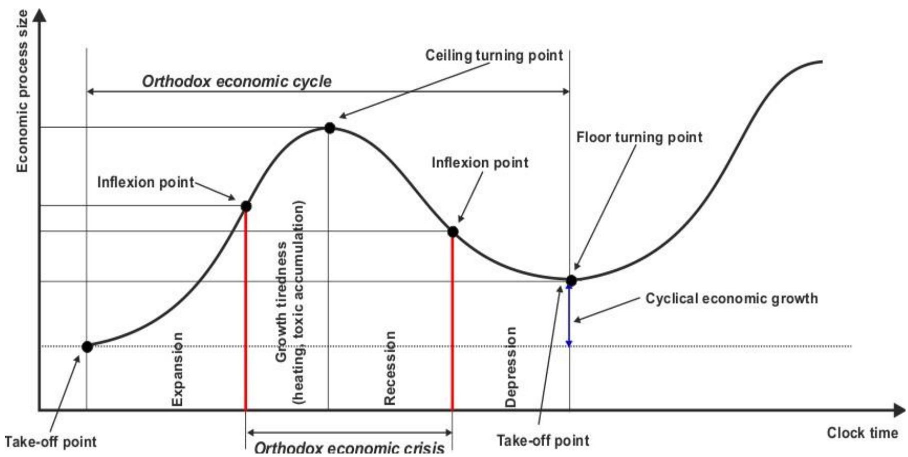

Figure 2. Abstract design showing the orthodox economic cyclicity

2. CRITICISM OF THE CURRENT FRAMEWORK AND SOME PROPOSALS

In our opinion, the orthodox models of the economic cycle can be reproached the following matters:

a. About the economic cycle structure

The most important problem of the economic cyclicity refers to its structure. The orthodox modelling of the economic cycle considers that the economic crisis is a component part (which covers a random time span) of the economic cycle (see Fig. 2). Well, in our opinion, the economic crisis has no such statute. In other words, the economic crisis doesn't belong to the structure of the economic cycle; actually it is a break, a discontinuity in this cycle. Furthermore, the economic crisis can not even signify a chronological separation between two successive cycles. The acceptance of such a "role" of inter-cycle chronological separation would mean to bestow a cyclic character to the economic crisis. Or, as mentioned previously, the economic crisis doesn't have a regulate frequency, not even according to the clock time, which means that the economic crisis is not cyclic. This statement has a particularly important conceptual consequence: the economic cycle is the rule of the economic process, while the economic crisis is an exception, an accident. Furthermore, the economic crisis has no causal connection whatsoever with the economic cycle, its aetiology lying outside the economic cyclicity, as we will subsequently show.

This means that an economic crisis can erupt any time within the economy, at any point on the "path"[5] of the economic cycle (remember the orthodox syntagmas of „economic crisis of overproduction", in which the crisis appears on the ascendant component of the economic cycle, as well as that of „economic crisis of underproduction", in which the crisis appears on the descendant component of the economic cycle). Therefore, the economic crisis will no longer play any role in the description (or worse, causal explanation) of the economic cycle, much less in the administration of the economic cycle. The outbreak of an economic crisis is a disaster for the economic cycle, which is disturbed, deformed, "turned around"[6] etc. The effect of an emerging crisis within the economic space is analogous with the effect produced within the physical space by the formation of a black hole (the analogy can go further to the huge consumption of economic resources, financial mainly, just like the black hole eats a huge amount of cosmic matter).

Once we have discarded the economic crisis from the phenomenology of the economic cyclicity, the economic cycle should have the following shape (Figure 3).

[5] Here, the term of path must be understood in a non-mechanicist meaning (for instance, it has no reversibility and, if it has one, it is characterised by hysteresis), rather as a metaphor which we can not get rid of when describing a dynamic economic system.

[6] See below the considerations on the turning points which appear in the matter of the critical thresholds between the real economy and nominal economy.

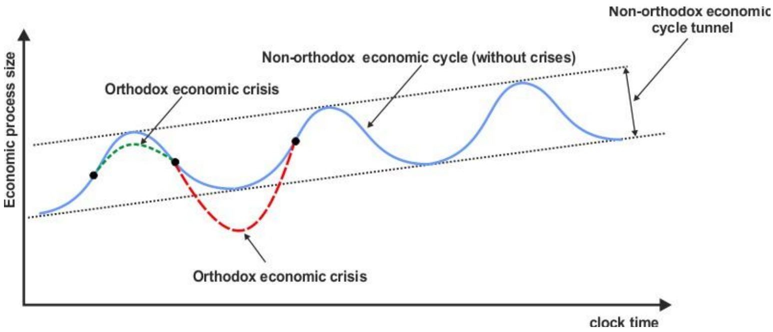

Figure 3. Abstract design showing the orthodox economic cyclicity vs. the non-orthodox economic cyclicity

b. About the economic cycle causality

If we accept an economic cycle with no economic crises (let us call such an economic cycle, an "adjusted economic cycle" – AEC), and if we take the economic cycle to be a "natural"[7] phenomenon, then AEC is the form of movement of the economic phenomenon (as macroscopic manifestation of the economic process). Therefore, AEC causality lies in the permanent, oscillating differences that may appear between the basic determinants of the economic cycle. Which are these basic determinants? In our opinion, they are: 1)the anticipated financial flows; 2) the current real flows (of goods and services); 3) the anticipated demand for workforce; 4) the current demand for direct investments.

Figure 4 shows a graphical description of our proposed understanding of the origin of the economic cyclicity (causality), of the economic crisis. Here are some comments:

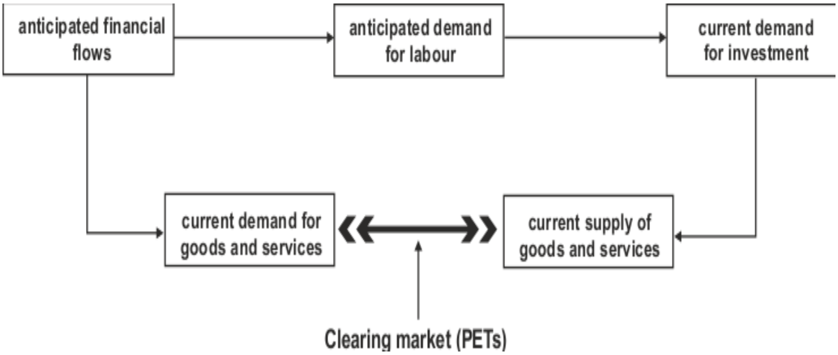

Figure 4. Abstract design of primary economic transactions (PET) generation

[7] We will show, subsequently, that the economic system is a logically vivid system, so the attribute of natural must be understood here with the meaning of logically necessary. So, neither the common meaning of non-vivid system, nor the more sophisticated meaning of phenomenon of equilibrium (present, for instance, in syntagmas such as "natural interest rate", "natural unemployment rate", "natural price" etc.) are acceptable within the context.

- The main economic transaction is accomplished via the relation between the real flows (goods and services created by the real economy) and the financial flows (exact monetary equivalent value of the real flows). The exchange rate of these transactions is the average transaction price. The average transaction price also includes the cost of the transaction generated by the search, by the information asymmetry and by the adverse selection. We will call this category of economic transactions: primary economic transactions (PET). Figure 4 shows the causality relation of TEP:
- The causality of both the economic cycle and the economic crisis lies within the general relation between the real economy and the symbolic economy[8]. The essential difference between the two categories of causalities (which justified our proposition to remove the economic crisis from the structure of the economic cycle) is that, while the economic cycle is
  generated by a mechanism of sway, of oscillation of the sign of the gap existing between the real flows and the financial flows, the economic crisis is generated by a mechanism of asymmetric accumulation, i.e. by a strong increase of the nominal flows in relation with the real stocks[9]. Figure 5 shows the distinction between the two types of causality:

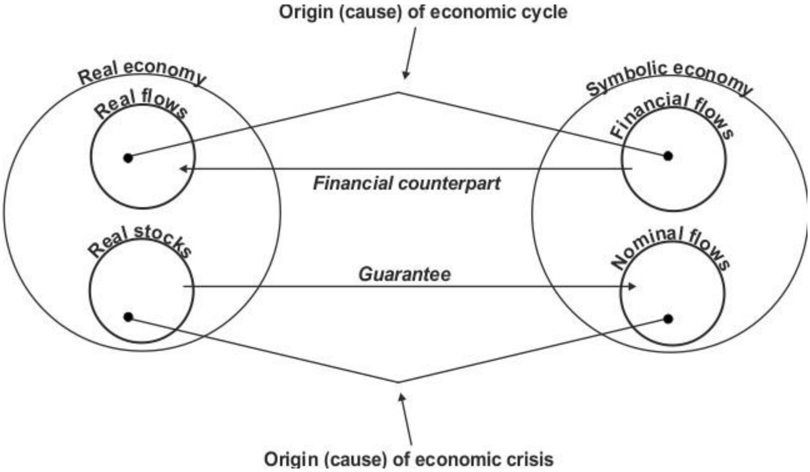

Figure 5. Origin (cause) of the economic cycle and of the economic crisis

[8] We maintain the hypothesis that the labour force market derived from the market for goods and services.

[9] Of course, there is a possibility that a nominal flow is not guaranteed by a real stock, rather by a future financial flow (for instance, a bank credit is guaranteed by the wage). In this case, the part of the total financial flow which forms the collateral for the reimbursement of the loan must be considered, conceptually, as a stock, not a real one, rather a financial one because, in principle, that part can not be used as equivalent value for the real flows, rather as source to reimburse the ban k loan. However, in most cases, the collateral for the nominal flows is in the nature of the real stocks.

c. About the economic crisis causality

We will develop briefly what we mentioned above regarding the causality of the economic crisis.

- As already shown, an economic crisis can develop, most times, independently of the phenomenology of the economic crisis. This statement relies on two arguments: 1) a theoretical argument – the nominal flows are not intended to cover directly a demand for goods and services; rather, they are intended to do it in an indirect manner (for instance by starting a business or by direct investments of portfolio which supply the financial flows used as equivalent value for the real flows). Even if, in some cases (for instance, the credit for consumption) the nominal flows can be used as direct equivalent values for real flows, these are isolated cases, not significant as proportion. Therefore, this argument says that, in principle, the nominal flows are autonomous (as amplitude or dynamics) in relation with the real flows; 2) an empirical argument – the economic crisis of 1929-1933 and that of 2007-2010 have shown that the main generative cause of these crises has been the huge and fast multiplication of the nominal flows in relation with the real stocks in support.
- Therefore, the economic crisis can start and develop in parallel with the development of the economic cycle. There is interference between the phenomenology of the economic cycle and that of the economic crisis. In this matter we propose the following evaluations: 1) structurally, the two phenomena are independent between them, as already mentioned before: the economic crisis is not a component (stage, phase) of the economic cycle; 2) strictly causally, there may be the following interference: the anticipation of the financial flows variation, which is also reflected in the variation of the demand for labour force (see Figure 4) and which leads eventually to the variation of the current demand for investments, generates the variation of the nominal flows (for instance, higher demand for bank credits, but there can also be influences in the current variation of the inflation, of the interest rate, or of the exchange rate). Therefore, we don't exclude a possible causal interface between the phenomenology of the economic cycle and that of the economic crisis, but, most times, it is not decisive, rather complementary to the autonomous dynamics of the nominal flows; 3) functionally, we will accept, nevertheless, a massive interference between the two phenomena. The onset of the economic crisis, which refers to the start of the accelerated multiplication of the volume of the nominal flows in relation with the volume of the real stocks, will affect both currently and by the formation of anticipations, the form and speed of the economic system on the path of the economic cycle. Actually, the outbreak of an economic crisis can change rather radically the phenomenology of the economic cycle[10].

d. About the measuring of the economic cycle

[10] We will refer in detail to this aspect at the time when we will propose a set of critical thresholds at which the economic crisis can interfere in the economic cycle.

At this point we will introduce our second suggestion to review the theory of the economic cyclicity (the first one was the causal and structural separation of the economic crisis from the economic cycle). In our opinion, the measurement (calculating) the economic cycle via the clock time is incorrect. The economic process develops its own pace of progress of the economic action. We suggest that actually only from the perspective of the clock time there is an economic cycle, while from the perspective of the economic time[11] the dynamics of the economic system is linear[12]. Figure 6 shows the appearance of the economic cyclicity by operating the clock time instead of the economic time:

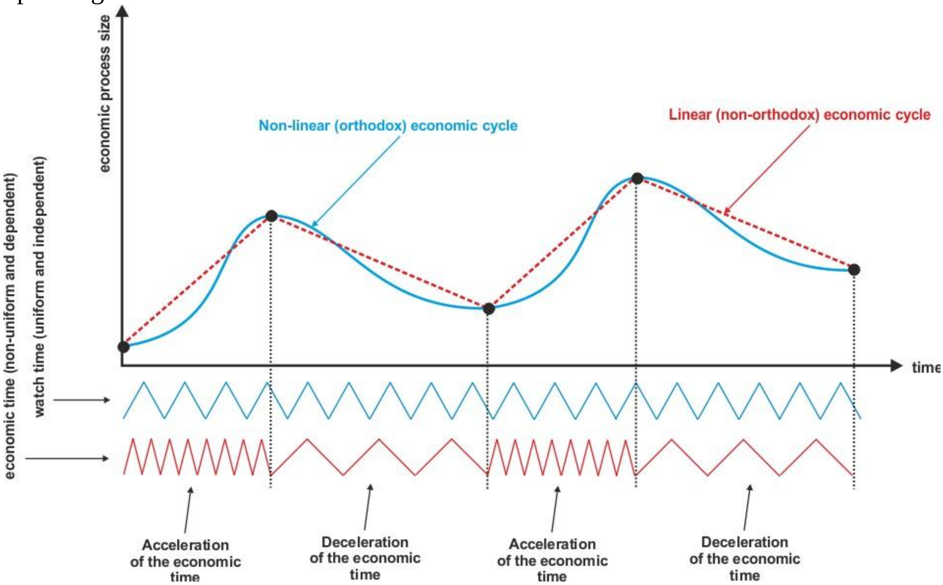

Figure 6. Linearization of the economic cycle when using the economic time instead of the clock time

The next theoretical step is to define the quantum of economic action (QEA) and to associate such a quantum to each unit of economic time (UET). On this basis we can introduce the accelerations of economic time in relation with the clock time simply by accepting the fact that several QEA occur within the same unit of clock time. As this is impossible once that we defined the QEA as the amount of economic action which can be produced in the UET, it results that the only way in which we can accept the occurrence of several QEA within the same unit of clock time is to accept the occurrence of several UET within the same unit of clock time, that is to say, to accept accelerations of the economic time in relation

[11] As it hopefully resulted from the introduction to this study, the economic time is the result of three categor ies of time (counters) of the economic process: the clock time (the natural time), the institutional time (introduced by the norms regulating the economic action), the psychological time (introduced by the presence of the subject within the economic process, which generates anticipations and interests).

[12] This conclusion might prove particularly important in terms of the necessity to ensure the epistemological simplicity not just for the theory of the economic cyclicity, but also for the methodology and technology to deal with this phenomenon.

with the clock time (in terms of the pace). On this basis we can introduce the following conjecture: the pace of the economic time is directly proportional to the intensity of the economic process which, in turn, is directly proportional to the density of the economic space. The distinction between the physical space (associated to the clock time) and the economic space (associated to the economic time) can lead to the idea of a continuum of the economic space-time (CEST). Both the space and the time of a CEST depend on the economic process, therefore they are no longer autonomous in relation with it. On this basis we can construct the economic "geodesics", the minimal "routes" of the economic action (production, distribution, consumption, saving etc.) on the basis of the principle of minimal action in nature (Maupertuis' principle). In the following studies we intend to develop this suggestion of a CEST so that we can adjust consequently the theory of the economic cyclicity.

3. THE CEST MODEL OF THE ECONOMIC CYCLE. SOME BASIC ASSESSMENTS

All the above allow us to make a synthesis of our proposed new vision on the theory and methodology of the economic cycle and on the theory and methodology of the economic crisis. Essentially, our proposition is as follows:

First, the economic crisis must be considered distinctly from the economic cycle, having its own causality, although there are some causal interferences (particularly functional) with the economic cycle. The economic crisis is not a periodical phenomenon, it doesn't have a inherent cyclicity. The cause of the economic crisis is the autonomous increase of the nominal flows in relation with the real stocks, due to multipliers much higher than the unit, working at high speed. The economic crisis can, however, disturb seriously the economic cycle when it bursts out, but economic cycles with no financial crisis are also possible.

Second, the adjusted economic cycle (AEC), the economic cycle where the economic crisis is not a structural part, is a rather symmetric oscillation from a trend which, according to the currently dominating economic culture, has an increasing course[13]. This structural trend or path is given by the succession of the inflexion points of the curve which describes, bidimensionally, the economic cycle (see Figure 3).

Third, we consider that there might be a second order cyclicity (by developing Kuznets's suggestion on the secondary economic cycle), which results from the trend of the first order economic cycle (the primary economic cycle). Following are some notes on the second order cyclicity:

a. The shape of the second order cyclicity is described by the curve joining the inflexion points of the first order cyclicity; in all the cases in which no economic crises occur within a first order cyclicity, the clock time

[13] According to the sustainability criteria, we may also accept, in our opinion, decreasing trends, as humanity will pass from the paradigm of the optimality (currently dominant and causal generator of the process of financial and economic globalization) to the paradigm of sustainability. Within this context, it becomes perfectly possible (or maybe even necessary) to have a theory of the sustainable economic ... decrease.

distribution of the inflexion points is "smooth", meaning that if follows closely the rate of increase of the particular economic activity (for instance, of the GDP);

b. Unlike the first order cyclicity, this cyclicity includes, as component part, the economic crisis; therefore, the second order cyclicity will influence the first order cyclicity, meaning that the disturbances which the economic crisis has on the location of the inflexion points of the first order cyclicity determines the new coordinates, in the clock time of the latter cyclicity; figure 7 shows a synoptic image of the interference between the first order cyclicity and the second order cyclicity. The mechanism shown in the figure describes the way in which the secondary economic cycle plays a role of pilot[14] for the primary economic cycle.

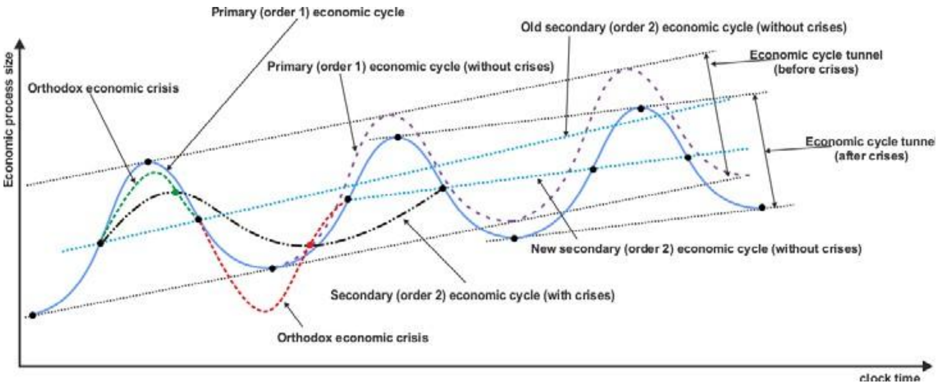

Figure 7. Interference between the primary economic cycle and the secondary economic cycle

Fourth, both the adjusted economic cycle (AEC) and the economic crisis must be quantified/measured and evaluated in the specific, proper time introduced above: the economic time. Based on the impossibility of a quantum of economic action to "take" more than one unit of economic time, we can draw the conclusion that the economic cycle measured in the clock time shows stages of growth or decrease, as the economic time is accelerated or decelerated in relation with the clock time. Actually, according to the economic time there are neither stages of growth, nor stages of decrease; the economic activity merely intensifies or moderates. Analogously, we will consider that an intensified economic activity generates a more dense economic space, while a moderated economic activity generates a more rarefied economic space. Therefore, the variation of the economic space density is equivalent with the acceleration of the economic time and vice versa. Within the context, we consider that we should talk of a continuum economic space-time (CEST), and all the trajectories which we associate to the economic cycles must be actually considered as geodesics of this CEST. On the basis of the economic rationality, we will therefore

[14] Practically, the secondary economic cycle builds permanently (by integrating the effects of the economic crises occurring at different moments in time) the configuration pattern of the next first order economic cycle.

admit that the economic activity describes economic cycles
(measured by the dimension of the economic activity) which simply follow these economic geodesics[15].

The utility, both conceptual and methodological, of introducing CEST in the description of the economic process is even more important when we discuss about the economic crisis. The autonomy of the nominal flows formation in relation with the economic stocks makes the acceleration of the economic time within the phenomenology of the nominal flows to be huge in relation with the clock time[16].

Fifth, as seen before, the continuum of the economic space-time is generated by the presence of the economic subject within the economic process, which introduces the third time within the equation, the psychological time, next to the clock time and the institutional time. The removal of the economic subject in the modelling of the economic process (inheritance of the wrong analogy of the economic science with the physics[17]) has been one of the major epistemological errors in the development of the economic science. The presence of the subject within the economic process turns the economic system into what we can call a logically vivid system. In relation with the standard system, the logically vivid system has two additional attributes: a) dissipativity (the capacity to preserve or reduce the entropy) and b) the structural and functional redundancy (replication principles for the structures/functions which "decay"). Bothe attributes are generated by the presence of the subject. Therefore, the economic cycle should be studied within the paradigm of the logically vivid system. Within this paradigm, the economic time will link to the two mentioned attributes of sufficiency and the resulting model of the economic cycle will by utterly different from the orthodox one. The living, non-human nature also has logically vivid systems but this kind of systems manifest with predilection within the systems where man is present as individual or as social group. Furthermore, if we add another two supplementary attributes to the logically vivid system: a) the exclusive existence of the hyper-cycles (any output of a subsystem is an integrated input of another subsystem or

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[15] Of course, this is a mere suggestion (largely relying on intuition). The formal part is yet to be constructed. The main problem at this point is: do we still need models of economic optimization, if the path of the economic process is "forced" t o follow the economic geodesic? Our answer is negative: the economic modelling will have to give up optimization and this is equivalent with the transition from the paradigm of optimality (current today, together with its methodological corollary: the homo oeconomicus model) to the paradigm of sustainability. This study makes only brief mentions a bout these more general aspects of the economic epistemology and methodology.

[16] An example at hand is the very empirical analysis of the economic crisis of 2007-2010. Both the explosion of the financial derivatives, on the one hand, and the insurances/reinsurances succession of the bank credits (with the resulting huge moral hazard) multiplied enormously the value of the nominal flows in relation with the value of the real stocks, which is equivalent, as shown above, with a staggering acceleration of the economic time in relation with clock time and a corresponding increase of the density of the nominal economic space.

[17] Also see the current development of a very epistemologically dangerous hybrid: the econophysics.

of a system within the environment of the particular system); b) dominance of the anti-cyclicity (any positive feedback is wrapped by a negative feed-back), then we will get an extremely important "institutional device": the sustainable system. Therefore, in the broadest paradigmatic framework (the most adequate, in our opinion), the economic cycle should be evaluated (described, explained, forecast etc.) within the context of the sustainable economic system. The proposition to remove the economic crisis from the structure of the economic cycle aims to bestow properties of sustainability to the economic cycle. Within this context, the economic crisis is an extraordinary, non-periodical, event which takes the economic cycle out of its tunnel of oscillation, out of its tunnel of sustainability (Figure 7). Actually, the intervention of an economic crisis disturbs the economic cycle, relocates its points of inflexion (as well as the points of cycle minimum and maximum) and yield, as shown, a secondary order economic cycle.

CONCLUDING REMARKS

The CEST model of the economic cycle involves the following basic logic consequences:

a. The separation of the generative causality of the economic cycle from the generative causality of the economic crisis. The economic cycle and the economic crisis are two distinct "institutional animals", despite some superficial resemblance. While the economic cycle is generated by gradients (quantitative gaps of gaps of dynamics) between the real economic flows and the counterpart financial flows to the real economic flows, the economic crisis is generated by gradients (quantitative gaps of between the real stocks and the nominal flows.

b. The mechanisms (regarding the economic cycle or the economic crisis) driven by gradients of amplitude (quantitative gaps) differ, both in terms of "anatomy" and in terms of "physiology" from the mechanisms drive by speed gradients (gaps of dynamics);

c. The clock time is not fitted for the quantification/measurement of the economic process in general thus of the economic cycle or of the economic crisis; the proper time for this quantification/measurement is the economic time, which is a resultant of the three categories of counters associated to the economic process: the clock time, the institutional time and the psychological time;

d. "Decreeing" the content of economic activity of a unit of economic time as always representing a quantum of economic action[18] allows drawing a basic conclusion for our endeavour: the economic time actually is linear. Its non-linearity, which is conveyed to the shape of the economic cycle, is generated by the acceleration of the economic time in relation with the clock time. The same considerations are valid for the evaluation of the economic crisis phenomenology; analogously, the economic space is "deformed" in relation with the physical space: it is

[18] The definition of the quantum of economic action remains one of the basic issues (actually the key-problem) of the CEST model of the economic cycle, respectively of the economic crisis.

more dense than the associated physical space when the economic time is accelerated in relation with the clock time, and it is more rarefied than the associated physical space when the economic time is decelerated in relation with the clock time; This continuum of the economic space-time generates the geodesics on which the economic process "moves" (actually we only have economic cycles if we measure the economic process with the clock time; if we would measure it with the economic time, we would see that the path of the economic process simply follows the economic geodesic);

e. The reintroduction of the economic subject within the economic process, in the economic cycle too, sets the epistemological conditions to use the paradigm of the logically vivid systems (or, broader, of the sustainable systems) to describe, explain and forecast the economic cycle, he economic crisis. The four ingredients which we propose for a new vision on the economic cycle (removal of the economic crisis from the structure of the economic cycle, the use of the continuum of the economic space-time, the introduction of the secondary economic cycle, the introduction of the paradigm of the logically vivid systems and of the sustainable systems) might bring, in our opinion, useful contributions to the new bases of the economic cycle theory within the current conditions of globalization.

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ELLIOTT WAVES IDENTIFICATION AND FINANCIAL INDEXES FORECASTING ON THE BASIS OF FUZZY LOGIC THEORY

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Abstract: There are developed the method and the model of identification and forecasting of financial indexes using methods of fuzzy logic theory taking into account a number of specific rules of price curve development from Elliott wave theory. The carried out analysis of forecasting results on real data confirms the reliability and high effectiveness of the developed method.

Keywords: fuzzy logic, financial time series, identification, forecasting, Elliott wave theory, membership function.

1 Introduction

Forecasting of financial indexes development is intricate and exceptionally important problem. The volume of investor’s income or losses depends on the rightness of prognosis. A fund market is representative in this sense, as every its participant makes transaction based on own prognosis of securities courses development. At forecasting of financial time series an investor aims to get a maximal profit but not minimal standard deviation as it’s accepted in the case of function approximation. And in most cases a trade result will depend exactly on the rightness of the predicted sign of course changes, in fact the player of fund market is aimed mainly on the receipt of income from the speculation for the rise and fall of prices. Therefore it has meaning to tune up the forecasting model exactly on the direction of change of price, instead of course value.

At the choice of mathematic tools for forecasting of securities courses a lot of various methods were put to the test by an author [10]. It was decided to renounce from the methods of extrapolation because of strong sensitiveness to the errors, in particular case on the edges of the examined interval. The methods based on the spectral analysis were found very sensible to the errors in initial data and that is why often resulted about the presence of regularities in the explored process which actually no exist.

At forecasting of securities courses it's desirable to apply such approaches which don't use the principle of averaging, as at the calculation of the expectation, moving averages (weighing and exponent) or regressive models. The usage of averages expects of stability of variables and invariance of environment. However, there are no bases for a hypothesis about variable’s stability on the selected sampling. It is possible to use some of these approaches only for the preliminary data processing before realization of forecasting model.

The method of neural networks turned out deprived of many noted failings at forecasting. Nevertheless, the prognosis of the next value of securities courses aims to repeat the value of previous day. Exactness of prognosis can be promoted by the inclusion in the model of different factors or by effective initial data processing and searching of optimum network configuration. But, in any case, precision of the predicted directions of course changing remains very low – little more than 50 %.

At that rate, with the purpose of income maximization from realization of purchase and sale operations at the fund market, it is useful to carry out tuning of model and do the proper prognosis exactly on the signs of securities courses changes. In this case the using of fuzzy neural network represents certain advantages on comparison with ordinary neural network. In particular, the process of tuning of model is simplified (unlike ordinary neural networks it is not necessary to fulfill research on determination of number internal layers and amount of neurons in these layers). The possibility of identification and forecasting of financial time series development in accordance with the set of rules appears with the use of theory of fuzzy logic.

2 Target setting

In general, the problem of financial time series forecasting with the use of various mathematical methods counts some centuries already. The series of the works devoted to forecasting of financial indexes with the use of fuzzy logic method appeared lately [1-3, 8]. In these works the forecasting is carried out by presentation of regressive functions in a fuzzy form and by processing of statistical material without taking into account the rules of time series development. It means that the difference from the classic methods of extrapolation is only in transition from the point values to the intervals, which are transformed by some functions of the specified type (membership functions).

To develop the identification model using fuzzy rules it has meaning to use other approach. There are a few methods to form the set of deciding rules. The automated extraction of rules from the initial time series can be carried out using the models of the Sugeno type [14]. Nevertheless, for such fuzzy models there is the problem of understandable interpretation of their parameters. For example, the made up of knowledge base with the parameters of tuning, which are the difficult functions of some specified type is not enough informing for a specialist in field of economy. As a result the user of the forecasting system loses the feeling of principles of its functioning that conduces to complication of process of the system tuning and of searching of the ways of the prognosis exactness increasing.

It is possible to set deciding rules on one's authority based on experience of experts in corresponding field. In this case for development of the forecasting system it has meaning to use the fuzzy model by the Mamdani type [9]. In the works [7, 15] the principle of forecasting on the basis of expert estimations of many influence factors is exposed. The set of deciding rules is formed on the base of these factors.

In the work [6] it's represented a similar example of exchange rate forecasting taking into account expert estimations of such factors, as a level of inflation, monetary reserves, state of fuel and energy complex, agriculture, environmental conditions, reflection and others. In any case there is the problem of adequate choice of set of factors, and also the estimations of experts are very subjective. In addition, these factors changing slowly and it is possible to carry out forecasting on their basis only for the long time intervals for the long-term capital investments.

Therefore there is the necessity of model development, which will avoid expert estimations of the selected factors and will functioning on such incoming data which will give possibility to forecast the price changes also for short time intervals. Author offers for forecasting of securities courses changes to use the Elliott wave theory [12], which can give the powerful instrument for establishment of the deciding rules set for the price curves of different time intervals. Let us construct the mathematical model of forecasting of securities courses changes based on fuzzy logic with the use of specific set of rules of the waves develop, in particular, waves with extensions.

3 Basic rules of the Elliott waves development

Bu the Elliott low every market decision is a result of significant information, and generates the significant information. Every transaction is included in the structure of market being the result and by transaction data transmission to the investors the one of the reasons of their behavior simultaneously. This feed-back is conditioned by mans public nature and generated some figures of certain forms. As figures recur periodically they have the forecasting sense.

Development of market prices finally adopts the figure of five waves with the special structure. Three waves from them, marked by numbers 1, 3 and 5 on a Fig. 1, really produce the directed motion and are the waves of motive (impulsive) style. They are divided by two countertrend interruptions – the waves of opposite direction, which are the waves of corrective style and are marked by numbers 2 and 4 on the Fig. 1. The important feature of this structure is an expressive tendency to the self repeating not only in time, but also in space – within the limits of itself.

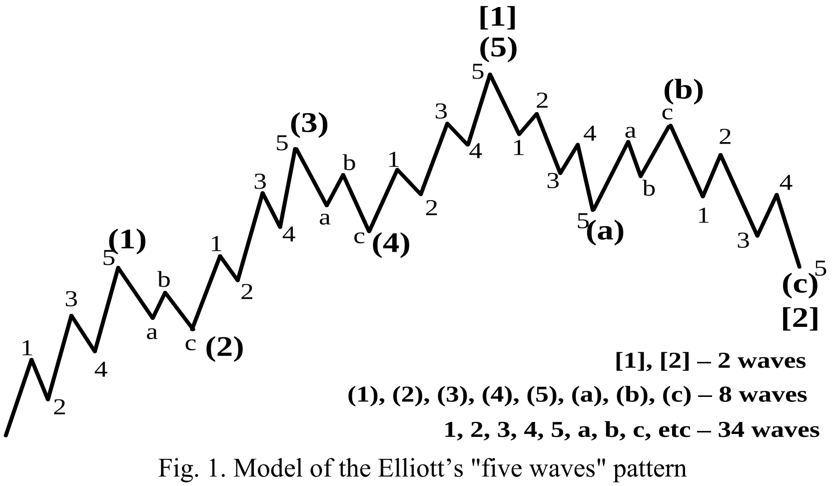

Fig. 1. Model of the Elliott’s "five waves" pattern

Most impulses contain the extension according to Elliott. Extensions are elongated impulses with exaggerated subdivisions. The majority of impulse waves contain an extension only in the one of their three actionary subwaves. In a nine-wave sequence it is occasionally difficult to say which wave is extended. However, it usually doesn't signify as since according to Elliott system a pattern of nine waves and a pattern of five waves have the same technical significance. For simplification of knowledge base forming we present the price curve as combination of waves with one extension of not identified type in every wave of upper wave level (e.g., see Fig. 2).

4 Forming of knowledge base of fuzzy rules

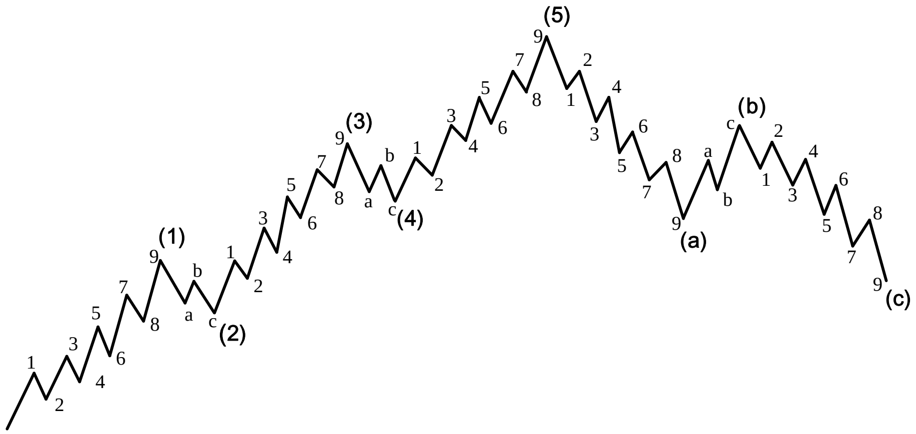

Fig. 2. Price curve with the extended impulses

At forecasting of price curves development to the entries of fuzzy neural network the several of the successive last values of the financial time series are given, and to the output of network a next value of series is given. At model tuning the optimization of its parameters is carried out by means of minimization of deviation between the forecast and real data for all time series.

To form the set of fuzzy rules it is necessary to formalize the input and output variables. Lets assign $x_1, \ldots, x_n$ – the set of values of input variables, y – the proper value of output variable. For description of prices behavior at the market we will use the aggregate of logical rules which characterize the direction of course changing and its conditional size. So, for estimation of the linguistic variables $x_1, \ldots, x_n$ we will use the integrated scale of terms: S – slump, MS is a moderate slump, MG – moderate growth, G – growth. To estimate the output variable y in addition to stated terms we will add yet two: CS – considerable slump and CG – considerable growth. We propose to make transaction of purchase and sale just at peaks of waves of high wave level taking into account of transaction charges.

Let us form a knowledge base on the basis of represented rules and additional instructions which describe the specific of market behavior. Every row of knowledge base (e.g., see Table 1) is a fuzzy rule for identification and forecasting of financial index development. Let us set of fuzzy rules taking as basis the model of Elliott market development (e.g., see Fig. 1, 2).

To form the base of fuzzy rules we will present the sequence of securities course changes (for input variables) for different types of extensions in the bull and the bear market with next value of course change (for output variable). In order to prevent the overload of knowledge base and also to avoid the use of all possible variants of the price curve development we don't distinction in description of different types of extensions. All inaccuracies of the knowledge base will be revealed and taken into account in the fuzzy neural network after its tuning on the real data. In this case for fuzzy rules forming we designate by the G term all motive waves of lower wave level, which develop upwards, and by the S term those waves which develop downwards. The slumping corrective waves of lower level we designate by the MS term, and increasing waves by the MG term.

As an example we will present the forming of fuzzy rule which specifies the further considerable growth of analyzed index course. We have first considerable course growth in the peak of wave (2) of high wave level of model (e.g., see Fig. 1). Accordingly, the output variable in this point takes the CG value. The slump of securities course precedes to this position on the graph that is x_8 has S value. Before it the eight-wave Elliott model specifies the moderate growth of price (x_7=”MG”). Earlier there was a slump (x_6=”S”), and before it was the course growth to the top level of wave 5 (x_5=”G”). In the same way the rule is described to the first variable x_1 and it is added to the row 11 of Table 1. The combination 12 specifies the considerable growth of course at the point (a) of Fig. 1.

The rule of knowledge base 13 specifies the considerable growth of course which is at the point (a) or (c) on Fig. 2 (in this case x_8 = ”S” as subwave 9 of corresponding wave, x_7 = ”MG” as subwave 8, etc.). As we have good profitable potential at point 2 of waves (1), (3) and (5) we may make forecast of further considerable growth of financial index. This situation is described in rules 14 and 15 of knowledge base in the Table 1. In the same way we form entire knowledge base of price curves development based on basic and specific rules of Elliott wave theory and inscribe it in the Table 1.

Table 1

Set of rules of price curves development with wave extensions

Number of input combination	Input variables								Weight of rule	Output variable
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	w	y
11	G	MS	G	MS	G	S	MG	S	w_1^{CG}	CG
12	G	MS	G	S	MG	S	MG	S	w_2^{CG}	
13	MG	S	MG	S	MG	S	MG	S	w_3^{CG}	
14	MG	S	MG	S	MG	S	G	MS	w_4^{CG}	
15	G	MS	G	S	MG	S	G	MS	w_5^{CG}	
21	S	MG	S	MG	S	G	MS	G	w_1^{CS}	CS
22	S	MG	S	G	MS	G	MS	G	w_2^{CS}	
23	MS	G	MS	G	MS	G	MS	G	w_3^{CS}	
24	MS	G	MS	G	MS	G	S	MG	w_4^{CS}	
25	S	MG	S	G	MS	G	S	MG	w_5^{CS}	
31	G	S	MG	S	G	MS	G	MS	w_1^{G}	G
32	MG	S	G	MS	G	MS	G	MS	w_2^{G}	

33	G	MS	G	MS	G	MS	G	MS	w_3^G	
34	MG	S	MG	S	MG	S	G	MS	w_4^G	
41	S	G	MS	G	S	MG	S	MG	w_1^S	S
42	MS	G	S	MG	S	MG	S	MG	w_2^S	
43	S	MG	S	MG	S	MG	S	MG	w_3^S	
44	MS	G	MS	G	MS	G	S	MG	w_4^S	
51	G	MS	G	MS	G	MS	G	S	w_1^{MG}	MG
52	G	S	MG	S	MG	S	MG	S	w_2^{MG}	
61	S	MG	S	MG	S	MG	S	G	w_1^{MS}	MS
62	S	G	MS	G	MS	G	MS	G	w_2^{MS}	

Lets us represent the mathematical form of fuzzy rules with the use of membership functions and weight coefficients for forecasting of value of course considerable growth (CG), that is the component part of complete model of identification and forecasting of the price development:

$$\begin{aligned}
&\mu^{CG}(x_1,\ldots,x_8) = w_1^{CG}\left[\mu^G(x_1)\cdot\mu^{MS}(x_2)\cdot\mu^G(x_3)\cdot\mu^{MS}(x_4)\cdot\mu^G(x_5)\cdot\right.\\
&\left.\cdot\mu^S(x_6)\cdot\mu^{MG}(x_7)\cdot\mu^S(x_8)\right]\vee w_1^{CG}\left[\mu^G(x_1)\cdot\mu^{MS}(x_2)\cdot\mu^G(x_3)\cdot\mu^S(x_4)\cdot\right.\\
&\left.\cdot\mu^{MG}(x_5)\cdot\mu^S(x_6)\cdot\mu^{MG}(x_7)\cdot\mu^S(x_8)\right]\vee w_3^{CG}\left[\mu^{MG}(x_1)\cdot\mu^S(x_2)\cdot\mu^{MG}(x_3)\cdot\right.\\
&\left.\cdot\mu^S(x_4)\cdot\mu^{MG}(x_5)\cdot\mu^S(x_6)\cdot\mu^{MG}(x_7)\cdot\mu^S(x_8)\right]\vee w_4^{CG}\left[\mu^{MG}(x_1)\cdot\mu^S(x_2)\cdot\right.\\
&\left.\cdot\mu^{MG}(x_3)\cdot\mu^S(x_4)\cdot\mu^{MG}(x_5)\cdot\mu^S(x_6)\cdot\mu^G(x_7)\cdot\mu^{MS}(x_8)\right]\vee w_5^{CG}\left[\mu^G(x_1)\cdot\right.\\
&\left.\cdot\mu^{MS}(x_2)\cdot\mu^G(x_3)\cdot\mu^S(x_4)\cdot\mu^{MG}(x_5)\cdot\mu^S(x_6)\cdot\mu^G(x_7)\cdot\mu^{MS}(x_8)\right],
\end{aligned}$$

where $\mu^y(x_1,\ldots,x_8)$ – the function of membership of input variables vector $x_1, \ldots, x_8$ to corresponding linguistic term y of output variable;

$\mu^f(x_i)$ – the membership function of variable x_i to term f;

w_p^y – the weight coefficient of rule p, $p = \overline{1, k_y}$, for linguistic term y (this coefficient clarifies the correctness of established rule and may have the value in interval from 0 to 1);

k_y – the number of rules of knowledge base which are relative to linguistic term y of output variable.

If we made an error in determination of turn point and the price curve has changed the direction before forecasted moment it’s necessary to discover an error to correct the system and make right purchase and sale transaction at the fund market. The recognition of the moment of course direction changing is possible by the ordinary methods of technical analysis. For example, one of indicators of course changing may be the crossing of the moving average with the price curve. This situation indicates the beginning of new wave of high wave level. But it is necessary to determine the optimal depth of series averaging.

The presented in Table 1 knowledge base of trend changes forecasting can be applied as for the short-term, as for the long-term investments. Nevertheless, this is only the "ideal model" and it's not necessarily that course of analyzed financial index will behave in so certain and predicate way. After model tuning on the real data it will be appropriated to every rule the own weight. And so the conclusive prognosis will be carried out taking into account the specificity of course development of each interesting securities.

The forecasting system of the financial index changes was developed with the use of fuzzy logic method on the basis of the constructed model, formed knowledge base and proper algorithms. Let us carry out the analysis of performances of this automated system.

5 Carry out the experiments for optimization of the system

As statistical material for fuzzy model checking the Standard & Poor's 500 (S&P 500) index was chosen, because such securities of exchange fund are the assets of the diversified portfolio, formed based on known and reliable financial instruments. Similar sort of investment allow to make the risk of losses lower in the case of one portfolio cost falling and decrease influence of market gossips and recommendations of financial analysts which are at the fund market.

Trading securities of exchange funds it's enough to have the general information for the condition and tendencies of financial markets and to use the basic methods of technical analysis [5]. In addition, during conducting of auctions with the securities S&P 500 there is possibility for the active investing, as indexes of exchange funds are determined not on the end of auction day, as for the opened mutual funds, but are counted during a day at every turn.

To analyze of the financial index forecasting results, model was tuned on the time series of index S&P 500 closing prices for period from July, 1 1995 to June, 30 2004. Verification of forecast was carried out on a test selection from July, 1 2004 to July, 1 2005.

One of the most essential stages of the forecasting system analysis is the determination of optimum type of membership functions, which are used for model constructing. This analysis revealed some interesting moments of functioning of the system which were not elucidated in the proper specialized literature earlier. So, at first the author offered to build the forecasting model of financial index price changing with the use of trapezium membership functions, as they allow fixing limits after which variable values don't belong to other term. In particular case it is important for a border between terms, which determine growth of course, and terms which indicate to its falling (for example, negative change of course is not conformed to the growth term). In addition, the form of trapezium membership function makes an opportunity to set the minimum size of oscillation, as it is represented on the Fig. 4.

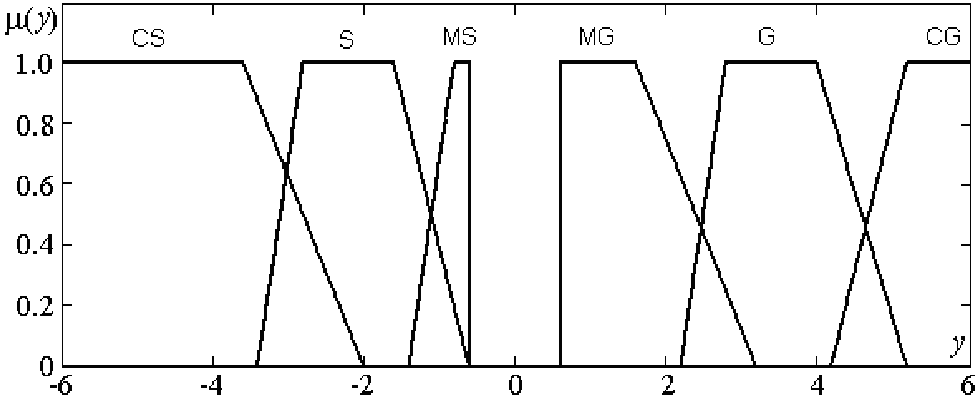

Fig. 4. Trapezium membership functions of variable output after model tuning

Nevertheless, at the forecasting both to and after tuning, the output value "0" of the system is often appeared, although such value is not among the possible values of output variable (e.g., see Fig. 4). This situation may be explained in this way. Each deciding rule of knowledge base is presented by the multiplication of membership functions of all input variables to the set terms that is usually replaced by operation of minimization. In any case, if current combination of input variables not absolutely accords to established rule (i.e., if any input variable does not accord to own term in the rule) then the value of one of membership functions become equal to the zero. In this case, as a result of multiplication operation the output of this rule also becomes zero.

And if among the logical rules, represented in a knowledge base, there is no one which completely corresponds to combination of input variables then the output of model also will be a zero as a result of maximization operation among the calculated values of all rules. This situation is not appropriate for the membership functions, which are flexible for all range of a function, because in this case the values of membership function always are positive and different from a zero, even in the distant points from the center of function. In addition, it should be noted that bad capacity for tuning is another great shortcoming of trapezium membership functions because of its undifferentiating. Therefore, it was accepted the decision to build a model without the use of trapezium membership functions.

Let us to consider the features of the differentiated symmetric membership functions for offered approach on the example of bell-shaped functions. At first, these functions practically eliminate the possibility to set the bounds of theirs changes. That is to say (e.g., see Fig. 5) as a result of model tuning two terms MG and G, which specify the growth of course, have values of membership functions in some neighborhood below of zero greater, than for the term MS, which is responsible for these values of output variable.

Moreover, the membership function of the G term passes with the high values between the functions of terms S and MS. It means that the fall of course for 1.5 %-2.0 % will be interpreted by the system as growth, as the G term has most value of membership function among all other linguistic terms for this interval. A similar situation meets after optimization of more simple models, for example, in the work [13], but the attention was not attracted to this there. Nevertheless, in our case this situation is critical and impermissible on account of capital loss. So, for example, for membership functions of output variable (e.g., see Fig. 5) the forecast of slump of index for 1.8 % will be recognized by the system as growth with require to purchase of securities.

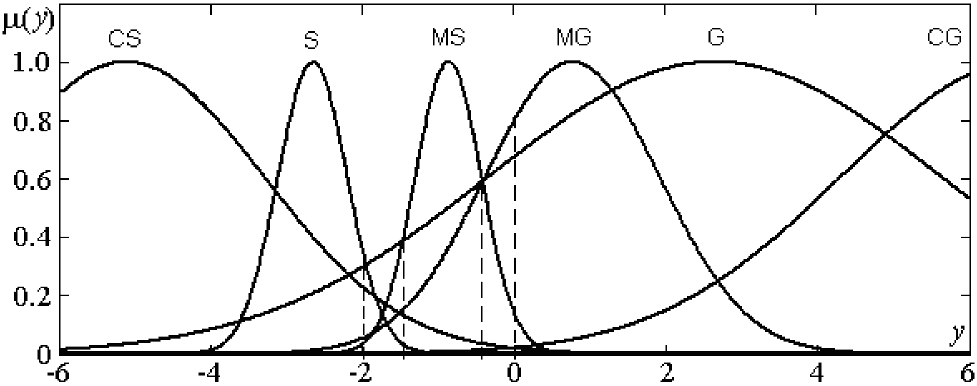

Fig. 5. Bell-shaped membership functions of output variable after model tuning

With the purpose to eliminate this glitches of mentioned functions, for the model development we will construct the membership function taking into account the best sides of the functions referred above. Therefore, we will form the membership functions for the use in our approach on the base of trapezium with the flank edges in the form of the proper sides of bell-shaped functions (3) with its own parameters. On the Fig. 6 this membership functions of output variable are represented after model tuning on the time series of the index S&P 500 closes for period from July, 1 1995 to June, 30 2004.

Our membership function is appropriate for tuning, because it is differentiable, and also it allows fix the bounds for all terms changes, which is very important for the features of task. The usage of membership functions of such type considerably curtail the time and improve the quality of tuning (substantially decrease the error of tuning), raise the profitability of the optimized system functioning. Below we will present the results of analysis of functioning of the developed fuzzy system, built with the use of the offered membership functions.

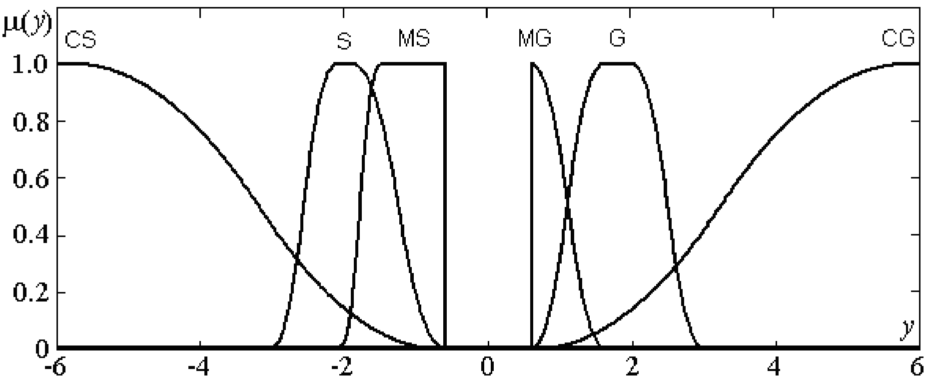

Fig. 6. Membership functions of output variable, formed taking into account the features of task

6 The analysis of efficiency of the developed system

When the verification of reliability and efficiency of the system was done, the purchase and sale transactions were carried out at every appearance of the CG and CS terms on the model's output without taking into account the transaction expenses, without optimization of volume of capital for investing. Although securities of the exchange fund S&P 500 can be bought with the use of broker credit (subject to future payments), this practice was not used at analysis of result of working of the developed system. When the output variable was equal to term of considerable growth (CG) of the S&P 500 index then the purchase of securities was carried out for entire sum of available monies. And at the forecasting of the considerable slump (CS) of index all set of securities was sold simultaneously. Calculations were conducted for an initial sum in $ 100 000 that enables practically to eliminate the influence of broker's charge to eventual result of the experiment because size of commission relies on the transactions volumes substantially.

During the auction year represented by a test selection the system made 42 transactions, from which 26 were profitable (the percentage of the profitable transactions is 61.9 %). Without taking into account transaction expenses general profitability of the system was 13.05 % of annual. This is a high enough effect on comparison with other trade systems and investment alternatives [4, 5, 11]. It is necessary to mark that a market was not attractive for a year, because profitability of the index S&P 500 for this period makes 5.8 % (although the average growth rate of this index for period from 1926 to 2001 was 11.3 % per annum).

Efficiency of developed system is more advantage in compare with results of functioning of various technical systems and indicators for forecasting of the exchange fund S&P 500 [5]. This research showed that most systems of technical analysis demonstrate a less profitableness than passive investing by principle „purchase and hold" (only two systems from thirteen showed profitability a bit greater than average margin of profit of the index). However, it should be noted that for saving of cleanness of experiments these systems were tested each taken separately, and in the real trade to increase the exactness of forecasting it is used not only one system, but certain of their combinations.

If at July, 1 2004 we passively allocated $ 100 000 into securities of the exchange fund S&P 500 than to July, 1 2005 we would have the sum of $ 105 802. The system, developed in the work, makes profit in size of $ 13 048 for the same initial capital, that means the state of the account is $ 113 048. We will notice that the system profitability was only 6.13 % before its tuning on the real data and an eventual sum on the account was $ 106 134 in the time of 29 profitable transactions from the 44 made (there is 65.9 % of the profitable transactions).

Such high percent of the guessed directions of course change with the use of model before its tuning indicates the correctly built of set of rules of market development, true election of the Elliott wave theory as a basis to form of knowledge base and introduction of oscillation minimum size for preliminary processing of financial time series. Nevertheless, profitability of made transactions with the use of model before its tuning is quite low, though and it is higher than market profitability. Model tuning on the real values of financial index allowed substantially promoting the profitability of made transactions which specifies by successful approach to construction of membership functions and by correctively built algorithm of model tuning on the real data.

If we make the purchase of securities of the exchange fund S&P 500 at every appearance on the model output of the CG, G or MG terms (that is, all terms which specify the further growth of course), and carry out the sale of the securities S&P 500 at appearance of the CS, S and MS terms, then profitability of the optimized system will go down to 8.78 %. It confirms the expedience of initial financial time series ecimation and of insignificant course changes not take into account at forecasting. If we do not fulfilled the ecimation (that is, to set the minimum size of the oscillation $R = 0$), profitable potential of the system declines to 6.11 % at 35 profitable transactions from 56 made (there is 62.5 % of the profitable transactions).

One way or another, the developed model for the speculation for the rise and fall of prices with the real money it is necessary to use only after comprehensive its verification at the various markets, different time intervals. Thereby it is possible to expose and remove of the model's defects. And if the tuning of the developed fuzzy model was carried out used one sequence of financial data, then the control of forecast quality must be carried out with use of subsequent data of the same time level. Let's notice that the developed approach demonstrated steady profitability for various time intervals, which specifies for possibility of its use for the real securities purchase and sale transactions at the fund market.

7 Conclusion

The method and model of identification of development of financial time series of the Elliott wave type and forecasting of financial indexes changes on the basis of fuzzy logic theory is the main result of the article. The specific rules of market development were taken into account at construction of model, in particular the wave extensions. The developed model enables to avoid the expert valuations about the selected factors in contrast to known approaches of financial time series forecasting based on fuzzy logic. Such approach reduces the subjectivity of eventual decision. Also our model allows to obtain the equally exact prognosis both for short-term and for the long-term investments.

The analysis of functioning of the developed system was carried out for finding of model's failings for further its improvement and determination of conditions for getting the biggest profit. Also it was carried out the analysis for determination of optimum type of membership functions for the model. As a result it was exposed the deficiencies of classic membership functions for the forecasting model and it was offered the approach for construction of new membership function taking into account the specific of the set task.

To improve the efficiency of the developed system it is necessary to associate it with different indicators and methods of technical analysis. It's possible by means of introduction to the system the additional analysis of transactions expedience for the purchase and sale of securities, predictions of current wave ending with the purpose of more exact determination of transaction moment, calculation of capital volume for investment etc. The carried out analysis of forecasting results on the real data with usage of the developed automated system testified high enough efficiency and confirmed authenticity of our approach.

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UNCERTAINTY ABOUT MAJOR CHALLENGES IN THE 21ST CENTURY

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Abstract.

What will come first – non-acceptable global warming or extinction of oil reserves? Both processes can bring substantial costs to the mankind, but their order has important economic implications. It is well known that global warming is driven by the growth of concentration of carbon dioxide in the atmosphere, but the translation of particular emissions into temperature change and further into costs to cope with that is highly uncertain. If we decide that this is the major challenge for the 21st century, we should stick to Kyoto-type agreements and to limit use of fossil fuels, replacing them by biofuels and other renewable energies. In this case the market value of resource-holding companies should drop since full exploitation of their reserves will bring global warming to non-acceptable level.

On the other hand, we have global scarcity of fossil fuels. The proven oil reserves are only for 40 years of exploitation, but there is uncertainty in the volume of non-discovered reserves and in the revealed reserve volumes of such countries like Saudi Arabia and Russia. In the case of business as usual (BAU) we will come eventually to global oil peak, where global oil production reaches its historical maximal level and then drops. The socio-economic consequences of it will be quite substantial, and the process of full oil substitution by alternative fuels can take several decades. If oil peak comes before substantial global warming and if oil replacement is slow, then the value of oil companies would skyrocket.

The presented model is about the expected value of global future output, when both costs are taken into account. An important policy implication comes when ex ante predictions about order of both challenges will not coincide with actual realization. In this case we can observe a bubble with value of oil companies, when their value first declines and then grows up (or visa versa). This market disequilibrium drive by irrational expectations (or false information) can be exploited by speculators in financial market, and the owners of resource companies should be aware about that.

1. Introduction

1.1. Vernadskij and Global Challenges

V.I.Vernadskiy was a great thinker who suggested the term of “noosphere” [1] as the product of interaction between human civilization and the nature. He suggested the dominance of Homo sapiens over other species, creation of global informational system and the control over global forces that can even change the shape of the Earth. The history has confirmed the validity of most of his predictions, but we also have got some problems from global changes of such scale. This paper is also about the global phenomena that are likely to dominate the global threats for mankind in the 21st century.

1.2. Global Warming

Global warming (GW) is currently considered as the major challenge of the 21st century. The global temperature index has increased by 0.5-1 degree C during the last century, being accompanies by substantial growth of CO_2 in atmosphere. If the trend continues, it will cause many damages to the nature: falling crops and desertification, melting mountain glaciers causing shortage of water, rising ocean level, etc. Scientists are uncertain about the temperature rise during the 21st century: it can be only 1.5 degrees in optimistic scenario and 4.5 degrees in pessimistic. The temperature rise also depends on the pattern of future emission reduction. However, Kyoto protocol was not signed by all countries, while emission trading permit scheme in Europe ended with price collapse, causing the destruction of incentives to reduce emissions. In order for the scheme to work (causing shift from coal to gas), the carbon price should be close to 30-50 euro per ton, but it is much lower today.

While emission reduction should be on international agenda, the level of its priority over other problems is still quite uncertain. This uncertainty comes from lack of scientific knowledge about both the physical consequences of growth of carbon in atmosphere and on the associated social cost of climate change. Countries are also asymmetric here: Russia may gain more benefits in Siberia, while Mediterranean countries can suffer desertification. While is it generally agreed that the total cost for the world of each degree added is positive, the absolute levels of associated costs are highly uncertain. And those cost can trigger the policy of how much fossil fuels (and of what type) should be extracted, where we should set the global limit.

We have quite substantial uncertainties about its dynamics, costs and possibilities to influence it. So far, the rise of global temperature during the last 50 years just by1 degree has been observed. Indeed, we observe more frequent climate extremes; but are they due to this 1 degree change? When the level of global ocean will start to rise substantially due to ice melting? When we get plus 3 or plus 5 degrees? Most scientists in climate change agree that many unacceptable changes will occur when the average global temperature will rise to 3-5 degrees. But how much fossil fuels should we burn to get there? In the recent IAEE conference in Dusseldorf (F.Holz) I heard the following numbers. There are 416 Gt of coal in reserves of conventional gas, 662 of oil and 1832 of coal. Besides, there is also non-conventional oil and gas. All this makes above 3000 Gt of coal. But catastrophic consequences for climate change will occur, if we burn 1000 Gt. So we should shift to renewable before all fossil fuels are extracted. But who has given this magic number of 1000? Yes, it is in literature, but literature of global warming is not yet based on solid research grounds. It may happen that this border of 1000 is fuzzy, and be something between, let say, 500, and 5000. Also we do not know what does “unacceptable change” mean exactly.

1.3. Scarcity of Fossil Fuels

However, the global scarcity of fossil fuels (FS) might be another challenge to the mankind. While the known reserves of all fossil fuels are more that for 200 years of use (at current consumption), known reserves of conventional oil are only for 40 years. But what will happen if Saudi Arabia and/or Russia overestimate its reserves? There is “oil peak” theory saying that oil production will peak globally, i.e., supply cannot be increased even if demand will grow. The term has been introduced by Hubbert King [3] who has correctly predicted the maximum of US oil production to occur in 1970. Since that time oil has peaked in many producing countries, and soon can peak globally. Salameh [4] says that conventional oil has already peaked in 2006, but not all scientists agree with that. In any case, there is also non-conventional oil (tar sands). When oil prices became above 70 $/brl, it became profitable for Canada to exploit them produced (but economic and environmental costs are high).

Will this peak be at 90, 100 or 120 bbrl/d, nobody knows. Also it is not clear if it will happen in 10, 20 or 30 years. There are also studies about potential severe consequences of oil peak as a global shock to economy with very high costs, if it comes unexpectedly. (see, for example, Robert and Lennert, 2010). Yes, we got rid of oil almost completely in electricity production and to high extent in heating, but we have no other fuel yet for air transport even on paper, and we need few decades to make transition away from oil in other transport.

1.4. Paper Goals

The goal of this paper is to show very important policy implications of the order and timing of those challenges. The transition to renewable energies and energy saving are considered to be a proper answer to both. However, the speed of those processes might be not sufficient. Suppose that full transition to renewable energies does not occur before either GW or FS will reach some critical level, implying non-acceptable social cost. The problem is that present science cannot exactly estimate neither the level nor the social cost of global warming; it can do only probabilistic estimate. On the other hand, we do not know the volume of non-discovered fossil fuels. Pessimists think that global oil reserves are just for 40 years, and oil peak will come soon.

The goal of this paper is to compare both dangers and to evaluate their influence on commodity markets, in particular, on price dynamics for fossil fuels. We are aware of great uncertainties in the structure of consequences of global warming and oil peak on the world economy. That is why we develop a highly stylized model that might be far from reality. It is also not clear whether the economic damage from both will be temporal in time or not. The role of discount rate is also very important for economic decisions.

2. The Model

In the baseline scenario it is assumed that no significant efforts will be taken in global reduction of CO_2 emissions to acceptable level and in global transition to renewable energies. How it is possible to compare the danger for the mankind from non solving those problems during acceptable time period? Such a danger definitely exists. While there are many talks about global transition to renewable energies, there is still some skepticism about the speed of implementation (Wirl & Yegorov, 2013).

The reduction of discovered oil reserves was predicted by King Hubbert [3] for a particular country (USA), and now there are theories about possibility of global oil peak in the coming years. However, there is an uncertainty about its exact timing: pessimists tell that it is coming already this decade, while optimists think that this will be in 20-40 years. The situation with natural gas is better: the current proven reserves are for 70 years, but there is still higher speed of new discoveries than depletion. Finally, shale gas revolution is doubling the known gas reserves, making it not scarce for at least a century. The known reserves of coal are for 200 years. While all these numbers are finite, high economic discounting of future (about 3-10% if we use banking interest rate) makes those problems too distant for "economically rational" present generation.

What about climate change? We have an observed increase of an average global temperature by 0.5-1 degree in the last 50 years, and most likely it has anthropogenic character. But we do not know exact hazard and its composition from the next degree of temperature increase, and so on. Most scientists consider the 2 degree increase barrier as acceptable, and derive the patterns of global reduction of CO_2 emissions that would satisfy this goal. However, others think that 3 or 4 degrees can be an acceptable limit. We also have quantitative uncertainty in the relationship between concentration of CO in atmosphere and global warming. And we do not know the threshold below which the global processes (like ice melting and an increase in the ocean level) are reversible.

Given all those uncertainties, it makes sense to deal with a simple model of hazard dynamics from global warming and fossil fuel scarcity. Since we use exponential discounting in future, it is mathematically easier to deal with exponential dynamics of hazard cost as well. First of all, it is increasing and convex function (C'>0, C''>0). Second, it does not add mathematical complexity.

2.1. Cost of Global Warming

None of economic models are correct, but some are useful. We have too little information today to suggest a good model, but we will demonstrate some economic policy consequences on a simple caricature of reality. The first question: what is finite over time? It is likely that any irreversible changes to climate are likely to have very long living economic implications in the form of cost for mankind.

That is why we assume that the social cost of global warming changes exponentially over time, but the power of exponent has some distribution:

$$C_{GW}(t) = A \exp(kt). \tag{1}$$

Assume also that the global economy has constant population (this is indeed not so unrealistic assumption today, when demographers have theories of its stabilization at the level of about 10 billion people) and that economic growth is b, giving the dynamics of value of output Y(t)=Bexp(bt) (this does not account for environmental cost). Assuming that r is discount rate, the overall (expected) utility is given by:

$$EU = E\int_0^{\infty} e^{-rt}(Be^{bt} - Ae^{kt})dt \tag{2}$$

Suppose that k is uniformly distributed in the interval $[k_1, k_2]$. Then the integral (2) is given by

$$EU = \frac{1}{k_2 - k_1}\int_{k_1}^{k_2} dk \int_0^{\infty} e^{-rt}(Be^{bt} - Ae^{kt})dt. \tag{3}$$

The inner integral converges for all trajectories with k<r and diverges for all trajectories with k>r. If k_2<r, it is always converges. We will focus on this case for the moment.

Let us calculate the expected utility.
$EU = \frac{B}{r-b} + \frac{A}{k_2-k_1}\ \ln\,[\frac{r-k_2}{r-k_1}].$

This utility is positive if B $(k_2-k_1)/(r-b)$ > A $\ln[(r-k_1)/(r-k_2)]$. Several factors can influence this outcome.

This happens when initial economic output B is relatively high comparing to initial cost of global warming, and this is the real case. If economic growth rate, b, is higher than the maximal growth rate of damage cost, then intertemporal utility will always remain positive, and the ratio of positive to negative term will grow This means that hazard from global warming is relatively low and can be easily compensated by economic growth. In this case the k can be misperceived and should be paid less attention in future than we believe today.

It may also happen that $k_1<b<k_2$. Then under some trajectories economic cost of global warming will offset economic output in future. Now the discount rate r plays crucial role. For high discount we simply do not care what would happen with the word in many years from now, since we enjoy more short term horizon where the balance of benefits and costs is still positive.

However, when the discount moves down and becomes below the maximal growth rate of cost from global warming, $r<k_2$, we face first mathematical problem related to nonconvergence of integral. In this case the growth of cost cannot be offset by discounting, and we get the utility of minus infinity. Even if this happens only for some possible trajectories, this already becomes nonacceptable, and we have to combat global warming. Note that we need really low discount for that.

It is useful to do some calibrations. We know from global warming literature that the global temperature can increase between 2 and 5 degrees Celcium over the 21st century. What level of k does it impose? No such estimations are known to us, and this gives additional uncertainty. Suppose that an increase by 1 degree will lead to growth of economic costs of global warming by x. Suppose also that temperature growth will be linear in time. Let us try to give some estimates of x. If harm is coming moderately, then we can assume 5 degrees increase can mean cost increase by factor e=2.71… . Since k=1 means increase by factor e in 1 unit of time (year), an increase by factor e in 100 years requires multiplier 0.01. Let us assume that in 100 years temperature increase will occur linearly I time and cumulatively on 5 degrees. Then k=a dT(t), and dT(t)=T(t)-T(0)=ct, where c=0.05 for pessimistic scenario of 5 degrees growth (and c=0.02 for optimistic scenario of 2 degrees growth) over a century. Since kt=act and kt=1 for t=100, we have k=ac=0.01. For c=0.05 this gives a=0.2. Now we can fix a=0.2 and find that for optimistic scenario with c=0.02 we get k=ac=0.004. Thus for our moderate assumption of damage growth we get uncertain k between 0.004 and 0.01. Adding uncertainty on damage growth can only increase this range, let say, to $0.002<k<0.02$.
Now let us compare this rough estimation of k with discount, r, that prevails in modern economy. Note that our calibration requires mapping of 1% annual interest rate (proxy for the discount) into 0.01. Indeed, 1% discount prevails in some developed economies at present, but the long term average is higher, close to 3%. In such economies like

Russia it is rarely below 10%, or 0.1. This means that the condition $k_2<r$ holds for global economy at present. What does this mean? It means that markets should not be sensitive to the future hazard of global warming, if we have rational perception of future hazard and its growth over time. Unless the global interest rate will drop below 1-2%.

2.2. The Hazard from Oil Peak

Now consider the influence of oil peak. Many models in environmental economics assume that oil substitute comes immediately when oil price hits some threshold. However, we did not observe it for any data in the past (including the peak of 148 $/barrel from summer 2008). Most likely, this transition to oil substitutes from renewable energies will take some time T (also uncertain, but we skip this complexity in our model) independently on oil price. Thus, the timing of oil peak t* (random variable, let say, with uniform distribution between 0 and 40 years) will create a period of costs to cope with oil peak. For the sake of simplicity, we assume this cost to be constant (K) and to have duration of T. Moreover, we assume that economic growth fully stops for this period.

Thus, the economic utility in the sub-model of oil peak has the following shape for a particular realization t*:

$$U_{op} = \int_0^{t^*} e^{-rt}\, Be^{bt} dt + (Be^{b\,t} - K) \int_{t^*}^{t^*+T} e^{-rt}\, dt + \int_{t^*+T}^{\infty} e^{-rt}\, Be^{b(t-T)} dt \;.(4)$$

We can calculate those integrals.

$$U_{op} = \frac{B}{r-b} - \frac{Bb}{r(r-b)} e^{-(r-b)t^*}(1 - e^{-rT}) - \frac{K}{r} e^{-rt^*}\,(1 - e^{-rT}).$$

2.3. Comparison of both hazards

It is important to compare this threat with one from global warming. Both utilities, U_{gw} and U_{op}, have the same positive term, B/(r-b), that describes baseline utility in the absence of global warming or oil peak, and the negative term, measuring the corresponding hazard, H_i. The negative terms in both expressions thus correspond to cumulative hazard from both challenges. The hazard from global warming,

$H_{gw} = \frac{A}{k_2-k_1} \ln\,[\frac{r-k_1}{r-k_2}]$,

depends crucially on the difference between the maximal growth rate of damages over time with economic discount. The hazard from oil peak,

$$H_{op} = \frac{Bb}{r(r-b)} e^{-(r-b)t^*}(1 - e^{-rT}) + \frac{K}{r} e^{-rt^*}\,(1 - e^{-rT}),$$

depends positively (loss is larger) on K (intertemporal cost of peak) and its duration (T). At the same time, it depends negatively on the time before peak, t^*. This hazard also depends on discount r in a complex way.

Fig. 1 shows hazard density with discount of 2%, while Fig. 2 does it for 10%. As we see from Fig.1 (relevant for OECD countries), the economic hazard from global warming has very low decline in time. This happens because of high variation of possible growth rates of damage over time. The hazard from oil peak is not much discounted, since it will come in few decades. But still it may happen that public opinion on the cost of global warming is higher. The values of cumulative hazard at Fig. 1 are comparable (0.228 vs 0.249). Still, it may happen that $H_{gw} < H_{op}$.

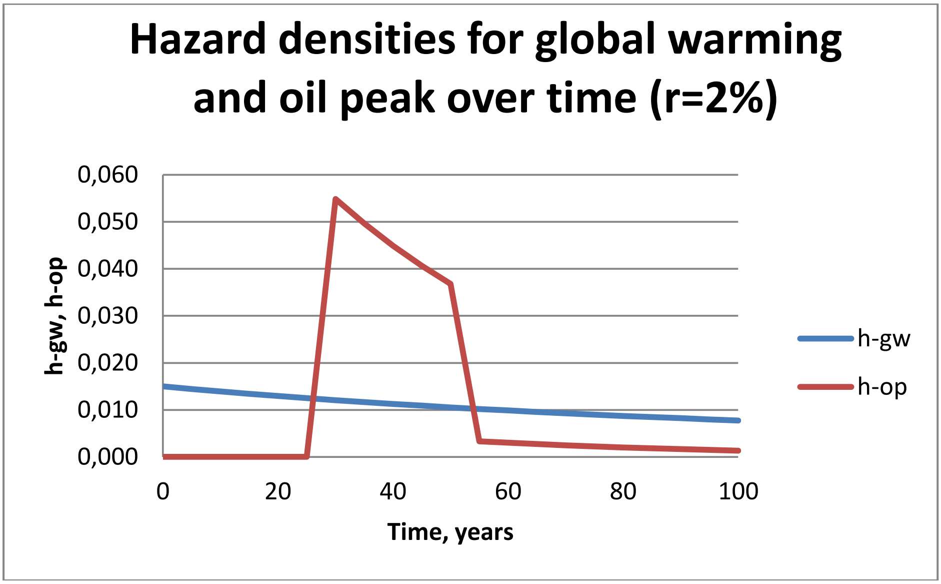

Fig. 1. Vision of future hazard density from global warming (h-gw) and oil peak (h-op) for annual discount r=2%.

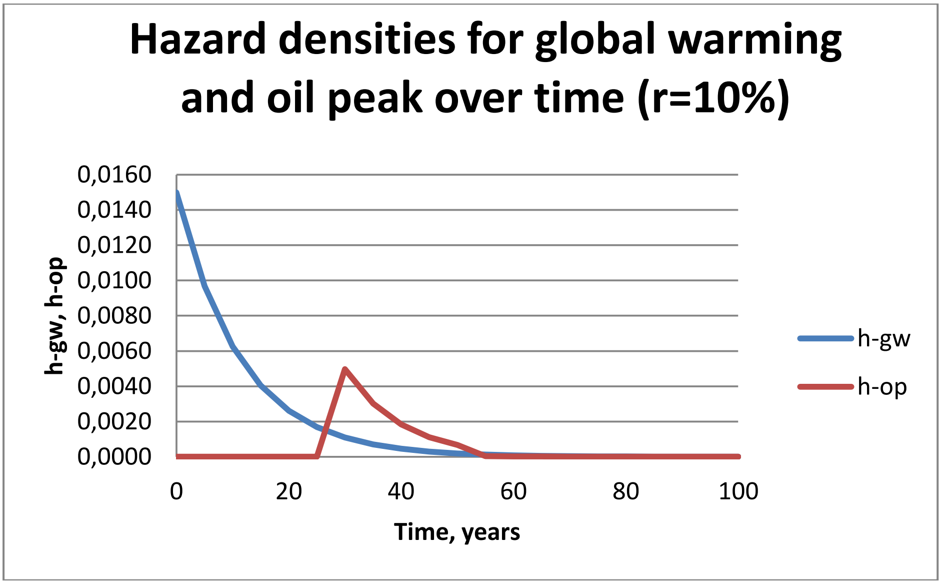

Fig. 2. Vision of future hazard density from global warming (h-gw) and oil peak (h-op) for annual discount r=10%.

The vision as on Fig.2 is typical for developing countries that are also exporters of oil. Contrary to the case of low discount (2%), the case of high discount sees little harm from oil peak (this is far away is time), but the hazard from global warming is also declining fast over time. Here we can have reverse: $H_{gw} > H_{op}$.

2.4. Economic Implications

Further we will analyze economic implications. First, we see that perceived expected harm from both oil peak and global warming depends on the believed path of future costs. We never experiences any of those yet (but when we will, it will be too late), but we might overestimate the danger from global warming. What will happen if the majority will believe that we should stop extracting fossil fuels (before they end) to combat global warming? The price of oil will not grow (or grow not much). In fact, developing countries (oil producers) will have in mind Fig.2 and panic more about negative consequences from global warming already in the coming 2-3 decades, when oil peak will not yet reveal itself.

Suppose now that our vision on global warming was too pessimistic (realized path of economic externalities grows not so fast over time), comparing to our vision of hazard from consequences of oil peak. Before oil peak we then treated oil as “bad”, and price of it was modest. Moreover, holders of oil reserves could have sold them to buyers (especially from OECD countries). They could rationalize such decision on the belief that some oil will never be extracted (due to global warming concerns). When (after nee information arrives) the world revises the hazards, it starts to value oil at high extent, and its price sky-rockets. At the short horizon, there is no substitute for oil in air industry (except from biofuels, but there price is also rising due to competition with agriculture for land), and R&D will require several decades.

3. Conclusions and Policy Implications

The idea of this paper was to show how the owners of oil reserves can be fooled, if they underestimate the danger from oil peak and/or overestimate the hazard from global warming. We are playing global lottery game, with a lot of uncertainties (and even information manipulation). We have to understand how resource owners can be stripped from their assets.

Another important issue (also highly relevant for Russia) is its high discount value. Even if we forget for a while about considered hazards, it is simply not rational to exploit oil in parity with Saudi Arabia, given that the latter have more oil resources in all scenarios. If Russian discount rate would jump from 10% to 2% (and even intermediate 5%), it would revise its production path, cut oil production and save it for future generations. Partly it will recover some current benefits from rising oil price.

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TOWARDS A COMPLEX-SYSTEMS ECONOMICS

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Abstract
Political economics, in developed societies, had its origin as a mere philosophical thought focused on social behavior; it was initially indistinguishable from theories concerning social organisation and ethics. In Europe, physiocratic doctrines, Liberalism, Socialism – for example – were substantially philosophical theories aimed at replacing religion in justifying political regimes or initiatives, social reforms and even revolutions, as a consequence of the transformation in the way of thinking that was brought about by the development of positive sciences. A number of European philosophers became convinced that both human history and societies could be investigated scientifically, like any subject of positive science, such as, for instance, chemistry, physics, medicine, etc. Such a belief is still hard to die, despite undeniable failures and tragic events undergone by billion people because of it. Social systems – as conditioned by local traditions, culture, resources and individual aspirations – are "complex systems", to mean phenomena that escape the human capability of final, objective and complete understanding**:** the perception of them is intrinsically affected by a predominant amount of uncertainty. A humble awareness joined to a cautious use of such a constraint might improve the effectiveness of our methods for understanding, with no ontological ambition and to the common benefit, what we can observe of complex self-organised systems. A simulation theory is here proposed to try an unbiased description of mental processes concerning the study of complex socio-economic systems.

1. Introduction

Political economics has its historical origin as a philosophy focused on the behavior of human commu-nities in developed societies, and was initially undistinguishable from theories concerning social organisation. Physiocratic doctrine, Liberalism, Socia-lism and other currents of sociological thought were substantially theoretical systems aimed at justifying political reforms and even revolutions, as a consequence of the transformation in the way of thinking that was brought about by Enlightenment in 18th Century, in conjunction with an extraordinary progress of positive sciences and development of technological innovation. From Locke, Smith, Fourier, Owen, to Marx, Comte, Mills and several others, a number of European philosophers became convinced that both human history and societies could be investigated scientifically, like any subject of positive science, such as, for instance, chemistry, physics, medicine, etc. Positivism and Neo-Positivism followed and dominated the philosophical debate for about half a century. The exceptional development of mathematics and statistics between the first half of the 18th Century and the first half the 20th Century have corroborated the socio-economists' conviction that sociology, econo-mics and even politics could be the subjects of scientific, objective and unbiased analysis. Such a belief is still hard to die, despite undeniable failures and social and economic disasters undergone by billion people in the world, as occurred because of the pretence to apply "scientific" criteria to the organisation and life of political communities. One example for all**:** Marxian analysis is still largely thought of, by its supporters, as an example of "scientific" socio-economic analysis, against which *more modern* (either Keynesian or neo-Keynesian or monetarist, for example) theories of political economics seem still inadequate. I doubt it is possible to apprehend, from either Marxian, post-Marxian or other contemporary schools of economics, of any movement of thought inclined to admit that theories of political economics do still persist in a philosophical sphere that has no connection with positive science.

The power of positive science consists in its capability to predict events, basically through calculation, with a high level of accuracy and to control physical phenomena to such a point to allow anyone, thanks to the technological output of applied sciences, to objectively benefit from the scientific achievements upon individuals' demand. In this connection, it is important to remark that positive sciences are used to deal with phenomena which can be represented, though schematically, with quite a limited – though experimentally sufficient – number of known parameters and variables**;** whereas the study of socio-economic systems, each with its own historical, cultural, sub-cultural, political and geographical identity, have proved impossible to be *summarized* and scientifically represented by means of a limited number of parameters and variables such as private and/or public investment capital, labor offer and employment rate, per-capita income, growth rate, monetary circulation, propensity to consume, marginal utility, population growth, demand elasticity, inflation rate versus unemployment, production functions, inno-vation impact, and a few other additional variables that complete the list conventionally addressed by theories and models of political economics.

Furthermore, most macro-economic indicators and parameters that are considered as significant is assessing aspects of developed and democratic countries, have quite often very low or no significance in underdeveloped countries, where development is more often an issue than a process, and political regimes have little or nothing to do with democracy and human freedom.

Thus, however high the theoretical persuasion power of economics, the facts are there, every day and especially today, to show that it is a field of studies unable both to make useful objective predictions on a secure theoretical basis and to control socio-economic processes at will.

What might appear paradoxical is that in recent decades economists have showed off theories and models built up by use of highly sophisticated mathematics**;** but such abilities are not *per se* sufficient to turn the mathematical reasoning based on arbitrary assumptions and abstract simplifications into scientific stuff.

Nevertheless, it seems also impossible to renounce any attempt to understand the behavior of human communities. Economics provides models for would-be effective interpretation of our common socio-economic behavior, because it is indispensable to understand something of what is going on, in a view to undertake any political as well as individual initiative.

Statistical economics first, and econometrics later, have been and still are reasonable ways to respond to our demand for understanding.

Certainly, the analysis of relationships between quantifiable events, i.e., the use and processing of statistical and other observational data regarding events and effects of social life form a more rational approach to the issue. Statistical analyses of economic processes are at least the best way to corroborate or – to the contrary – question and confute theoretical models proposed by economists. However, I do not know how many economists are aware of that statistics does inevitably introduce a crucial component of uncertainty in the analysis and interpretation of the study subject.

What I mean is that human communities are very complex *systems of interactions* between local institution, between institutions and individuals, and between individuals, all in turn heavily conditioned by local history and individual stories, culture, tradition, beliefs and expectations, climate, geography and much more, so that statistical analysis and data processing and econometric models, wherever possible, cannot avoid to omit too many significant *as well as fickle* aspects of each particular community.

Pre-selected theoretical options, cultural formation and prejudices do always bias the way in which we tend to represent the reality we observe, even when our basic purpose is to avoid any reference to general philosophical criteria. Yet, the situation would be even worse if theories and models concerning human societies could identify, assess and incorporate all imaginable variables and parameters**:** an insoluble problem would arise, because of the impossibility of establishing in principle the correct way to put them in relation with each other. Attempts of a similar kind, which regard another group of *complex systems*, characterise nowadays models of ecosystems, aimed at predicting the destiny of our planet's climate. The result consists in a remarkable confusion (strongly and obviously denied by the model builders), according to which almost *anything* – and the relevant opposite – may be *predicted*.

Unfortunately, as a regional planner involved in the preparation of development programs for several countries in different continents, I have had more than one occasion to be amazed by the dullness of "experts" from schools of economics of Western Countries**:** "experts" used to go – firmly relying on their pre-made conceptual tool-kits – to advise governments, especially of the Third World, affected by critical socio-economic conditions.

Shall we remind ourselves, for example, of the economic disasters that followed "advices" (they were actually imposed conditions) given by IMF or World Bank "expert economists" to Mexico, Argentina, Ethiopia, Russia, countries of South East Asia and others in recent years?

No eco in those experts' minds of the severe public self-accusation made by Milton Friedman[1] in 1972:

"In our capacity of economists we have caused major damages to the whole society and to our profession too, in promising more than that we can give. We have encouraged politicians to make odd promises and to infuse groundless hopes, since the results [*of the policies suggested*], though sometimes acceptable, remain far from the economists' Promised Land".

2. The spirit of econometrics

Econometrics tries to respond to the need for a closer approach to the study subject through any available quantitative measurement method and data processing instrument. Econometrical analysis focuses on the "quantities" identified both as statistical data and measurement results. Basic instruments of econo-metrics are multivariate analysis and correlations, with the relevant search for possible links between hypothesized causes and respective effects. The limits of these particular instruments are in the intrinsic "passivity" of the analyses, especially concerning macro-systems**:** any possible diagnosis or projection is – on the one hand – based on a

[1] M. Friedman, *Have monetary policies failed?,* in The American Review, 1972, LXII

drastically subjective data selection, generally with reference to particular theories or models of economics; and – on the other hand – econometric instruments have a substantially weak predictive power, because of the unjustifiable assumption that things tend to indefinitely follow the path they have gone so far.

Certain theoretical choices – such as, for instance, the criteria for data selection or the assumption of trend continuity – give almost immediate evidence to the observational and computational omissions.

Now, experience teaches that "complex systems" are just those systems on which the classifiable information is too little, fuzzy and intrinsically unstable for allowing analysts to make *deterministic* predictions**:** which is equal to say that "stable laws of behavior", capable of connecting precise causes to definite effects, are for such systems impossible to identify. *Evolution* of complex systems is something much more complicated than a *tendency* to behave.

After the monumental work of statistical economics carried out by Simon Kuznets, the first important attempt to take econometrics to the level of a comprehensive and unbiased methodology was made by Wassili Leontiev through his macro-analysis of the inter-sector relationships of a national economic system. The method sticks to the *measurement* of the interaction flows between sectors of a given economic system, with no other "prejudice" than the criterion adopted for identifying the various inter-related sectors.

To note: such a criterion alone is already sufficient to undermine the full objectivity of the analysis though.

The only hypothesis (and the crucial technical limit) of the method is that the *input* of each sector is made directly proportional to the respective *output*. In principle, concerning the production system, the hypothesis is hardly questionable**:** everybody would agree, for instance, that the amounts of coal, mineral materials, labor, energy, capital money, transport loads, etc. are directly proportional to the amount of steel produced**;** and so on for other sectors. The practical problems in applying the method arise when each of the identified sectors does not consist of a single type of production plant, but – because of an inevitable need for simplification – gathers the output of several different activities, which are *akin* but not *identical* to each other. So that the inter-sector transactions cannot be measured in homogeneous product units (e.g., in tons, or cubic meters, etc.) but only as transaction flows expressed in monetary units. Additional practical difficulties intervene when the analysis aims at long term predictions, which cannot necessarily account for the immanent *disturbing* role of technological innovation and unforeseeable changes in the price/cost of some inputs or in governmental policies.

Notwithstanding the inherent practical difficulties, Leontiev's conceptual approach to the economic macro-analysis is revolutionary, in that it does not break down the study system into selected conventional economic categories (labor, capital, investment, marginal utility, demand, offer, market equilibrium etc.)**:** instead, the analysis limits itself to identify and account for transactions between different *activities*, intrinsically and objectively measurable irrespective of their nature and of any cause or end that determine them. The methodological scheme, in other words, may be applied to any society and economic system, provided that the basic assumption is verified, i.e., that a certain degree of inter-dependency between the different activities exists. In itself, Leontiev's inter-sector analysis has no reference to any particular school of economic thought. Beyond all possible criticism, it is an important attempt to free macro-economics from philosophical speculation, with a view to keeping the observation of a complex system within a *least-biased* conceptual reference frame.

As known, after the original scheme proposed by Leontiev, the method has undergone a significant number of improvements and adjustments, and the *input-output* inter-sector analysis has been adopted by many governments for managing national accounts. It is a fact that the method, despite the approximations associated with the hypothesis of linear dependency between the system's activities, provides analysts with a useful *calculation* instrument to get credible short term indications about the expected impact on the whole system caused by possible alterations in the activity of one or more of its sectors. No other model can provide the analysts with a credible *objective* indication of what impact, for instance, on clothing industry could be expected from an increased investment in automobile industry, or what impact on fishery production could be connected to a decrease in the family savings.

Actually, the method constitutes the first *usable* instrument of *complex systems analysis*. The observation and measurement of interactions between human activities, along with the identification of the functional nature of the relationships, accounts for *all* that motivates and determines the behavior of the members of a self-organised human society, including the *chaotic* set of individual intentions, prejudices, errors and superstitions. *All this* is completely, as well as indistinguishably, expressed by the intensity of the measurable transactions.

3. A further step

The methodological jump made by Leontiev in addressing macro-economic issues is an encouraging suggestion to go further along the conceptual path he has indicated.

Leontiev's inter-sector analysis, as already remarked, is affected by one ill-working *functional* hypothesis, the one regarding the "technical coefficients" of direct proportionality between inputs and respective outputs. The analytical need for aggregations of various *akin* different activities makes the direct proportionality not only questionable, but systematically unstable with time, mainly – but not only – because of frequent alterations in the price set of the production factors along with unforeseen productivity changes in some of the activities considered. The method would be quite adequate, especially as for short run projections, if the "technical coefficients" would be constant quantities. Unfortunately, experience has widely shown that it is not so. This fact has actually implied a complicated and endless work of formal adjustments of the method together with a continuous activity of updating of the set of values forming the matrix of technical coefficients.

In years Seventy and Eighty of the past century, my professional work of regional analyst and planner has led me to re-consider Leontiev's methodological approach to complex systems from a more general point of view. I thought it was appropriate to exploit the fundamental importance of each interaction flow, this viewed as the conveyor of *all* the information inherent in the specific relationship it represents and expresses.

Upon the only assumption that not all the interaction flows between well identified components of a complex system are random flows, i.e., assuming in general that part, if not all, of those interaction flows are caused and motivated by specific ends (which I dub "intents"), I found it is possible to configure a mental image of the study system in a *quasi-neutral* way. Such a "neutrality" is affected only by the limits of the language we use both to describe what we observe and to process the findings of our observations. However, every language is the inherited basic instrument generated by the culture we live**;** it is used not only to represent but also *to understand* the reality we experience.

In other words, the perception of any object or set of objects occurs *both* through a physical contact (i.e., through senses and instruments) *and* through languages that can represent and describe the object perceived. It's just through the language that one can determine the modes of concentration and distribution of his attention.

The *linguistic institutions*, which pre-exist individuals and generations, not only determine a shared communication medium between different observers, but also - to a very large extent - *a shared way in which the world is perceived*. It's a physiologic *datum* that transcends individual mental attitudes and induces many to believe *naturally* that each of the terms and concepts, which belong to the languages used, are *objectively* corresponding to *things*, these being therefore perceived as objects that *pre-exist per se*.

The above premise intends to introduce the awareness that the identification, the definition and the description of whatever "system" is substantially a linguistic operation of a subjective nature.[2]

Complex human systems emerge and evolve because of "local constraints" that prevent possible interactions between members of a human set from being all random and meaningless. This statement implies that our mental activity inclines to use a concept like "degree of order" in observing "anything" we are able to classify as "system". Any system is such, in our view, to the extent to which we do not perceive it simply as a chaotic set.

The description of the behavior of a system depends principally on the criteria adopted for identifying its components.

The identification of the components does also determine the representation of the system's configu-ration, i.e., the seized distribution of the interactions within the system.

The salient property of any system is that all its components are *active*. The interactions that regard any system are both those that occur between different components of the system and those of each component with itself (*self-interaction).*

In this connection, it's worth remarking that also the "external universe", though theoretically not clearly identifiable in its own features, shall nevertheless be considered as existing and included in the set of the system's components, whatever the system. Also such "external component" generates (and confines in itself) a *self-interaction,* which consists in the amount of activity supposedly due to its relationships with the other well identified components of the study system.[3]

Under the condition that the interactions between the system's components are all identifiable and measurable, the description of the system's behavior becomes simpler than one could expect.

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[2] M. LUDOVICO, *Syntropy. Definition and Use,* in online magazine www.syntropy.it, December 2008, No.1, p. 158

[3] For the determination of the *self-interaction* of the system's "external component" see M. LUDOVICO, *L'evoluzione sintropica dei sistemi urbani" (Syntropy and Evolution of Urban Systems),* Published only in Italian language by Bulzoni, Roma 1988 (2nd ed. 1991), pp. 208-223.

In analysing any system, the observer is used to focus his attention only on those interactions that are significant for the study's purposes.

Then, in describing a particular behavior of the system, it is obviously supposed that the interaction flows, as observed in a given time unit, are methodically quantified by use of a measurement system that renders all interactions homogeneous quantities, in order to make them comparable to each other.

A subsequent important consideration is that all the individual interaction flows, if these are *not stable* by hypothesis, can be converted into *interaction probabilities*, which also enable the analyst to exploit some mathematical properties of a probability distri-bution.

Besides any possible discussion on the *meaning* of this kind of probability distribution, the *percent values* expressed by such probabilities (i.e., the ratio of each flow to the total amount of flows in the system) are significant enough to justify the relative use in the analysis. In particular, interactions expressed in the form of "probabilities" are useful to the purpose of expressing the intrinsic amount of informational *uncertainty* associated with the system's states.

As a matter of fact, the subjective assessment of a probability distribution depends only on the information with which the analyst is provided by the measurement of the interaction flows.

If we now apply the set of concepts expressed above to the representation of a regional or national economic system we do actually adopt the Leontiev's approach to the macro-analysis of the system. The intervening differences are in the supposed nature of the observed activities, and in the identification and description of the *objective constraints* that chara-cterise the economic system as a self-organised system. Basically, economic inputs and outputs are both viewed as *transactions,* i.e., as *action flows* moved by the *intent* to achieve quantifiable *benefits,* whatever the nature of these.

The new approach is no more deterministic, but *probabilistic***:** sector inputs and outputs are supposed to be *possible* events, each occurring with a variable probability**.** The relevant probabilistic nature consists in that such flows are *not* considered as *stable events,* but only as *possible events* whose *temporary* intensity is detected through surveys conducted during states of *precarious equilibrium* of the system's configuration.

The theoretical scheme outlined in the next paragraphs may be considered as applied to a *closed* economic system. The economic system becomes "closed" by subdividing the "final sector" of Leontiev's scheme into two sectors: (1) the "families sector", viewed both as "labor provider sector" and as "consumer sector"; (2) the whole set of foreign countries viewed both as source of the system's imported products and destination of exported products.

In this way, the transaction matrix of the economic system is a square matrix of NxN transactions, N being the number of the different specific sectors of the system identified, there including the "external sector", which is the origin and destination of the main system's imported and exported products, respectively.

4. **Basic theoretical features of a new method**

The theoretical framework of the new method recalled here is as follows.

This *simulation theory* regards only a *mental representation* of any possible complex self-organised system. It is *not* the representation of any *real system.*

Of course, any economic system may in particular be thought of as a complex self-organised system.

The theoretical description of such a *mental representation* of complex system rests on a few basic assumptions.

The first assumption is that the *interaction flows* between elements of the considered system are *physically measurable*.

The second assumption is that a quantifiable *expected intent* is associated with each interaction.

The third assumption is that *all that is known* about interaction flows is expressed

(i) by the physical measurement of the flows,

(ii) by the formal definition of the relevant "intents", and

(iii) by those relationships, between any flow and other flows, which can be identified and expressed formally.

On the basis of these three major assumptions, the theory derives the fundamental equation that puts every interaction (or transaction flow) into a mutual univocal relationship with the "intent" that motivates the same flow. The equation is:

$$\textbf{[1]} \qquad T_{jk} = \frac{hO_j D_k e^{m_{jk}}}{T}\,; \qquad \text{(valid for any } j \text{ and } k\text{)}$$

where:

T_{jk} is the measurement of the ***flow,* i.e.,** the **transaction per time unit,** which is originated by system component j and bound for component k ;

m_{jk} is the measurement of the *expected intent* associated with the same flow. As shown ahead, these intents are completely determined by the given distribution of the system's transactions.

O_j is the total amount of transaction flows generated by j in the time unit considered (in an economic system it represents the *total output* of sector j per conventional time unit);

D_k is the total amount of the system's transaction flows that are bound for k during the given time unit (in an economic system it represents the *overall demand* of sector k for commodities and services per conventional time unit);

T is the overall amount of transactions generated by the system in the same time unit (i.e., the system's total output);

h is a coefficient that depends on the system's equilibrium state, if any.

The following formal definitions concern some of the quantities introduced above:

$$\textbf{[2]} \qquad O_j = \Sigma_k T_{jk}\,; \qquad \text{(valid for any } j\text{)}$$

$$\textbf{[3]} \qquad D_k = \Sigma_j T_{jk}\,; \qquad \text{(valid for any } k\text{)}$$

[4] $T = \Sigma_j\Sigma_k T_{jk} = \Sigma_j O_j = \Sigma_k D_k$.

To shift from flow absolute measurements to *flow probabilities* it is sufficient to divide Equations **[1]** by total flow *T*, to obtain:

[5] $$P_{jk} = \frac{T_{jk}}{T} = hP_i Q_k e^{m_{jk}}$$; (valid for any *j* and *k*)

which expresses the probability of a transaction flow from *j* to *k*, once defined

[5a] $P_j = O_j / T$ **;** $Q_k = D_k / T$ **;** (valid for any *j* and *k*).

P_j is the probability for component *j* to generate a unit flow (of output) during the fixed time unit, while Q_k is the probability for component *k* to be the destination of any unit flow generated (demanded) by the system during the same time unit.

It is also immediately seen, because of relations **[4], [5]** and **[5a]**, that

[6] $\Sigma_j\Sigma_k P_{jk} = 1$.

This equivalence indicates that the set of the flow probabilities associated with the system may be considered as a *probability distribution*.

Equations **[1]** are obtained by maximizing the probabilistic uncertainty *E* associated with discrete probability distribution **[6]**, under all the quantifiable constraints that affect this probability distribution. The constraints are expressed by equations **[2], [3], [4],** and by the following equation

[4a] $\Sigma_j\Sigma_k u_{jk} T_{jk} = U$

in which u_{jk} is *the mean effect expected* in association with transaction T_{jk} . Concerning an economic system, quantities u_{jk} may be viewed as the *mean economic benefit expected* in association with one unit of transaction T_{jk} . *U* is the expected overall benefit per time unit associated with the system's activity.

At this point in the discussion, it is of a fundamental importance to draw attention to the fact that *definitions* **[2], [3], [4], [6]** and *assumption* **[4a]** constitute *all* that the analyst is supposed to know for sure about the study system. All other possible information is too fuzzy and uncertain to be clearly formulated and steadily associated with the complex system's activity, so that *no additional assumption* can in general be clearly formulated and proposed as *systematically true.* Therefore, apart from the four definitions and the hypothesis **[4a]** mentioned above, the analyst's uncertainty is maximum as to the indefinite myriad of contingencies upon which the system's activity forms and develops.

In other words, the uncertainty in describing the system would be maximum (i.e., the analyst's information about the system would be nil) if there were no constraint to limit the randomness of the interaction distribution between the system's components, as otherwise would be perceived by the analyst.

"Intent" m_{jk} is the relative *expected effect* u_{jk} multiplied by constant λ , which is a Lagrange multiplier determined through the constrained maximisation of the probabilistic uncertainty associated with the probability distribution defined by equation **[6].**

Lagrangian multiplier λ is a positive constant quantity that depends on the measurement system adopted by the analyst. In this connection, however, it is worth observing that in most practical applications the numerical determination of λ is not necessary, "intent" $m_{jk} = \lambda\, u_{jk}$ being already in itself quite a significant indicator.

As already indicated, intent m_{jk} is a measurement of the mean unit "economic purpose" associated with the respective flow T_{jk} . For the theory, the value of m_{jk} may vary between $-\infty$ and $+\infty$.

The complete set of values m_{jk} (i.e., the NxN matrix $\{ m_{jk} \}$) determines the *system's structure.* It is the system's network of *expectations*.

The concept of *probabilistic uncertainty*[4] is sub-stantially the concept of *information entropy* defined by Shannon and Weaver in 1949, and is expressed here by

$$\textbf{[7]} \qquad E = -\Sigma_j\Sigma_k P_{jk}\,\mathbf{Ln}P_{jk}$$

"**Ln**" is the symbol for natural logarithm.

Function **[7]** (*uncertainty* or *entropy)* is then the quantity to be maximized (by *Lagrange multipliers method*) under the constraints – as indicated above – which can be written to express *all that is known* about the considered interaction flows.

Probabilistic uncertainty, or entropy, is a positive quantity which is *always* associated with any probability distribution and can be expressed *only* through function **[7]**.

Given the measurement of all the interaction flows, an important implication of Equations **[1]** is that the quantification of the *expected intent* associated with each flow – as previously announced – is also univocally determined. In fact, from **[1]** one obtains:

$$\textbf{[8]} \quad m_{jk} = \mathbf{Ln}T_{jk} - \mathbf{Ln}(O_jD_k\,/\,T) - \mathbf{Ln}\,h\,; \quad \text{(for any } j \text{ and } k\text{)}$$

and it can be proved that

$$\textbf{[9]} \quad -\mathbf{Ln}h = 2\mathbf{Ln}(N/T)+(1/T)\,\Sigma\,(O_i\,\mathbf{Ln}O_i + D_i\,\mathbf{Ln}D_i).$$ [5]

N is the number of components that form the system. Coefficient *h* has no physical dimension and pertains to any "intrinsically unstable equilibrium state" in which the system can be described by Equations **[5]** (whereas *transition phases* - which are inherent in *transformation cycles,* are described by subsequent Equations **[21]** to **[26]**).

Parameter *h,* whose value may vary between 0 and 1, can be thought of as the probability for the system to change its state.

From the theoretical point of view, it is important to remark that Equations **[8]** and **[9]** imply that the interactions between the components of ***any*** system may in general – at

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[4] C. SHANNON & W. WEAVER**,** *The Mathematical Theory of Communication,* University of Illinois Press, Urbana, 1949.

In its original formula, uncertainty – or entropy – includes a constant coefficient that depends on the logarithm base**:** it has here been assumed as equal to 1 .

[5] The whole mathematical discussion concerning the theory summarised here, with the relevant theorems and proofs, is in my book, *L'evoluzione sintropica.."*, op. cit.

least to some degree – be viewed as *intentional*, considering that $m_{jk} = 0$ means *no intent*. If all the elements of matrix $\{ m_{jk} \}$ are nil, then $h = 1$, necessarily.

Instead if $h = 1$, it is easily proved that

$$T_{jk} = (O_j D_k / T) = T / N^2 , \quad \text{(valid for any } j \text{ and } k\text{)}$$

Actually, $h = 1$ characterises the *extremely unstable equilibrium state* of maximum disorder, as it is expressed by maximum uncertainty $E_{Max} = 2\mathbf{Ln}N$.

In this connection, h is interpreted as the probability for the system to change its state. For any "system", the state of "total disorder" is by definition meaningless and therefore impossible.

4.1 - Other basic definitions

The maximum value of uncertainty (disorder) is associated with probability distribution **[6]** if all probabilities P_{jk} are equal to each other. In this case, the flow probability between any pair of components is a constant expressed by $P = 1 / N^2$. That is why, because of Equation **[7]**, the maximum value of uncertainty is expressed by

$$\textbf{[10]} \qquad E_M = 2\mathbf{Ln}N = H \,.$$

However, as remarked above, E_M cannot affect any system, since "systems" may form only if $E \neq H$.

The theory considers uncertainty E as a measurement of perceived disorganization (*disorder*) in the system and, therefore, H expresses a theoretical *limit state* of the system, about which nothing remarkable can be said except that it is extremely unlikely or – better – substantially impossible. Such a limit state is also referred to as the system's *entropy potential.*

Relation **[10]** indicates that quantity H depends only on the number of the different components that form the system. This fact draws attention to the importance of the criteria used for identifying-describing the system.

In any perceived state of the system, the difference S, between entropy potential H and uncertainty E, is taken as a measurement of the system's degree of *order* or *organization* in that particular state, and is defined as the system's "*syntropy*". Therefore, *syntropy* is

$$\textbf{[11]} \qquad S = H - E \,.$$

Syntropy[6] provides a means for measuring the degree of organization (*order*) in the system, and any change in the system's syntropy gives an indication on the overall "improvement" or "worsening" undergone by the system upon simulated (or recorded)

6 The term "syntropy" was first introduced by matematician Luigi Fantappie' in 1945, to mean that "*quid*" which brings organisation in any physical (especially biological) process, in an apparent contrast with the Third Law of thermo-dynamics. In the same year, for analogous purposes, Max Plank suggested the term "negentropy", but the relevant concept differs from that inherent in "syntropy".

alterations in its hypothe-sized (or observed) states, under the conventional assum-ption that *order* is better than *disorder*.

In this connection, a significant indication from the simulation theory[7] is the relation between the system's syntropy S and the overall benefit U associated with the system's activity. The relation is given by

[11a] $$S = \lambda \frac{U}{T} = \lambda u$$

T being the sum of all interaction flows**.** λ is the Lagrangian multiplier whereby "intents" are defined, and u is the expected mean benefit per transaction unit. The relation draws attention to the very close relationship between the concept of "degree of order/organisation in the system" and the concept of "expected benefits" associated with the transaction flows that characterise the system.

Going back to the probability distributions, consider now that also the probability distributions expressed by relations **[5a]** imply a probabilistic uncertainty associated with each of them. The set of quantities **[2]** and **[3]**, (i.e., *outputs* and *demands* in an economic system) or, alternatively, the set of the two respective discrete probability distributions $\{ P_j \}$ and $\{ Q_k \}$, is here called "base of the system", and the sum of the relevant "uncertainties", as defined by

[12] $$E^* = -\Sigma\, (P_i \mathbf{Ln} P_i + Q_i \mathbf{Ln} Q_i)\,,$$

is the "base entropy" of the system. In general, E^* differs from E.

If, for any j, is $P_j = Q_j = 1/N$, then the base entropy becomes $E_M^* = H$, and therefore no system exists for the observer.

Thus, analogously to definition **[11]** above, it is also possible to define the "base syntropy" of the system as

$$S^* = H - E^*.$$

Base syntropy S^* is in general different from syntropy S, though the following relationship is constantly true**:**

[13] $$E + S = E^* + S^* = H\,.$$

The following equivalence is also true of any unstable *equilibrium state* of the system:

[14] $$S^* = -\mathbf{Ln}\, h\,,$$

and justifies the name of "stability" for base syntropy S^*. If $h = 1$, "stability" becomes nil, which occurs – as already seen – if the system's entropy $E = H = 2\mathbf{Ln}N$ is maximum.

It is worth observing that Equation **[14]** leads, through Equation **[8]**, to express every flow intent m_{jk} also as a function of the system's stability S^*.

(Note: This conclusion conflicts with the properties conceptually associated with the maximum entropy of thermodynamics. According to classic thermodynamics, any

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[7] M. LUDOVICO, *L'evoluzione sintropica…,* op.cit. pp.225-226

isolated system – and the Universe itself – tends to a final equilibrium state that establishes at the maximum entropy level, because such a state – from the thermodynamics point of view – is the most probable one. Instead, in the theory presented here the maximum entropy state is extremely improbable *for any system*, to the extent to which maximum entropy H implies neither equilibrium nor existence at all for the system. At the opposite extreme, also $h = 0$, i.e., a definitively stable equilibrium, is impossible, since it would imply a system consisting of an infinite number of components. See Equation **[20]** ahead).

4.2 - A cardinal theoretical assertion

Equations **[8]**, the number of which is N^2 (the square number of the system's components), together with Equation **[9],** show that all the needed information concerning the study system can be expressed through functions of the interaction flows.

From a practical point of view, this means that the significant amount of information concerning the state of the system can be obtained through any appropriate collection, interpretation and processing of the data that quantify the flows.

However, and this is the fundamental methodological statement, all the information obtained from the theoretical analysis depends strictly on how the system has been identified and described. The simulation theory does not provide any *true picture* of the reality to which the analysis refers, but *only* the logical implications of *a mental representation* of it.

5. A description of the system's evolution

The most important equations provided by the theory are those which enable the analyst to simulate the system's evolution process. This is described by a sequence of "transformation cycles", each cycle developing through discontinuous "transition phases", which are changes in the system's state, each phase being described by a different distribution of the interaction flows.

In every transformation cycle, the condition of the system is expressed by a set of parameters (*state* and *phase parameters*), amongst which *entropy, syntropy* and *stability* are the most significant ones.

Any transformation cycle starts from an "initial phase" (also called "phase zero"), which is determined by any change **–** however small **–** in the original flow configuration that modifies the system's *base entropy* defined by Equation **[12].**

The *initial transition phase* of an evolution process is not the ***original state***: this is only the system's first configuration that could be described through a direct survey of the transactions flows, which also provided the first set of observed data.

Instead, the *initial transition phase* is supposed to be the *initial change* in the system's flow distribution *observed* (or *introduced*) after the original one.

Thus, the *initial phase* (also dubbed ***phase zero***) is supposed to be connected with the *original state* through a sequence of *virtual transition phases* that represent the virtual "past story" of *phase zero.*

"Phase zero" is viewed as the *initial phase* of an observed *transformation process,* which is defined by a sequence of *actual transition phases*, each representing a section of the system's simulated future.

It is worth remembering that a sequence of *actual phases* describes *one transformation **cycle***, which may – or may not – be followed by further transformation cycles.

During each cycle, the continuity in the identity of the study system rests on two sets of quantities, dubbed "*structure potentials*" and represented by letters X_k and Y_j , whatever j or k.

Y_j and X_k are non-negative values that remain constant with the system's structure $\{ m_{jk} \}$ during each transformation cycle, and may be considered as obtained from the solution of the two following linear equation systems, respectively:

[15] $$\Sigma_j Y_j \, e^{m_{jk}} = 1 \; ; \qquad \text{(valid for any } k\text{)}$$

[16] $$\Sigma_k X_k \, e^{m_{jk}} = 1 \; ; \qquad \text{(valid for any } j\text{)}$$

where "structure potentials" Y_j and X_k are the unknown terms.

It is also proved that all the $2N$ "structure potentials" Y_j and X_j verify the following relations**:**

[17] $$\Sigma\, Y_j = \Sigma\, X_j = h \, .$$

The *structure potential* values range between 0 and 1.

Moreover, it is proved true that , in equilibrium states,

[18] $$h = X_j / Q_j = Y_j / P_j \, , \qquad \text{(valid for any } j\text{)}$$

which is a useful calculation instrument in simulating a transformation cycle.

Note:

[19] if $E = E_M = H$, then $h = 1$ and $Y_j = X_j = 1/N$, for all j .

The simulation theory does also prove that

[20] $$h = e^{E^*} / N^2 \; .$$

which explains why no system, in no state, can enjoy permanent stability (see definition **[12]** for E^*, and **[14]** for "stability" S^*), unless the system consists of an *infinite* number of components.

The state of absolute maximum *syntropy*, according to definitions **[10]** and **[11]** above, is also expressed by
$S_M = 2\mathbf{Ln}N = H$, while the respective comple-mentary entropy is $E = E^* = 0$. However, from **[13]** and **[14]** we derive that $h = \mathbf{e}^{E^*} / N^2$. Thus, $h = 0$ only if $N = \infty$, in which case the system has zero probability to change its state. Instead, if $N < \infty$, as it is for the *normal consistence* of systems, it is $h > 0$ in all cases, whatever the value for E^*. This means that any identifiable system has always a probability to change its state.

Therefore, logic arguments prove that no system can attain its pertinent maximum syntropy or entropy state. Such maximums must only be considered as asymptotic limits.

Because of Equations **[17]**, ratios Y_j /h and X_j /h (for any j) define formally two probability distributions. According to a reasonable interpretation, such ratios represent the probability for each element of the system to remain in its state of flow *generator* or *flow attractor,* respectively.

During a transformation cycle, the varying flow distribution relevant to each transition phase of the cycle is identified by indexes between parentheses. For example, $P_j(f\text{-}1)$ represents the "output probability" of component j in the phase $(f\text{-}1)$, which precedes phase (f)**;** instead, as another example, $Q_k(f+1)$ represents the "demand probability" of component k in the phase $(f+1)$ immediately subsequent to phase (f)**;** and so on also for any other varying distribution of flow probabilities proper to the various transition phases of the cycle.

The equations that determine the probability distributions in the *virtual phases* ("the past story of phase zero") are as follows:

$$\begin{cases} P_j(f-1) = \Sigma_k P_{jk}(f) = Y_j \Sigma_k Q_k(f)\, e^{m_{jk}} \quad ; \text{ (valid for any } j\text{)} & \textbf{[21]} \\ P_{jk}(f-1) = P_j(f-1) X_k\, e^{m_{jk}} \quad ; \quad \text{(valid for any } j \text{ and } k\text{)} & \textbf{[22]} \end{cases}$$

Instead, the equations of the probability distributions in the *actual phases* ("the possible future") are:

$$\begin{cases} \Sigma_j P_j(f)\, e^{m_{jk}} = Q_k(f-1) / X_k \,\mathbf{;} \quad \text{(valid for any } k\text{)} & \textbf{[23]} \\ P_{jk}(f) = P_j(f) X_k\, e^{m_{jk}} \, ; \quad \text{(valid for any } j \text{ and } k\text{)} & \textbf{[24]} \end{cases}$$

$$\begin{cases} \Sigma_k Q_k(f+1)\, e^{m_{jk}} = P_j(f) / Y_j \, ; \quad \text{(valid for any } j\text{)} & \textbf{[25]} \\ P_{jk}(f+1) = Q_k(f+1) Y_j\, e^{m_{jk}} \quad ; \quad \text{(valid for any } j \text{ and } k\text{)} & \textbf{[26]} \end{cases}$$

5.1 – Meaning and use of the transition equations

Equations **[21]** and **[22]** simulate the *most probable* way-back (or "past story") towards the system's original configuration recorded through the original survey by which the original unstable equilibrium state of the system has been identified.

In those two sets of equations, the *unknown terms are on the left-hand side,* whereas the polynomial expressions on the right-hand side are known, starting with the data pertaining to the *initial transition phase* (phase "zero"), in which an *alteration* in the *original* ***base*** *of the system has either been detected or hypothesised.* Since phase "zero", because of the introduced alterations, is a *transition phase* of a *transformation cycle,* it must accordingly be supposed that it is preceded by a series of *antecedent transition phases.*

It must also be remarked that the 2N *structure potentials* X_j and Y_j remain unchanged during the "wayback" to the *original phase*, because such potentials are just the ones that inhere in the *original state* of unstable equilibrium.

Instead, in identifying the alteration occurred, for any reason, in the *initial phase* ("phase zero"), the structure potentials are bound to change at the conclusion of the transformation cycle.

Important to note**:** all the solutions found for Equations **[21]** and **[22],** which regard *virtual transition phases,* are *positive* probability numbers**;** which is compatible with the concept that the "past story" of the system's *initial transition phase* is certain and traceable. The number of the virtual phases depends only on the number of significant decimals used to approximate the probability values.

Equations **[23]-[24]** and **[25]-[26]** regard the most probable series of the system's *future* configurations as described by the *actual transition phases*. Also in those equations the unknown terms are on the left-hand side, whereas the known terms are on the right-hand side. Alike for the *virtual phase equations,* the initial known terms are provided by the configuration of the *initial phase*.

Looking at the mathematical form of all the transition equations, one can observe that the *semi-bases* $\{ O_j \}$ and $\{ D_j \}$, as well as $\{ P_j \}$ and $\{ Q_k \}$, respectively, are closely inter-related**:** in no case it is possible to modify, for instance, semi-base $\{ O_j \}$ (or, correspondingly, $\{ P_j \}$) without implying necessary changes in semi-base $\{ D_j \}$ (or, correspondingly, $\{ Q_k \}$) . And vice-versa. To mean that the values of each semi-base may not be changed independently from one another.

From the simulation point of view, this fact entails that alterations affecting the two complementary semi-bases of the *initial transition phase* (with respect to the *original state*) must be mutually compatible, according to equations of type **[23]** or **[26].** Otherwise, it shall be necessary to opt for that of the two semi-bases in which the alterations are considered as more credible and/or significant. The problem regards in particular the *observed or hypothesised alterations* that the analyst introduces in the *original state.*

There is also to allow for possible cases in which the flow distribution and relative probabilities of the original state (which is in an unstable equilibrium) change without involving any changes in the system's base. Such cases regard intrinsic fluctuations in the values of the configuration's elements, which are not sufficient to start a transformation cycle. In other words, the simulation of a transformation cycle can start only if there are permanent modifications that concern also one of the system's semi-bases.

In principle, alterations detected through surveys should always show mutually compatible semi-bases, at least at an acceptable degree of approximation that take into account inevitable uncertainties inherent in the survey and measurement methodology. In this connection, the conduction of appropriate surveys might work as a significant test on the reliability of this simulation theory. To be born in mind, however, tests of the kind should not regard *original states* of precarious equilibrium, but only observed transition phases of transformation cycles.

As an example concerning national economic systems, the planner (or the simulation operator) should opt either for modifying the *output* (production) semi-base $\{ O_j \}$, or the *demand* (input) semi-base $\{ D_j \}$ of the system in its original unstable equilibrium. Any one of the two options implies the mathematical determination of the other one, which therefore goes to determine the respective semi-base of the subsequent *transition phase* (i.e., *phase one*) of the transformation cycle.

At variance with the *virtual phases,* the solutions of the equations from **[23]** to **[26],** which regard the *actual phases,* **do not** necessarily provide *positive probability values.* There is always an actual phase of the cycle for which the equations give (at least one) *negative* solutions. As soon as any *negative solution* appears in the configuration of an *actual transition phase,* the same phase must be considered as the *barrier* that stops the transformation cycle. In practice, it makes no sense accounting for *negative* probabilities, also because of intervening logarithms that would turn the negative values into *imaginary quantities.*

Therefore, such a phase is considered as that of the *system's disappearance* (sort of *decease or collapse of the system),* unless a transformation of the system occurs (or is established) on the basis of the phase configuration (i.e., the transaction flow distribution) immediately preceding the *decease phase.* This last phase of the system's life is dubbed "agony phase".

The transformation that avoid the system's collapse consists in a change of its structure – i.e., as to the simulation – in the re-calculation of its *structure potentials,* which express the intervened change in the network of expectations that motivate the actions of the system's components and allow the system's "survival". The re-calculation of the changed *structure potentials* X_j and Y_j uses the flow distribution of the "agony phase" as basic data, as if they were the findings of a survey conducted on a **new** *original state* of *unstable equilibrium.* Obviously, the *new equilibrium state* may also be considered as the *original state* of a further transformation cycle**;** and so on.

(It might be interesting to know that – according to a number of experienced applications of this simulation procedure – the "agony phase" is in most cases the one in the cycle that shows the highest degree of organisation (syntropy) achieved by the system, which seems to "claim" at that stage a change in its structure to avoid disintegration.

As a general observation, the simulation appears better balanced and more significative when the alterations shown in the phase zero, with respect to the original state, are of a moderate amount. For the purpose of simulating the probable effects of major alterations, it seems better to introduce these by small instalments, one by one, in subsequent unstable equilibrium states achieved during the simulated evolution of the study system).

6. Summary of the conceptual framework

Simulations are possible only if a complete set of "original" interaction flows is given.

The observed original flow distribution provides the whole set of data that is necessary and sufficient to carry out analyses and to start simulations.

Conventionally, this simulation theory considers any flow distribution obtained from surveys (or other observation operations) as the representation of an *original equilibrium state* of the study system. This original equilibrium state is intrinsically unstable, and the relevant observed configuration shall be taken as a *mean configuration* about which the system fluctuates precariously.

"Intrinsic instability" means, in fact, that reversible fluctuations in the flow distribution within the original configuration are inevitable, and will sooner or later determine an *irreversible permanent* alteration. *Any minimal* ***irreversible*** alteration in the equilibrium flow distribution (or in the relevant probability distribution), *which modifies also the base entropy of the system,* generates a corresponding particular *initial* phase of the system's "transformation cycle",.

All simulations have to start with an initial transition phase, or "phase zero" ($f = 0$).

The *initial transition phase* of a simulation is an alteration – known by hypothesis – in the original equilibrium state**:** the resulting configuration is the *initial phase* of a transformation cycle, which comes from the *original equilibrium state* and will inevitably conclude with either a change in the system's structure or – in an alternative – in the system's disintegration. In fact, the conclusion of any transformation cycle may either consist in a *new* unstable equilibrium state achieved by the system, or in the *collapse* (and disappearance) of the system.

If a new equilibrium state concludes the cycle, another transformation cycle is expected to originate from it.

Any irreversible alteration – however small – in the system's equilibrium, as quantified by the relevant base entropy, is the necessary and sufficient condition to identify only *one particular* transformation cycle amongst an infinite number of possible cycles.

There is an important remark that concerns the different kinds of instability that characterise the *equilibrium states*, which identify both the *original state* and the *conclusive state* of any transformation cycle, with respect to the instability of the transition phases internal to each cycle.

The instability of *equilibrium states* depends on stochastic events, which may or may not happen, and whose nature and intensity are intrinsically unpredictable. Instead, once any irreversible change modifies such equilibrium states, the subsequent phases of every transformation cycle, with relevant degree of instability, are necessary and univocally determined.

From a logical standpoint, any transformation cycle includes the transition phases that have *virtually* preceded the *initial phase* (i.e., *phase zero*), since this one is conventionally considered, by logic consistency, as the consequence of antecedent "virtual" transition phases that have just led to the irreversible alterations showed by the configuration of transition "phase zero".

In simpler terms, any given *initial configuration* or *phase* is only a *transition phase* with its own *given* past history.

That is why, from a theoretical point of view, the state described by the survey (i.e., the *original* configuration) is never seen as the *initial phase* of the evolution process, but only – with respect to the analyst – as the "conventional unstable original equilibrium state" of a *possible* evolution of the study system.

Therefore, in this context, the adjectives "original" and "initial" have quite different meanings. From the *original state* infinite different *initial* transition *phases* of different transformation cycles may alternatively and arbitrarily be identified or defined, upon an infinite number of possible different alterations in the given *original state* (or *original configuration*).

As a conclusion, in this simulation of an evolution process *original state* and *initial phase* do never coincide.

If a system "survives" at the conclusion of a series of transformation cycles, then the series of undergone "transformations" (which characterise the system's overall evolution) consists in a series of changes in the values that express the "intents" (see Equations **[8]**). These "intents" form the "structure" of the system.

During each transformation cycle (which develops between the original equilibrium *state* and the next equilibrium state, if any), the structure of the system is supposed to remain unchanged, while the flow distribution varies phase by phase, up to a critical phase, in which also the structure must change to allow the system to survive. The critical phase (*agony phase*) is that given by the last set of non-negative solutions obtained from Equations **[23]** or **[25]**.

The *leitmotiv* of the simulation logic is as follows**:** solutions **{** $P_j(f)$ **}** of *actual phase* *f* Equations **[23]** (i.e., the transition phases that form the "future section" of the cycle) provide the numerators of the known terms (right hand side) of phase *(f+1)* Equations **[25],** whose solutions **{** $Q_k(f+1)$ **}**, in turn, provide the numerators of the known terms of subsequent phase *(f+2)* equations, and so on, until the *agony phase* is attained, which is the actual phase of the cycle that provides the last set of *non-negative solutions.*

Since negative solutions, in terms of both flows and flow probabilities, are with no physical significance, it is conventionally assumed that the represented system *cannot exist further* in the phase that follows the *agony phase*.

Under the assumption that the system will instead survive, the new structure of the system is normally re-calculated in function of the flow distribution inherent in the "agony phase", by use of Equations **[24], [26]** and **[8].**

The assumption of survival, however, is not logically necessary**:** It may or may not be adopted, according to the nature and purposes of the simulation.

Note: At variance with the equations regarding the *actual transition phases*, Equations **[21]** and **[22]**, which relate to virtual transition phases, do always and necessarily provide *non-negative* solutions for *all virtual phases.* The solutions (configurations) obtained from these virtual-phase equations tend to regain the configuration of the *original equilibrium state* of the system, through a reverse-time phase sequence. The number of phases of this virtual sequence is practically determined by the number of significant decimals adopted for the values of the solutions obtained from **[21]** and **[22]** as well as for the values of the *original state*.

The process described by the simulation is that of activities generated by a system of expectations (the *intents*), which tend to be conservative but are necessarily modified, through feedback reactions, by the development of the overall system activity.

Once each cycle is concluded by a transformation that brings the system into a new unstable equilibrium state, a new cycle may start. The cycle that follows assumes the flow distribution resulting from the preceding transformation as a new *original* equilibrium *state* in the system's evolution.

In the same way, further cycles may follow in describing the system's evolution

If, at a certain point in the evolution, the necessary structure transformation does not occur, then the system exits from the area of the conventional reality, and the simulation stops.

It is worth pointing out that a theoretical potential maximum syntropy (level of organisation) inheres in any defined system. The maximum syntropy value coincides with the corresponding "entropy potential" **[10]** of the system**.** In approaching its maximum syntropy, the system's evolution may substantially enter stationary conditions, which actually block any further development.

Further evolution (or involution) is then possible only if the system undergoes a *mutation.* A mutation occurs when the system must be re-defined because of major changes in its features, the nature of which *involves an increase or decrease* in the number of its components (or "sectors", if it is an economic system).

A process of progressive (or regressive) *functional differentiation* within the range of the system's components (or "sectors") may involve a sequence of "mutations".

The main use suggested for the simulation process illustrated here is that proper to a "sensitivity analysis", which is often necessary to test the suitability of planning or political initiatives.

CONDITIONS FOR INDETERMINACY AND THRESHOLDS IN NEOCLASSICAL GROWTH MODELS

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Abstract This paper considers externalities and investigates which general properties and respectively their magnitudes induce complex behavior like thresholds and indeterminacy (and cycles). The objective is to obtain general mechanisms within a general setting in order to complement the much advanced but model specific literature. It turns out that these arithmetic and simultaneously economic conditions for thresholds or indeterminacy require complementarity and non-moderate dynamic social influence. In other words, complexities can be related (necessary and sufficient) to a familiar static property that social interactions turn the (stationary) demand for the private stock into a Giffen good. Indeterminacy requires in addition 'low' subjective discounting and proper interactions between control, stock and externality. These conditions provide an easy way to construct models that allow for the targeted outcome, stability, thresholds or indeterminacy.

Keywords Complementarity and non-moderate dynamic social influence · Thresholds · Cycles · Indeterminacy · Arithmetic and economic conditions · Neoclassical growth · Labor market · Pollution

JEL Classification C61 · C62 · D62

1 Introduction

This paper investigates the relation between the standard (unique turnpikes) and two different cases—history dependence or indeterminacy—and their linkage to externalities or social interactions. The contribution to this already huge literature is to depart from a general framework and to focus on general conditions (arithmetical as well as economical) instead of investigating a variation of a particular model class like the familiar two-sector growth model, modifying preferences, etc. However, some restrictions are imposed on this otherwise general framework: the economies must converge to finite steady states and the agents have only a single control. Of course, removing any of these assumptions provides a topic for future research. For completeness, pure adjustment cost models are also excluded because they are already treated in Wirl and Feichtinger (2006).

The related literature works, by and large, around familiar growth models specifies the functional forms (mostly familiar ones such as variations of power functions for utility and productions), and addresses different kinds of externalities. Examples are Benhabib and Farmer (1994) using a one-sector model while most papers use a two sector growth model, e.g., Benhabib and Perli (1994), and more recently Bosi et al. (2007) and Benhabib et al. (2008); related are also the papers in the recent issue of the *International Journal of Economic Theory* on macroeconomics in honor of Jess Benhabi, e.g., Mattana et al. (2009) on multiple balanced growth path in the model of Lucas (1988) and Eusepi (2009) on production externalities; Graham and Temple (2006) applies empirically the two sector growth model and finds that around a quarter of the world's economies corresponds to the low equilibrium. In contrast the following paper will derive general conditions, more precisely about second order derivatives, the degree of dynamic social interaction, the shape of the steady state conditions, etc. This allows two things: Firstly, to obtain economic mechanisms responsible for complexities, e.g., it is shown that dynamic complexities are equivalent to the Giffen property for the stationary demand for the (capital) stock. Secondly, to construct examples that are (a) off the beaten track and (b) can be very simple. This is demonstrated with three examples, one for the labor market à la Krugman (1991) and then in the context of growth models, first for positive and then for negative stock externalities. Only very few papers choose a similarly general route. An early exception is Benhabib and Rustichini (1994) in an attempt to summarize endogenous growth models in a special issue of the *Journal of Economic Theory*, another one is Shimokawa (2000) in a discrete time setting and a very recent one is Gliksberg (2009) about monetary economics. However, these papers do not provide arithmetic or economic conditions that are applicable to a general framework and at the stage of model building in order to determine whether complexity and in particular indeterminacy is possible. In all fairness, a reason for this focus on real business cycle models models seems to find empirically plausible models that can be subject to data rather than indeterminacy perse.

2 Framework

The following framework departs from Wirl (2007). A competitive agent (each agent has measure zero, all agents are identical and have measure 1 so that individual, aggregate and average coincide) solves a general intertemporal optimization problem,

$$\max_{\{u(t)\in\mathbb{R}\}} \int_0^\infty \exp(-rt)\pi(x(t), y(t), u(t))\,dt, \tag{1}$$

$$\dot{y}(t) = u(t), \quad y(0) = y_0, \tag{2}$$

where $r > 0$ is the constant discount rate, $\pi(x, y, u)$ represents the agent's instantaneous benefit, $y(t) \in \mathbb{R}$ is the agent's private stock in period t, $u(t) \in \mathbb{R}$ is the agent's control given by net adjustment of the private stock, and finally $x(t)$ is the aggregate or average of private stocks such that,

$$x(t) = y(t) \quad \forall t \in [0, \infty). \tag{3}$$

The future path $\{x(t),\ t \in (0, \infty)\}$ is a given datum for individual agents, but endogenous within the model due to (3). That is, this externality or social influence does not involve another dynamic relation (on this see Wirl (1997, 2002)) but is dynamic in the sense that it is linked to a stock and not to a flow.[1]

Assumptions 1. $\pi \in C^2$ is jointly concave in the individual variables (u, y): $\pi_{uu} < 0$, $\pi_{yy} \leq 0$, $\pi_{uu}\pi_{yy} - \pi_{uy}^2 \geq 0$.
2. $\pi_u(x, y, u) \mid_{x=y, u=0} < 0$.
3. Each agent (having measure zero) takes the evolution of the externality $\{x(t), t \in [0, \infty)\}$ as given but predicts rationally its future evolution.
4. The economy does not grow boundless.

The first and third assumption are standard. The fourth excludes perennial growth and has been already mentioned in the introduction. The second assumption excludes pure adjustment cost models ($\pi_u(x, y, u) \mid_{x=y, u=0} = 0$) and restricts the number of cases to those in which $(-\pi_u)$ corresponds to the marginal costs of expansion of a positively valued stock, $\pi_y > 0$; a negatively valued private stock requires $\pi_u > 0$ and a change of signs in some propositions below (in particular in Proposition 2).

Definitions Dynamic social influence (**DSI**) is defined by

$$\mathbf{DSI} := \left(-\frac{\pi_{xy}}{\pi_{yy}}\right) - \left(\frac{\pi_{uy} + \pi_{ux}}{\pi_{yy}}\right) r, \tag{4}$$

and is called moderate (**MDSI**) iff $|DSI| < 1$ and positive (or complementary) and non-moderate (**PNMDSI**) iff $DSI > 1$.

Proposition 1 *A perfect foresight, competitive and symmetric equilibrium is given by the following pair of differential equations* [*listing all arguments except time and with* U *implicitly defined below in* (7)]:

$$\dot{y} = U(y, y, \lambda), \quad y(0) = y_0, \tag{5}$$

$$\dot{\lambda} = r\lambda - \pi_y(y, y, U(y, y, \lambda)). \tag{6}$$

[1] The case of a flow externality, $x(t) = u(t)$, for which conspicuous consumption is an example (see e.g., Fisher and Hof Fisher and Hof (2000)), cannot explain complex patterns.

This proposition results from the agents' first order optimality conditions,

$$H_u = 0 \Longrightarrow u = U(x, y, \lambda), \quad U_x = -\frac{\pi_{ux}}{\pi_{uu}}, \quad U_y = -\frac{\pi_{uy}}{\pi_{uu}}, \quad U_\lambda = -\frac{1}{\pi_{uu}}, \tag{7}$$

$$\dot{\lambda} = r\lambda - \pi_y, \tag{8}$$

in which $H \equiv \pi(x, y, u) + \lambda u$ is the Hamiltonian and λ is the costate, both in current values. The assumptions about π allow for a competitive equilibrium and ensure the concavity of the Hamiltonian with respect to the control u and the private stock y such that the first order conditions are sufficient if a limiting transversality condition holds. Substituting the identity between individual and aggregates (3) yields (5) and (6). □

At an interior steady state of (5) and (6), the following must hold,

$$-\pi_u(x, y, u)\mid_{x=y,u=0} = \frac{\pi_y(x, y, u)\mid_{x=y,u=0}}{r}, \tag{9}$$

i.e., the marginal cost for expanding the stock ($-\pi_u$) must be equal the net present value of the marginal benefit from the stock. Hence, the intersection of the rate of substitution between the private stock and the control evaluated at a steady states (defined below as scalar function $\psi(y)$) with the discount rate,

$$\psi(y) \equiv -\frac{\pi_y(y, y, 0)}{\pi_u(y, y, 0)} = r, \tag{10}$$

determines all interior steady states. Moreover their stability properties depend on the sign of the slope of ψ.

Proposition 2

$$\det(J) = \left(\frac{\pi_u}{\pi_{uu}}\right)\psi', \tag{11}$$

$$sgn(\det(J)) = -sgn(\pi_u)sgn(\psi') = sgn(\psi'). \tag{12}$$

Proof Let $(y_\infty, \lambda_\infty)$ denote an (interior) steady state of the competitive equilibrium described by (5)–(6). The Jacobian,

$$J = \begin{pmatrix} -\frac{\pi_{ux}+\pi_{uy}}{\pi_{uu}} & -\frac{1}{\pi_{uu}} \\ -(\pi_{yy} + \pi_{xy}) + \pi_{uy}\frac{\pi_{ux}+\pi_{uy}}{\pi_{uu}} & r + \frac{\pi_{uy}}{\pi_{uu}} \end{pmatrix}, \tag{13}$$

has the eigenvalues

$$e_{12} = \frac{\operatorname{tr}(J) \pm \sqrt{\operatorname{tr}(J)^2 - 4\det(J)}}{2}, \tag{14}$$

where determinant and trace are as follows:

$$\det(J) = -\frac{(\pi_{uy} + \pi_{ux})r + (\pi_{yy} + \pi_{xy})}{\pi_{uu}}, \tag{15}$$

$$\mathrm{tr}(J) = r - \frac{\pi_{ux}}{\pi_{uu}}. \tag{16}$$

Rearranging (15) and using the definition of ψ verifies the claims using the assumption $\pi_u < 0$. □

As a consequence, the function ψ defined in (10) allows not only to compute steady states but to check their stability as well: $\psi' < 0$ implies the usual saddlepoint property, while $\psi' > 0$ implies either a stable (and thus an indeterminate outcome) or an unstable steady state leading to thresholds and thus to dependence on history and/or expectations. Furthermore, these stability properties are linked to the degree of intertemporal social interactions.

Proposition 3 $\det(J) < 0$ (*and thus local saddlepoint stability*) *at an interior steady state iff the dynamic social influence is either positive and moderate or negative,*

$$\left(-\frac{\pi_{xy}}{\pi_{yy}}\right) - \left(\frac{\pi_{uy} + \pi_{ux}}{\pi_{yy}}\right) r < 1, \tag{17}$$

while $\det(J) > 0$ *iff it is positive and non-moderate* (***PNMDSI***),

$$\left(-\frac{\pi_{xy}}{\pi_{yy}}\right) - \left(\frac{\pi_{uy} + \pi_{ux}}{\pi_{yy}}\right) r > 1. \tag{18}$$

Remarks 1 1. This proposition, which follows directly from (15), stresses the similarities and differences between MDSI and the characteristic of moderate social influence (MSI),

$$\left|\frac{\pi_{xy}}{\pi_{yy}}\right| < 1, \tag{19}$$

introduced in Glaeser and Scheinkman (2003) in static games where each player

$$\max_y \pi(x, y).$$

More precisely, both criteria MDSI and MSI ensure stability and that the (absolute) slope of the stationary 'reaction' is less than 1. In the static game since $-\pi_{xy}/\pi_{yy}$ determines the slope of the reaction function. Similarly in the dynamic case, because the total differential of the steady state condition ($r\pi_u + \pi_y = 0$),

$$r(\pi_{uy}\,dy + \pi_{ux}\,dx + \pi_{uu}\,du) + \pi_{yy}\,dy + \pi_{xy}\,dx + \pi_{yu}\,du = 0,$$

implies for $\frac{dy_\infty}{dx}$ the left hand side of (17) since $du = 0$ and $dx = dy$. The difference is that MDSI accounts in addition for the indirect impacts of the control of each individual agent on private and external stock, i.e., the term $(\pi_{uy} + \pi_{ux})$ in (17) and (18).

2. Based on the definition in (10), ψ can be interpreted as the stationary marginal willingness to pay for the stock (i.e., inverse demand[2]), which of course, must be equated to its price, the discount rate (r). Therefore saddlepoint stability is equivalent to downward sloping demand while complexities, i.e., $\det(J) > 0$, require that the social interactions twist this demand relation sufficiently until it is upward sloping (at least locally, i.e., $\psi' > 0$). Viewed this way, the issue of dynamic complexities, indeterminate or history dependent outcomes, are equivalent to the condition that the stationary demand for the stock has the characteristic of a Giffen good, which is a usual suspect for an unconventional outcome.

Re-writing the inequality (18),

$$(\pi_{yy} + \pi_{xy}) + (\pi_{uy} + \pi_{ux})r > 0, \tag{20}$$

and accounting for concavity, in particular for $\pi_{yy} \leq 0$, suggests the following classification of routes towards positive non-moderate dynamic social influence:

Proposition 4 *A positive sign of at least one of the following mixed second order derivatives of π is necessary for $\det(J) > 0$ (or positive non-moderate dynamic social influence,* ***PNMDSI****), which is itself a requisite either for a threshold or for indeterminacy*:

1. $\pi_{xy} > 0$, *i.e., complementarity between private and aggregate stocks.*
2. $\pi_{ux} > 0$, *i.e., a higher aggregate stock reduces the marginal costs of expansions due to the assumption $\pi_u < 0$.*
3. $\pi_{uy} > 0$*; the interpretation is as above but with reference to the private stock.*

Defining the marginal benefit of the stock (evaluated at $x = y$ and at the steady state, thus $u = 0$)

$$MB^y \equiv \pi_y(y, y, 0), \tag{21}$$

and similarly of the control (negative since control is costly),

$$MB^u \equiv \pi_u(y, y, 0), \tag{22}$$

the inequality (20) can be arranged in terms of the (total) derivatives of these marginal benefits leading to the following criteria, which allow for economic interpretations.

[2] This point was raised by one of the referees and I am very grateful for drawing my attention to this otherwise overlooked and very useful interpretation.

Proposition 5 *In terms of the marginal benefits defined in* (21) *and* (22),

$$\det(J) > 0 \Longleftrightarrow \frac{MB^{y\prime}}{r} + MB^{u\prime} > 0, \tag{23}$$

Therefore, at least one of the following two conditions is necessary for ***PNMDSI***:

1. $MB^{y\prime} = \pi_{yy} + \pi_{xy} > 0$, *i.e., positive and non-moderate social influence as defined in* (19) *for static games.*
2. $MB^{u\prime} = \pi_{uy} + \pi_{ux} > 0$, *i.e., a higher stock reduces the marginal costs of expansions after accounting for the spillover due to the assumption* $\pi_u < 0$.

The necessary and sufficient condition (23) means in economic terms that the impact of an (infinitesimal) increase of the stock (accounting for the social interactions by computing total derivatives) on the net present value of the (marginal) benefit from the private stock y must exceed the effect on the marginal benefit of the control ($-MB^u =$ marginal costs). In other words, the direct effect (evaluated on net present value basis) of a change in the stock is larger than the indirect one operating through the control. The first route for **PNMDSI**, $MB^{y\prime} > 0$, requires in economic terms that the marginal benefit of the stock is upward sloping due to sufficient social spillover opposite to the usual marginal benefit, which is downward sloping with respect to the own stock, $\pi_{yy} \leq 0$, by the assumed concavity of π. This 'wrong' sign of the slope of this marginal benefit, $MB^{y\prime} > 0$, reminds of the Giffen interpretation rendered to ψ if the steady state is not a saddlepoint. The difference is that the demand property ($\psi' > 0$) is necessary and sufficient while an increasing marginal benefit, $MB^{y\prime} > 0$, is helpful but not necessary for **PNMDSI** since $MB^{u\prime} > 0$ can achieve $\det(J) > 0$ too. The second route, $MB^{u\prime} > 0$, states that an increase of the stock (individually and socially) lowers the costs (since $MB^u = \pi_u < 0$). This second route in Proposition 5 emphasizes that violating the inequality in (19) = non-moderate social influence in the degenerated static game is neither necessary nor sufficient for **PNMDSI**. Indeed, a consequence of Proposition 4, item 3 is (with some abuse of language) that $DSI \neq 0$ even absent any social interaction ($\pi_{xy} = 0 = \pi_{ux}$) up to the point of inducing positive and non-moderate dynamic social influence (**PNMDSI**). And such examples exist (e.g., in concave optimization problems according to Wirl and Feichtinger (2005), which by definition lack a social interaction).

Given these explicit results, it is easy to write down sufficient conditions after making some additional assumptions, like $MB^{y\prime} > 0$, which is equivalent to positive and non-moderate social influence according to (19). Instead Fig. 1 sketches the domains of **PNMDSI** differentiating whether $MB^{y\prime}$ is positive or negative rendering immediately sufficient conditions, e.g.,: *Assuming* $MB^{y\prime} > 0$ *then* $\pi_{ux} > 0 \wedge \pi_{uy} > 0$, *or weaker*, $\pi_{ux} + \pi_{uy} > 0$, *are sufficient for* **PNMDSI**; *if, however,* $MB^{y\prime} < 0$, *then simple sufficiency criteria are lacking*. Moreover, Fig. 1 links the criterion of **PNMDSI** with the two different outcomes of either an unstable or stable steady state according to Propositions 6 and 7 below.

If **PNMDSI** holds at a steady state it still remains to differentiate between unstable and stable while all neighboring steady states must be saddlepoints (due to Proposition 2 and $\psi' < 0$ at these steady states). An unstable steady state implies a threshold

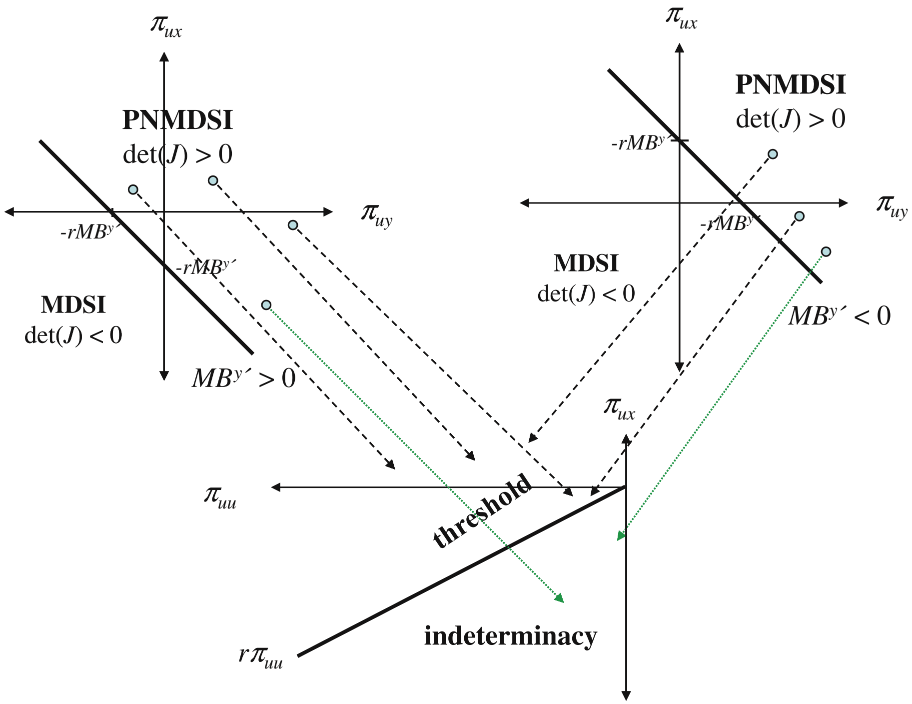

Fig. 1 Conditions for positive and non-moderate social influence (**PNMDSI**) and for thresholds or indeterminacy versus mixed second order derivatives ($\pi_u x$, $\pi_u y$) and the slope of the marginal benefit (MB^y)

that separates the domains of attractions of the neighboring steady states, while a stable steady state induces indeterminacy (at least locally). The differentiation between these two different cases depends only on the sign of the trace of the Jacobian.

Proposition 6 *An unstable steady state is characterized by a positive determinant,* $\det(J) > 0$*, and a positive trace,* $tr(J) > 0$*. The latter is equivalent to*:

$$tr(J) = \frac{r\pi_{uu} - \pi_{ux}}{\pi_{uu}} > 0 \Leftrightarrow r > \frac{\pi_{ux}}{\pi_{uu}} \equiv \textit{social influence on marginal 'costs'}. \quad (24)$$

Therefore, $\pi_{ux} > 0$*, i.e., a higher social aggregate reduces the marginal 'costs'* $(-\pi_u)$ *of control, is sufficient for* $tr(J) > 0$ (*and for instability if* $\det(J) > 0$ *holds simultaneously*)*. If* $\pi_{ux} < 0$*, then this social effect must be small relative to the own effect* π_{uu} (*in absolute terms*) *times the discount rate, more precisely,* $-\pi_{ux} < -r\pi_{uu}$ *for* $tr(J) > 0$.

Proposition 7 $\det(J) > 0$ *and* $tr(J) < 0$ *are necessary and sufficient for a stable steady state, which characterizes indeterminacy. The second condition requires 'small' discount rates,*

$$tr(J) < 0 \Leftrightarrow r\pi_{uu} - \pi_{ux} > 0 \Leftrightarrow r < \frac{\pi_{ux}}{\pi_{uu}}, \quad (25)$$

more precisely, below the social influence on the marginal costs [*defined in* (24)]. *Combining* (25) *with* $\det(J) > 0$ *yields*

$$\pi_{xy} > 0 \wedge \pi_{ux} < 0. \tag{26}$$

as necessary condition for indeterminacy.

Proof (25) follows from (16) as well as $\pi_{ux} < 0$, which in turn implies

$$\pi_{xy} > 0 \quad \text{or} \quad \pi_{uy} > 0 \tag{27}$$

in order to facilitate $\det(J) > 0$ in (15) or PNMDSI in (20). Multiplying (25) with r, then adding to (20) and re-arranging yields,

$$\left[r^2\pi_{uu} + 2r\pi_{uy} + \pi_{yy}\right] + \left(\pi_{xy} - r\pi_{uy}\right) > 0. \tag{28}$$

The squared bracket is a negative definite quadratic form since π is concave in (u, y). Hence, the second bracket must be positive so that either $\pi_{xy} > 0$ and/or $\pi_{uy} < 0$. Assume indirectly that $\pi_{xy} \le 0$, then $\pi_{uy} < 0$ which contradicts (27). □

A pair of eigenvalues with negative real parts implies that any choice of an initial value of the costate (i.e., of λ_0) from an open set around the steady state corresponds to a competitive evolution. Such an *indeterminate* outcome differs from the ambiguity between two different evolutions addressed in Proposition 7 and is impossible in pure optimizations. Thus it is a consequence of the social interactions while thresholds can arise absent social interactions (and even under the law of diminishing returns, see e.g., Wirl and Feichtinger (2005)). As mentioned, Fig. 1 links the conditions for a positive determinant = **PNMDSI** with the sign of the trace that differentiates conditional on $\det(J) > 0$ between unstable (leading to a threshold) and stable (implying indeterminacy) steady states. Assuming **PNMDSI**, $\pi_{ux} > 0$ is always mapped into a threshold according to Proposition 7. Assuming $\pi_{ux} < 0$ requires first of all $\pi_{xy} > 0$ (trivially satisfied for $MB^{y'} > 0$) to meet **PNMDSI** and both combined ensure then indeterminacy.

Remarks 2 1. $\pi_{ux} < 0$, i.e., a higher aggregate stock increases the marginal costs of expansion $(-\pi_u)$, is necessary for a negative trace and thus for indeterminacy, which reduces the three pathways to a positive determinant to two, $\pi_{xy} > 0$ and $\pi_{uy} > 0$.
2. Indeterminacy requires in addition to $\pi_{ux} < 0$ 'low' discount rates. More precisely, the corresponding inequality (25) demands that the net present value effect of an increase of x on the marginal costs $(-\pi_u)$ is larger than the consequence of an increase in the control (and thus the flow), $|\pi_{uu}| < |\pi_{ux}/r|$.
3. Hopf bifurcation as a route to limit cycles requires the existence of a pair of purely imaginary eigenvalues, thus

$$tr(J) = 0 \Leftrightarrow r = \frac{\pi_{ux}}{\pi_{uu}} > 0 \Longrightarrow \pi_{ux} < 0. \tag{29}$$

Such a bifurcation can only arise if a stable focus (i.e., the real part of the complex eigenvalue is negative) turns into an unstable focus, i.e., the local characterization that applies also to thresholds.[3] Thus Hopf bifurcations are located between indeterminacy—i.e., complete stability of (5) and (6)—and an instability leading either to a threshold or to (stable) limit cycles. A corresponding example will be sketched below.

3 Applications

3.1 Thresholds without social interactions

As indicated above, one can induce $\det(J) > 0$ for a strict concave optimization problem that lacks any social interaction. Thus even the optimization of concave welfare objectives allows for thresholds and corresponding examples are given in Wirl and Feichtinger (2005); the much more explored field of thresholds for convex-concave problems originates from Sethi (1977), Skiba (1978), Dechert and Nishimura (1983) with many applications following covering inter alia growth (Skiba 1978), public economics (Brock and Dechert 1985), public choice (Brock 1983), and environment (shallow lakes, e.g., Mäler et al. 2003).

3.2 Krugman's labor market: thresholds versus indeterminacy

Krugman (1991) familiar labor market model is chosen as a starting point due to its simplicity and because it allows for a threshold, and after some modifications for indeterminacy. Identical competitive agents allocate their time between employment in agriculture $(1 - y)$ that pays a fixed wage w_a (per unit of time) and manufacturing that pays $w_m(x)$ depending positively on aggregate employment $(w_m' > 0)$ satisfying $w_m(0) < w_a$, $w_m(1) > w_a$, for simplicity linear,

$$w_m(x) = a + bx, \quad a < w_a, \quad a + b > w_a. \tag{30}$$

Moving between these sectors causes adjustment costs. Assuming linear[4]-quadratic costs of adjustment (u) yields for the agents' objective,

$$\pi = y w_m(x) + (1 - y) w_a - \left(u + \frac{c}{2}u^2\right). \tag{31}$$

[3] This is surprising since Feichtinger et al. (1994) finds that limit cycles and thresholds are exclusive properties in dynamic optimizations, which holds also for a competitive economy of the adjustment cost type but affected by a dynamic externality, Wirl (2002).

[4] In order to satisfy Assumption 2, because pure adjustment cost models as in Krugman (1991), $\pi_u = 0$ at $u = 0$ rule out some of the complexities that are subject of this paper. The purpose here is solely to document the easy and straightforward applicability of the above Propositions.

The above propositions imply immediately that an interior steady state is unstable (either a node or a focus), since

$$\pi_{xy} = b > 0, \quad \pi_{uy} = 0, \quad \pi_{ux} = 0 \Longrightarrow \det(J) = b/c > 0. \tag{32}$$

Hence, this model yields a threshold and thus implies history dependence (the threshold is at the unstable steady state if the eigenvalues of the Jacobian are real) possibly depending in addition on expectations (if the eigenvalues are complex such that the saddlepoint paths to the boundary steady states overlap).

Karp and Thierry (2007) considers an interaction between agriculture and polluting manufacturing within this labor market that allows also for indeterminacy. The purpose of the following extension is however to introduce minimal changes in order to demonstrate the easy applicability of the above propositions, and to address also the issue of limit cycles. All what is needed to render the (unique) interior steady state stable and thus the outcome indeterminate is to assume that the adjustment costs increase with respect to employment in manufacturing (x). This is not entirely implausible, because it is getting harder to compete for a spot in manufacturing with many already working in manufacturing, or respectively, the job qualifications demanded by manufacturing increase as the industry expands and matures raising thereby individual entry costs. This leads to the following objective,

$$\pi = y w_m(x) + (1-y) w_a - \left(u + \frac{c}{2}u^2\right)\phi(x), \quad \phi' > 0, \tag{33}$$

and a linear choice, $\phi = 1 + \alpha x$ and $\alpha > 0$, is of sufficient 'complexity' to do the trick. Note that π is concave in (u, y).

The following computations stress the applicability and the power of the above pathways. Computing the relevant derivatives,

$$\pi_{xy} = b > 0, \quad \pi_{uy} = 0, \quad \pi_{ux} = -\alpha < 0, \tag{34}$$

all evaluated at the unique interior steady state, $y_\infty = \frac{w_a + r - a}{b - r\alpha}$, complementarity ($\pi_{xy} > 0$) must provide the route to **PNMDSI** while $\pi_{ux} < 0$ is the clue for indeterminacy. And given these insights, it is very easy to construct examples with stable interior steady states and thus with indeterminate outcomes. Figure 2 shows the (stable) flows in the phase space for a particular numerical example.

Furthermore, it also easy to generate limit cycles in this labor market via the Hopf bifurcation theorem following the above remark 2, item 3. Critical parameter values, i.e., where the eigenvalues are purely imaginary, can be explicitly calculated for (33), e.g., for

$$\alpha^{crit} = \frac{b + (a - w_a)cr \pm \sqrt{(b + (a - w_a)cr)^2 - 4bcr^2}}{2r}.$$

Indeed stable limit cycles exist [computed with Mathematica 7.0 and their stability is in addition verified with LocBif, see Khibnik et al. (1992)] and Fig. 3 shows a particular numerical example of such a cycle.

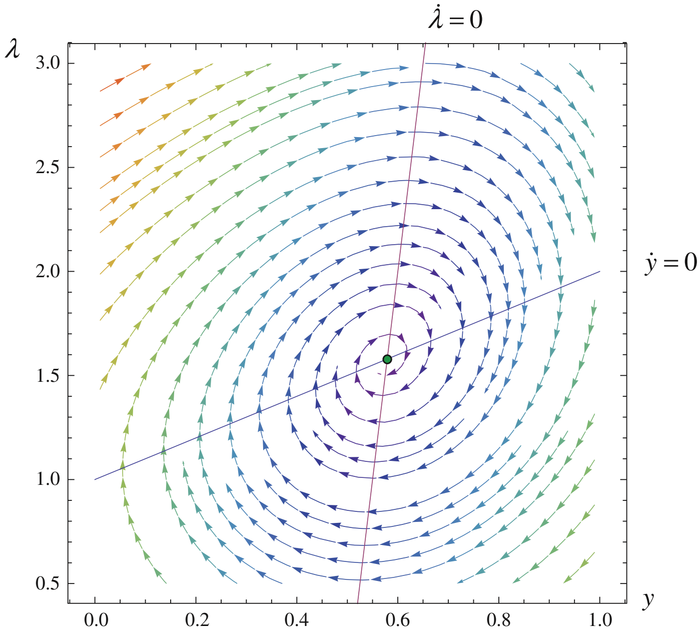

Fig. 2 Krugman model with a stable interior steady state for extended adjustment costs (33), $r=0.1$, $a=1$, $b=2$, $w_a=2$, $c=2$, $\alpha=1$

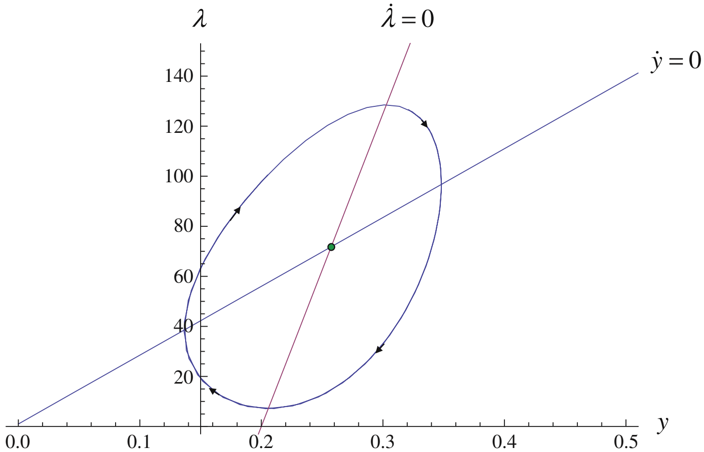

Fig. 3 Limit cycle in the extended Krugman model, $r = 0.2$, $a = 50$, $b = 250$, $w_a = 200$, $c = 20$, $\alpha = 275$ ($\alpha^{crit} = 228.0776\ldots$)

3.3 Neoclassical growth models

Consider the following class of neoclassical growth models with consumption as the only control and spillovers linked to a single stock (capital, physical or human, or a renewable resource): A representative (and competitive) agent solves the following

Ramsey-type optimal control problem,

$$\max_{\{c(t)\in\mathbb{R}_+\}} \int_0^\infty \exp(-rt)\phi(K(t), k(t), c(t))\, dt, \tag{35}$$

$$\dot{k}(t) = F(K(t), k(t)) - c(t), \quad k(0) = k_0, \quad k(t) \geq 0, \tag{36}$$

where $K(t)$ is exogenously given to each individual but identical to $k(t)$ for all t along a symmetrical equilibrium as in (3). The only particular feature is that the utility function ϕ includes not only consumption but also the private and the aggregate level of capital. This is not implausible, e.g., if absolute and relative wealth matters to the agents of the economy.

The transformations

$$k \to y, \quad K \to x, \quad c \to F(x, y) - u, \tag{37}$$

define the instantaneous payoff in the setup of (1)–(3)

$$\pi(x, y, u) := \phi(K, k, F(K, k) - u), \tag{38}$$

and imply the following derivatives:

$$\begin{aligned}
\pi_u &= -\phi_c,\ \pi_x = \phi_K + \phi_c F_K, \quad \pi_y = \phi_k + \phi_c F_k, \\
\pi_{uu} &= \phi_{cc} < 0,\ \pi_{ux} = -\phi_{cK} - \phi_{cc} F_K,\ \pi_{uy} = -\phi_{ck} - \phi_{cc} F_k, \\
\pi_{xx} &= \phi_{KK} + 2\phi_{cK} F_K + \phi_c F_{KK} + \phi_{cc} F_K^2, \\
\pi_{xy} &= \phi_{Kk} + \phi_{ck} F_K + \phi_{cK} F_k + \phi_{cc} F_k F_K + \phi_c F_{Kk}, \\
\pi_{yy} &= F_{kk}\phi_c + \left(\phi_{cc} F_k^2 + 2\phi_{ck} F_k + \phi_{kk}\right) < 0.
\end{aligned} \tag{39}$$

Both kinds of complexities—indeterminacy as well as thresholds—demand a positive determinant, thus (18). The denominator in (18) is negative, $\pi_{yy} < 0$, by the usual assumption of concavity of production and of utility (thus the term between brackets is a negative definite quadratic form) in the private variables (c, k). Complexities require a positive determinant and thus positive non-moderate dynamic social influence (**PNMDSI**) and these (equivalent) conditions can be arranged in a way that facilitates the economic interpretations below,

$$\begin{aligned}
&\left(\phi_{cc} F_k^2 + 2\phi_{ck} F_k + \phi_{kk}\right) \\
&\quad + \phi_{Kk} \\
&\quad + (F_{Kk} + F_{kk})\,\phi_c \\
&\quad + (F_K - r)\,\phi_{ck} \\
&\quad + (F_k - r)\,\phi_{cK} \\
&\quad + (F_K F_k - (F_K + F_k)\, r)\,\phi_{cc} > 0.
\end{aligned} \tag{40}$$

Proposition 8 *The determinant of the Jacobian associated with the neoclassical growth model* (35)–(36) *is positive iff* (40) *holds. Therefore at least one of the five terms following the negative definite quadratic form in the first line in* (40) *must be positive. This leads to the following five potential pathways for complex* (*either threshold or indeterminate*) *outcomes*:

1. $\phi_{Kk} > 0$.
2. $(F_{Kk} + F_{kk}) > 0$ *since* $\phi_c > 0$.
3. $(F_k - r)\phi_{cK} > 0$.
4. $(F_K - r)\phi_{ck} > 0$.
5. $(F_K F_k - (F_K + F_k)r) < 0$ *(since* $\phi_{cc} < 0$*).*

This proposition establishes general arithmetic conditions or ‘mechanisms’ for complex evolutions—history dependency (including the dependence on expectations in case of an ‘overlap’) or indeterminacy—that contrast the related literature that focuses on particular models and particular mechanisms, some of them blending the mechanisms above. The advantage of this proposition is that it is applicable at the stage of modeling, in particular models that lack any of the above five interaction terms does not allow for complexity. Positively, one can now easily construct examples off the trodden paths of power functions by departing from other (e.g., linear-quadratic) specifications and choosing the coefficients in ways that are compatible with the signs addressed in Proposition 8. A disadvantage is that each of these five conditions is only necessary and only the consideration of all five together with the negative definite quadratic form (in the first line of (40)) provides then a sufficient condition, which is hard to check at the modelling stage.

Remarks 3 The arithmetic conditions 1–5 in the above Proposition 8 allow for the following corresponding economic interpretations:

1. $\phi_{Kk} > 0$ requires complementarity of the preferences between own and aggregate capital. It is quite plausible to use own capital as a signal of wealth, thus $\phi_k \geq 0$, e.g., farmers buying ever larger and more powerful tractors. Indeed it is hard to conceive that one does not appreciate capital on its own, unless it carries a negative externality. However, $\phi_{Kk} > 0$ demands that this effect increases as the neighbors’ wealth/capital increases, which is counter to the usual status effects, $\phi_{Kk} < 0$, since richer neighbors diminish the benefit from individual wealth. Therefore, this route is rather unlikely, except for strong network economies that affect preferences directly.
2. $(F_{Kk} + F_{kk}) > 0$. Since $F_{kk} < 0$ this inequality can only hold for complementarity again, $F_{Kk} > 0$, and moreover sufficiently strong interaction in the production function such that the spillover effect outweighs the own effect on the margin.
3. $(F_k - r)\phi_{cK} > 0$. This condition, which requires endogenous [Ryder and Heal (1973) is the seminal work] and moreover socially influenced preferences over consumption, can be met under two different scenarios depending on the sign of ϕ_{cK}. Firstly, assume that (c, K) are substitutes within the preferences, $\phi_{cK} < 0 \implies F_k < r$. This deviation from the Ramsey rule can be achieved e.g., by positive private benefits from individual capital ($\phi_k > 0$). Secondly, assume complementarity between own consumption and aggregate wealth, $\phi_{cK} > 0 \implies$

$F_k > r$, which requires that the marginal product of capital is above its rental price, e.g., due to private costs associated with own capital (e.g., pollution).

4. $(F_K - r)\phi_{ck} > 0$. The interpretation is similar to above except that the growth effect from spillovers (F_K) is here crucial and coupled with the interdependence between marginal utility from consumption and own capital. Hence again, two scenarios are conceivable: $\phi_{ck} < 0$ requires $r > F_K$ ($F_K > 0$ is not necessary); $\phi_{ck} > 0$, requires positive and strong marginal spillovers that exceed the discount rate, $F_K > r$, which will presumably induce divergence (at least locally).
5. $(F_K F_k - (F_K + F_k)r) < 0$, i.e., the product of the marginal products must be less than their sum times the interest. Again two routes enable this. First assuming that both marginal products are positive, thus in particular a positive spillover, $F_K > 0$, their product must be less than the interest on both marginal products. Second, a negative and relatively small spillover, $F_K < 0$, resulting for example from pollution.

Assuming that the determinant is positive and thus that the dynamic social interactions are positive and non-moderate (at least locally around a steady state), the discrimination between the two opposite characteristics of indeterminacy (stable = two eigenvalues with negative real parts) and a threshold (unstable = two eigenvalues with positive real parts), depends on the sign of

$$r\pi_{uu} - \pi_{ux} = (r + F_K)\phi_{cc} + \phi_{cK}.$$

Proposition 9 *Assuming a positive determinant of the Jacobian at a steady state, this steady state is stable* (*two eigenvalues are either negative or have negative real parts*) *rendering indeterminacy* (*at least locally and transiently*)*, iff the marginal utility from consumption is sufficiently affected by the spillovers, more precisely,*

$$\phi_{cK} > -(r + F_K)\phi_{cc}. \tag{41}$$

Otherwise, it is unstable and induces a threshold.

What kind of spillovers are necessary for indeterminacy? It seems to be almost a contest about the weakest spillover still allowing for indeterminacy. While no externality is needed for $\det(J) > 0$, an externality is a conditio sine qua non for indeterminacy. Ignoring the requirement of a positive determinant, the crucial condition for indeterminacy is (41), which can be re-written as,

$$\frac{\partial c^*}{\partial K} = -\frac{\phi_{cK}}{\phi_{cc}} > r + F_K, \tag{42}$$

i.e., *an increase of the aggregate stock (capital) must increase consumption by an amount that exceeds the discount rate plus the marginal product of aggregate capital.*

Since a positive determinant is a requisite for indeterminacy, let us link condition (41) with the above five pathways. First, assume $\phi_{cK} > 0$, which is helpful irrespective whether the right hand side in (41) is positive (then it is even necessary) or negative (then any non-negative ϕ_{cK} satisfies (41)). This rules out the third pathway for a positive determinant. The other pathways 1, 2 and 4 require all additional spillover effects ($\phi_{Kk} > 0$, $F_{Kk} > -F_{kk}$, and $\phi_{ck} < 0$ for $F_K < r$) and the fifth is insufficient for indeterminacy on its own. In contrast, $\phi_{ck} < 0$, e.g., due to a negative spillover of own capital on utility, requires for indeterminacy that the marginal external effect exceeds the rate of discount, $r < -F_K$; more on this below.

How do these conditions relate to those obtained for endogenous growth models in the related works of Benhabib and Rustichini (1994), Shimokawa (2000) and Gliksberg (2009)? Benhabib and Rustichini (1994) focuses on eigenvalue properties of balanced growth paths and stresses the role of cross terms for indeterminacy by the mean of an example. Shimokawa (2000) uses a discrete time model and is concerned about periodic solutions and applies the indirect method of constructing a utility function delivering the cycles. Gliksberg (2009) focuses on monetary economics and thus, not surprisingly, the condition for indeterminacy—a passive monetary policy such that an interest rate increase with respect to inflation is sufficiently small (below an explicitly calculated threshold)—is restricted to such models.

The growth literature is filled with many examples, but most allow for more than one control, e.g., endogenous labor supply accounting for leisure preferences, investment into human capital, etc. in addition to consumption and focus on balanced growth paths. Furthermore, most of these examples off the shelf are rather computation-intensive, yet an objective of the following example is to demonstrate how easy it is to construct from the above derived properties models that allow for indeterminacy. Therefore, consider the following example of positive spillovers on preferences, more precisely, on consumption as argued above in order to satisfy (41), and production in a simple Ramsey model in order to allow for $\det(J) > 0$. Very simple and familiar neoclassical specifications are sufficient to do the trick and in fact the aim of the following example is to construct a very simple one:

$$\phi(K, k, c) := \frac{c^{1-\theta}}{1-\theta} + acK + bk, \tag{43}$$

$$F(K, k) := (f + gK)k, \quad f < r. \tag{44}$$

Preferences consist of the standard utility from consumption and include a positive interaction term (acK) and a wealth effect (bk, $b > 0$). The spillover gK can cover positive or negative (pollution) externalities depending on the sign of g. Production is for simplicity linear in own capital and the externalities are positive, ($g > 0$) as throughout the endogenous growth literature. However, the total marginal product of capital must be below the time preference rate (for all relevant states),

$$f + gK < r \Longrightarrow k, \quad K < \bar{k} = \frac{r-f}{g} \tag{45}$$

in order to meet Assumption 3, i.e., to allow for finite steady states, because an economy with returns on capital exceeding the discount rate can lead to unbounded growth. However, inequality (45) is necessary but not sufficient to exclude growth, because the value attributed to private capital per se can induce growth if this evaluation is sufficiently strong and if the initial conditions are above the threshold. On the other side,

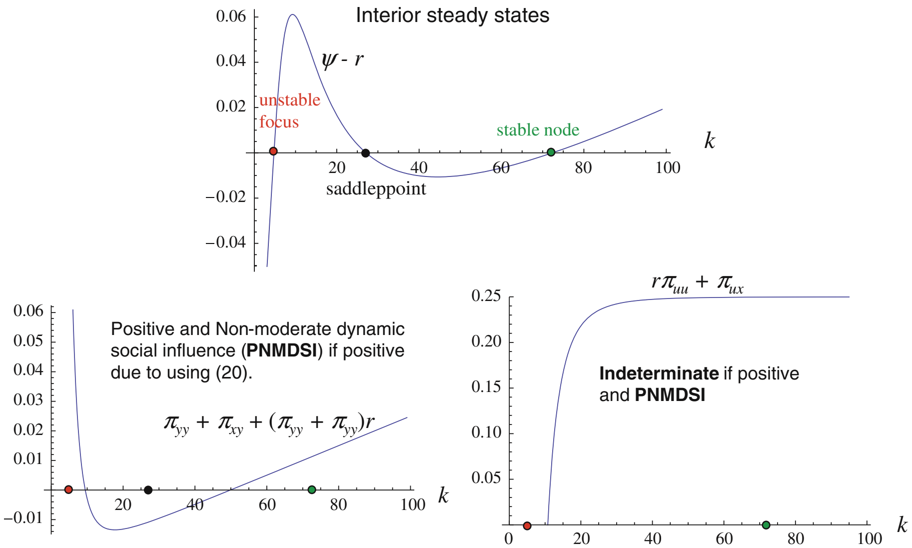

Fig. 4 Ramsey-type model with production affected by spillovers/externalities according to (44) and preferences (43) over consumption, individual wealth and a social interaction, $r = 0.2$, $\theta = 2$, $f = 0.10$, $a = 1/4$, $b = 1/2$, $g = 0.001$

this positive wealth effect can counterbalance the small marginal product of capital allowing for interior steady states (otherwise, $b = 0 \Longrightarrow k \to 0$ for $k < \bar{k}$, i.e., the lower bound is the only steady state). This specification satisfies only the second of the five conditions listed in Proposition 8, the term $gkK > 0$ in the otherwise linear production function. The interaction within the preferences ($acK,\ a > 0$) is required to induce indeterminacy according to Proposition 9. Therefore, this model has the potential for complex outcomes.

Indeed, one can easily construct numerical examples for the specification (43) and (44) that allow for multiple steady states, thresholds and indeterminacy. Figure 4 shows the construction of the steady states via the function ψ and the crucial criteria for **PNMDSI** [i.e., for $\det(J) > 0$ using (20)] and for indeterminacy [i.e., inequality (25) for $tr(J) < 0$]. Figure 5 shows the corresponding phase diagram with the peculiarity that $\dot{k}$ exists only above the indicated line (the feasible set below $\dot{k} = 0$ is thus very thin unless k is small). The multiple steady states (in ascending order) are: the limiting but (saddlepoint stable) boundary outcome, $k \to 0$, $\lambda \to \infty$, an unstable focus inducing a threshold ($\det(J) > 0$, $(r + F_K)\phi_{cc} + \phi_{cK} < 0$), a saddlepoint ($\det(J) < 0$), and a stable node leading to indeterminacy. The boundary outcome (=outcome absent spillovers since $f < r$) is attained along a saddlepoint path (in order to satisfy the limiting transversality conditions) for initial conditions below the threshold, which is located close to the unstable steady state but that is not explicitly determined. The interior saddlepoint stable steady state can be reached along the stable manifold and thus only for initial conditions on the indicated saddlepoint path. One half of the unstable manifold—which is shown by a dashed line—heads directly and contrary to its label to the upper and stable steady state. Therefore, this example

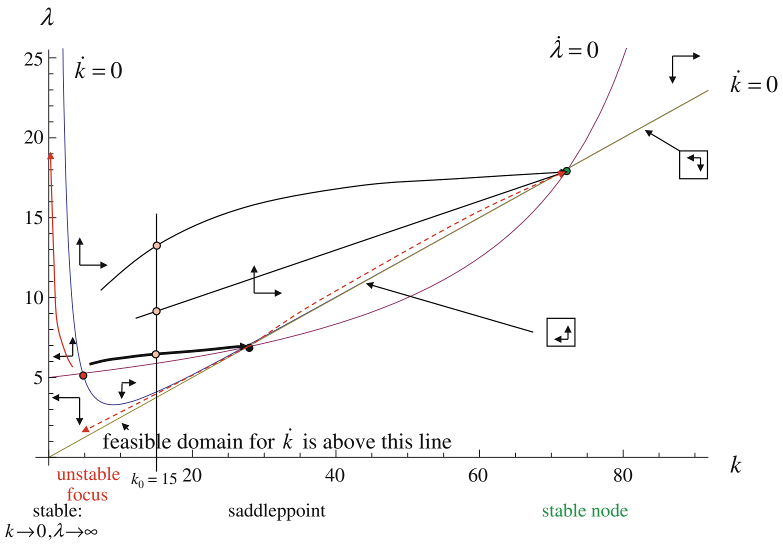

Fig. 5 Phase diagram—Ramsey-type model with preferences (43) and production (44), $r = 0.2$, $\theta = 2$, $f = 0.10$, $a = 1/4$, $b = 1/2$, $g = 0.001$; *boxed* motion indicators refer to the small part below $\dot{k} = 0$ and the feasibilty constraint

renders local (around the highest steady state due to negative real eigenvalues of the Jacobian) and global indeterminacy since both longrun outcomes are attainable for k_0 below the saddlepoint stable steady state depending on the choice of λ_0 (and thus of initial consumption) either on or above the saddlepoint path as indicated in Fig. 5 for $k_0 = 15$; however, k_0 exceeding the saddlepoint stable steady state (at $k = 27.1$) leads along one of the infinitely many routes to the highest steady state. As mentioned above, a higher appreciation of private capital, e.g., $b = 1$, induces a single and unstable steady state at $k = 3.14$ such that growth becomes possible despite returns on capital below the discount rate (substantially, e.g., starting at $k_0 = 5$ the return on capital including the spillover is just $0.105 < r = 0.20$). This growth mechanism is outside the familiar AK-model, but not further explored here due to the emphasis on economies with finite steady states.

Of course, one may argue that it is well known that externalities are capable to produce indeterminacy. Hence a crucial test is whether the proposed weaponry can derive indeterminacy in a class of economic problems where this is much less familiar. The following example will not only do that but will again demonstrate how easy it is to construct examples in so far less explored areas. Opposite to the growth models are environmental models, because the spillover from capital (own and/or aggregate) is negative on preferences and/or on production, i.e.,

$$\phi_K < 0, \quad \phi_{cK} \leq 0, \quad F_K < 0, \quad F_{kK} \leq 0. \tag{46}$$

Although many environmental models fall into this category, almost none of these growth models addresses the issue of history dependence, an exception is Wirl (2004) on a variation of a model of Ayong Le Kama (2001), and also very few and only recent papers investigate indeterminacy in models with negative externalities (pollution). Itaya (2008) investigates how environmental taxes affect balanced growth paths and the possible emergence of indeterminacy in increasing returns, learning-by-doing models of the Romer (1986) type with endogenous labor supply. Chen et al. (2009) show that if public abatement is substantial, indeterminacy may occur even in the absence of a positive labor externality (and for separable preferences). Although related to the framework in this section, none of these papers falls into the above model class, because of their emphasis on balanced growth paths.

The following example uses the above characterization of negative externalities due to pollution linked to the capital stock. The purpose is not so much to arrive at a 'realistic' model but to show how the above derived conditions—1–5 in Proposition 8 and the criterion (41) in Proposition 9—allow to construct examples, to choose corresponding parameters to yield indeterminacy, and how easy all this can be done. How can one meet the first criterion of $\det(J) > 0$ given the inequalities in (46)? The first route (according to the numbering in Proposition 8), $\phi_{Kk} > 0$, is not convincing for $F_K < 0$ and the second is ruled out by the last inequality in (46). The third route holds if coupled with the growth condition $0 < F_k < r$, which can be achieved by positive direct benefits from private capital, as used above in (43), and $\phi_{cK} < 0$. The fourth route is also applicable but demands $\phi_{ck} < 0$, i.e., an increase in own capital reduces the marginal utility from consumption which is somehow at odds with route three (private capital yields a positive spillover) such that one of the two routes must be chosen. Finally, the last route is suitable for negative aggregate capital stock externalities, unless to large. All in all there seems to be a substantial degree of freedom—either route 3 or 4 and for route 50—to obtain $\det(J) > 0$. However, how to satisfy (41) with $\phi_{cK} \leq 0$? Clearly, the limiting case, $\phi_{cK} = 0$, i.e., marginal utility from consumption is not affected by the pollution generated from the aggregate capital stock, is the best choice. Given a positive determinant, all what is then needed is that the negative externality in production exceeds the discount rate, $F_K < -r$. Furthermore, the total marginal product of capital $(F_k + F_K)$ must be positive for an interior steady state. Thus $F_k > r$, which in turn requires $\phi_k < 0$, i.e., negative effects of private capital on individual utility; this eliminates route 3 to $\det(J) > 0$.

In order to keep matters simple, the specifications in (43) and (44) are varied by changing by and large only signs (and using Greek letters for better differentiation):

$$\phi(K, k, c) := \frac{c^{1-\theta}}{1-\theta} - \beta k, \quad \beta > 0, \tag{47}$$

$$F(K, k) := (\varphi - \gamma K)k, \quad \varphi > r, \quad \gamma > r, \tag{48}$$

i.e., private capital negatively affects utility (ϕ) and aggregate capital imposes external costs on the firm's production process. The simple choice of production allows as above an explicit application of the above criteria, the calculation of the isoclines, etc. From the five routes to $\det(J) > 0$ only the last is suitable for the above specification, thus

$$-(\varphi - \gamma K)\gamma k - r(\varphi - \gamma K - \gamma k) = -(\varphi - \gamma k)\gamma k - r(\varphi - 2\gamma k) < 0 \tag{49}$$

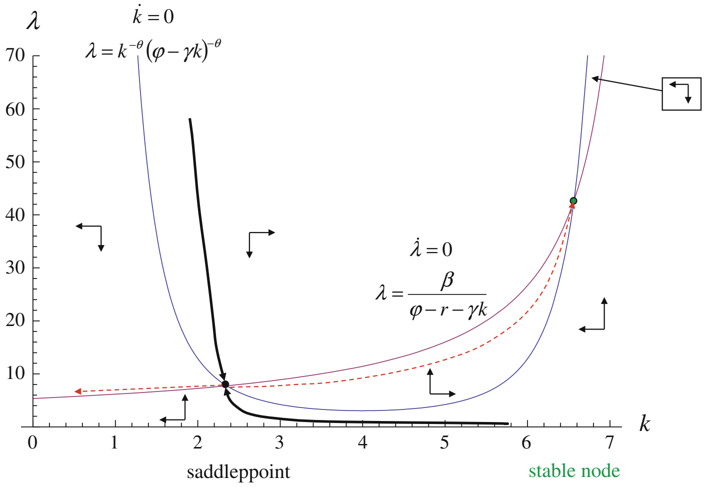

Fig. 6 Phase diagram—pollution model with preferences (47) and production (48), $r = 0.025$, $\theta = 5$, $\beta = 2$, $\varphi = 0.40$, $\gamma = 0.05$, thus route 5 in Proposition 7 is crucial for det(J) > 0

or respectively assembling all items of the Jacobian

$$k(\varphi - \gamma k)^{-\theta}[\gamma k(\gamma k - \varphi) + \theta(\varphi - 2\gamma k)(r + \gamma k - \varphi)] > 0 \tag{50}$$

for steady states $k_\infty < \varphi/\gamma$, which constrains the feasible domain for k, while the condition (41) for indeterminacy demands,

$$(\gamma k - r)(\varphi - \gamma k)^{-(1+\theta)} > 0. \tag{51}$$

Finding parameters that satisfy these two crucial requirements (50) and (51) at a steady state turns out to be very easy. A numerical example with two steady states, the lower one is a saddlepoint, the higher one is a stable node, is shown in Fig. 6 (again as a phase portrait, including the explicit expressions for the isoclines). Again, the unstable manifold is included in this chart and as in Fig. 5 (but much better visible here) one half of this unstable manifold heads directly to the stable steady state. Otherwise, the interpretations concerning local and global indeterminacy (between the saddlepoint and the stable steady state) are similar to the ones in Fig. 5.

4 Concluding remarks

The objective of this paper is to derive arithmetical and simultaneously economic conditions that provide pathways to complex evolutions within neoclassical growth models, i.e., to economies that remain finite. The crucial condition for any kind of complexity is that such an economy must exhibit positive non-moderate dynamic (social) interactions, where social is put between brackets since this condition can be met by models without social interactions in the narrow sense [i.e., in pure intertemporal optimization problems, see Wirl and Feichtinger (2005)]. In neoclassical growth models, a number of characteristics are helpful for thresholds and indeterminacy. More precisely, if none of these characteristics are present any steady state of this economy is attained along a unique saddlepoint path.

1. Complementarity between own and aggregate capital: either in the preferences or sufficiently strong complementarity in production that exceeds the own effect (or combined).
2. Preferences are either socially influenced and/or result from the individual accumulation of capital. In both cases, these endogenous effects on preferences must be linked to the marginal products of capital. The first subclass due to aggregate or social influence requires proper deviations from the standard stationary capital accumulation rule ($F_k \neq r$): $\phi_{cK} > 0 \Longrightarrow F_K > r$ (e.g., due to negative effects of individual capital on utility, i.e., $\phi_k < 0$) while $\phi_{cK} < 0 \Longrightarrow F_k < r$ (e.g., due to benefits from own capital, i.e., $\phi_k > 0$). The second subclass of preferences affected from the individual accumulation of capital requires analogous inequalities on the marginal product but here of aggregate capital ($\phi_{ck} > 0 \Longrightarrow F_K > r$) or ($\phi_{cK} < 0 \Longrightarrow F_K < r$).
3. Finally, the product of the marginal products (of private and aggregate capital) must be less than their sum times the interest.

The additional differentiation between a steady state that is an unstable node or a focus and the stable counterpart (again node or focus) depends crucially on how aggregate capital affects the marginal utility from consumption relative to the spillovers in production. If this condition is not met, any steady state with the property of positive non-moderate social influence must give rise to a threshold. Indeterminacy demands that the ratio of the sensitivities of marginal utility from consumption with respect to aggregate capital and own consumption, i.e., $(-\phi_{cK}/\phi_{cc})$, exceeds discounting plus the marginal product of aggregate capital $(r + F_K)$. Hence, $\phi_{cK} < 0$, e.g., due to a negative externality on preferences, requires for indeterminacy that the spillover on production is large and negative, $r < -F_K$ and given this property, indeterminacy can arise (and a simple example was constructed). In contrast $\phi_{cK} > 0$ proves helpful (although it complicates to obtain $\det(J) > 0$) and examples can be easily constructed. This arithmetic approach allows to draw a more general picture and to uncover more general economic mechanisms for indeterminacy than the common approach that departs from particular by and large familiar model structures. Indeed, the shown examples demonstrate how easy it is to construct examples.

Numerous papers stress the interaction of controls (often resulting from endogenous labor supply) in order to obtain indeterminacy, e.g., Pintus (2006) in a discrete time model. Therefore, an obvious candidate for future research is the extension to multiple controls. As long as an additional control amounts to a static decision rule, all findings remain valid after substituting the result of such static optimizations and then re-defining the objective and the production function. Although one expects that multiple controls should not have too much effect, after all, all motions are in the (k, λ) plane, the necessity of a matrix inversion to determine the partial derivatives of the controls with respect to state and costate (from the maximum principle) complicates the algebra considerably and is thus left for future research. Finally, the example of neoclassical growth points towards a growth mechanism—private appreciation of capital—which can trigger growth at returns on capital below the discount rate in contrast to the familiar AK-model. This may justify further research, including empirical investigations, of this so far overlooked mechanism for take off.

Acknowledgments I am grateful for the very helpful comments from two anonymous referee. I have also benefitted from many discussion with Franz X. Hof on related topics.

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FROM KUZNETS THEORY TO NEW GLOBAL GOVERNANCE, USING A MATHEMATICAL CONCEPT OF RELATIONS BETWEEN WEALTH CREATING KUZNETS CYCLES AND A KONDRATIEV-INSPIRED SYSTEM OF MANAGEMENT

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Abstract:

We recall that the debates around cycles since Kuznets, Kondratieff, Tugan-Baranowski, are coming closer to a unifying economic theory, borrowing concepts, ideas and models from the neo-Marxist, monetarist and / or neo-classical schools of thought. The theory of cycles perceived a structure in the economy, from which we can put into perspective statics and dynamics. Kuznets cycles would help us calculate a permanent new leadership in the global economy, whereas Kondratieff cycles will make understanding to abound, in order to exit of long periods of crises. That way we may get a top-down model for governance. This includes both global and institutional objectives. In order to allow future convergence by acting both empirically and theoretically using such a method, we also have to join a third direction, in the genesis of economic thought: how the various sources of European economic thought, before the divorce of the 1930s in econometrics.

In a book, French essayist Alain Minc qualifies the economists as “prophets of happiness” (Minc 2004). It is true that discipline has emerged among Scottish moralists such as Adam Smith (1999) and David Hume (1946) to the Era of British and French Enlightenments, as well as among the lawyers of gentle trade, supposed to spread happiness and harmony among peoples, as David Ricardo (1817) stated. That science also developed an early shifted reflection to dimensions related to the social contract (Locke 1985, quoted by Berthoud 1988; Locke 1979, quoted by Giacometti 1984, Dang 1997; Hobbes 1651; Rousseau 1963; Turgot 1970, quoted by Giacometti 1984), in the same time in the West when it kept under wraps an oldest source of reflection in Spain after a religious thought and concept of attrition with the reflections of the School of Salamanca[1]. These Atlantic pathways were crossed by a tension at the outset[2]. They had one day to confront a more continental approach, making the social subject, not polemic, but a main focus of research: it was the School Vienna and the value of human action (Von Mises 1985), the Social Liberals in Frankfort (Eucken 1989, 2004), Marx

(1983) even from the 19th century, as well as thinkers of cycles: Lachmann (1937, 1938, 1939, 1940, 1943, quoted by Longuet 1999) in Austria[3], Tugan-Baranowski (1894-1904 quoted by Makasheva 1993) in Ukraine, Chuprov (1889 quoted by Makasheva 1993) in Russia, etc.

There was also a day, a founder of the debate dispute to develop further the theories of cycles: the debate (see Kleinknecht and Van der Panne 2006) between the Russian Kondratieff and Ukrainian Kuznets. But, paradoxically, given the scope of the topic, the dominant economy, and the main representatives of since 1945 have been American scholars ... Kuznets himself who chose to immigrate to the United States ...: are relatively diverted from that path. It made its way as one of main paths of economic heterodoxy, providing much evidence of a sociological construction of economic institutions since year 1000 in the World and in Europe (Polanyi 1984), which ultimately means to integrate the phenomena of dependence in the modelling of economy (Kalecki 1987).

That way resurfaced strongly today in an unexpected way – while some researchers (Diebolt et *al*) have called "Monster of Loch Ness" (Diebolt 2002, 11–16)... At the same time, the very idea of the existence of cycles had actually released too little small in the 19th century. It was looked at as a strange object of observation, providing reflections on the phenomena of national economic development and their ties with the institutions of the economy, such as international trade, or the establishment of central banks, or education, in cycles (of) human life[4]. Hence, the first vague appellations starting from the "cycle of trade and business", that put us on the path of commercial or financial insurance techniques of merchants. This can affect the company – which rarely brags – but also the analytical framework of the national model, which guided the long path leading to social progress, before the financial globalization is tempted to take over. The objective is to integrate the phenomenon of a cycle of ten years, finding the right words to describe the cloud of side effects (legal risk, risk of bankruptcy… business interruption, loss of business, etc.)… In order to protect his business with care and discretion, in case you should suddenly face unexpected losses. This cycle is basically a market cycle and the beginning of the term – between seven and eleven years for the stock exchange, certainly a bit longer, twelve or thirteen years perhaps for real estate – from which the actors as intellectuals will try to establish assumptions of expectations of significantly more complicated and involving more distant time, integrating various registers of rationality (administrative, strategic, political, economic, etc.). At the same time begins the myth of the "End of Civilization" (from the late 19th century), its foreseeable consequences in the vocabulary of economics. These debates show, more and more: a thought of the "Great Depression" or "Great Depression, End of the Century" ..."The evil of the century" by poets, physicians and sociologists! A century after the first Great Depression observed in Europe – estimated by most historians to the years 1873-1896 – the genius of the late twentieth century is a century later, speaking at the outset of "crisis of civilization" (Amin 1980, 1988, 1991; Morin 1977, 1980, 1986, 1991, 2001; Modelski 1983) side by side with the dependence (Cardoso 1977; Herrmann & Tausch 2001; Tausch 1986, 1989, 1993, 1997a, 1997b, 1998a, 1998b, 2002, 2003, 2004a, 2004b, 2006a, 2006b), but in a context of rising risks (Beck 1986, 2001), which implies the need for new analytical frameworks (institutional economics, theories of regulation, multiple statistical analysis submitting their criteria to varied relationships with their games and challenges) in order to lay a sound and solid basis for a new beginning, a new system - richer, globalized financialized ... and crippled with multiple risk more or less properly taken into account and support today. The delay for such a project, that betray the anxieties (Ayres 2006: 55–71; Mensch 2006: 80–90; De Greene 2006: 10–21), are not fatal to the extent that we speak of a future system that will be very readily expanded in complexity and size by compared to the old. So beware of the risk of "a Tsunami!" This direction we will focus will enable us to quickly highlight the contribution of a Nobel Prize Winner in Economics [in 1971], Professor Simon Kuznets, particularly in terms of the famous academic dispute (see Kleinknecht and Van der Panne 2006: 118-125; quoting Kuznets 1940) between him (Kuznets 1930: 10) and Joseph Schumpeter (1939), but this with the profile given by a new situation, some eighty years after the famous dispute.

Since the first observations and empirical statistics of the phenomenon of cycles: (Parvus 1901, Tugan-Baranowski 1894-1904) ... We see that the phenomena of "strange attractor" of this uncontrolled theory use the starting time of a Great Depression (1929-1953) to provide an explanation of the mass phenomenon, both theoretically and statistically: the way Kuznets states that the existence of cycles of 18 years and the Kondratieff cycle, which sees 54 years. We will show that both approaches are not inherently contradictory essence of the first showing leadership in the creation of wealth, the second indicating the direction of the administration of this phenomenon.

1. **Theory.**

A) Yesterday

As Kuznets wrote (Kuznets 1940: 267) about the Kondratieff cycle theory:

To establish the existence of cycles ... one must first demonstrate that we take here fluctuations which occur lasting approximately regular, simultaneously with the movements representing various important aspects of economic life ... and secondly, there should be an indication that external factors or features of the economic system would be sufficient to account for these recurrent fluctuations. As the old framework of analysis of economic life will be in effect, the concept of such a cycle could be accepted without questioning the cohesion of economic life in general ... If the second condition, theoretically, is not met, we cannot establish a link between the findings on empirical observations concerning a certain type of cycles ... with the broader context of existing knowledge. Neither of these two conditions has never been satisfactorily completed in terms of Kondratieff cycles ... The prevalence of cycles of fifty years has not been demonstrated in the volumes of production or employment, or more in the physical commercial part, ... no satisfactory theory has been offered to explain these swings of 50 years that are supposed to return …

B) Today

For the Kondratieff theory, it was picked first by the pedagogy (Schumpeter 1939) which sees innovation the main source of growth which is organized along the Kondratieff cycles, depending scheduling with a pendulum effect. Then the relay is taken by Mensch (1979) which always assigns the role of innovation main causal factor explaining growth, but renewed pedagogy-in-fills, by saying that innovation occurs constantly, whether it or during periods of rapid growth or during periods of "depression" where growth is slowing, which he believes could one day help overcome the inevitable long depressive phenomena.

New education also helps to mask other issues skillfully indirectly related to the economic function of society: hegemony (the explanation of long waves by political factors like hegemony), World System (the Centre overlooking the Suburbs), training and organizational planning issues, etc. This "recovery" may be a factor by the other, including a factor of a nature (economic system) by a factor of another completely different nature (economic policy), can lead to two important practical consequences for the formalization of research: there may be displacement of the theory of a research area to another, especially from economics to move more toward political science. This risk seems rather limited because we live in an age of interdisciplinary research. So we allow ourselves to address these aspects, because it is a way for us both to report the presence and the existence of complexity and a normal way to address this complexity and scientific processing. However, the issue is important in econometrics. We cannot just say that since econometrics has grown massively, since roughly the 1970s, it was used first to conduct tests to compare theories and classify them in order to advise economic policies ... and that now he would need a new historical narrative accepted by all in order to refund the methods and the use of statistics. It should work simultaneously on the theory (which may include input from what history teaches us facts like this that can teach us the history of ideas), but also on the methodology, which opens also new epistemological questions, which do not necessarily arise only possible prior to any reflection, but may also arise during (to reflect new insights in response to the use of statistical tests and the results made) and parallel All-in-long research. This is the guarantee of a search process based on solid foundations, in a cultural context of the 21st century which we prefer to postulate the open minded aspect. We must then report the consequences of this on "maintenance" of econometric systems. Traditionally, if we perform statistical tests in order to "classify" the theories against each other, it emphasizes that the analysis of "facts" statistics – forecasts, and review of policy implications in terms of recommendations. Gold is always the problem of causes – and cause – that the econometrician cannot always deal face to a very high level of complexity.

Theoretical choices are then a paramount. We chose to focus on the following assumption, under an interpretation of long cycles of a monetary nature (Jourdon 2010a, 2010b). These long monetary cycles are backed by Management Systems of Property Rights, which in each new cycle indeed have to offer a new social project. Each new key currency to succeeding the previous one… is constituted as an attribute to defend such a social project. The end of the Kondratieff cycle is always manifested by a loss of coherence, whose practical consequences are manifested through foreclosure effects felt by all stakeholders in solidarity with the System. Here we see, with the end of the long cycle of $, the phenomenon of tax evictions: tax and social competition in Europe, and increasingly sophisticated financial technologies to avoid having to pay taxes: refuge in tax havens whose number continues to grow, refuge in sectors such as real estate shelters (so that the subprime crisis), the energy which suggests geopolitical shocks and a difficult transition to a long cycle to another, we may evoke financial wealth grabbing along with a number of private information or innovations also deprived of small businesses by Multi-National Companies organized to pay less tax by bypassing all intermediaries and advice – including institutional and / or public – and capture that way for organizational monopoly (3rd stage) overwhelming majority of access to resources, infrastructure financing enabling both to have market power and a power called off the market ... and finally the social consequences of these phenomena: indebtedness, loss of access to bank credit ...Particular we join the analysis (Mensch 2006) saying the long Depression is not necessarily inevitable. These phenomena can juxtapose with disastrous forms of social and even economic – or political – innovation such as participatory democracy (Jourdon and Tausch 2009), sustainable development (Chistilin 2006: 100–8), the Dialogue of Civilizations, Global Democratization, and Global Development. But this only confirms one thing: the development must be globally administered.

That is why we believe the debate between Kuznets and Kondratieff must now take a completely different form. Kuznets said that Kuznets cycles existed anywhere except in China. It was the era of confrontation between two political models also politically defending their economic efficiencies: the U.S. capitalists and the Soviet Union of Real Socialism. But even among the Communist powers of the time opposed two approaches: the industrial and planner towards the future of the USSR, with China emerging from underdevelopment… stating the necessity of walking on two legs: agriculture, industry. However, almost all the theories in development economics: are based on a structuralism reading of the economy, with core areas and sectors more or less industrialized as well as infrastructure and finally the domestic or informal economy. Kondratieff's theory is also sectorial according to the following teachings of Schumpeter. But his structuralism is different in nature: statistical structure seems to be in a prism through which one must pass if one wants to read a secondary reality. If we adopt today a comprehensive approach to development without separating brutally the developed and developing countries, we should compare the views to: adopt the perspective of the Dialogue of Civilizations. Clearly, then, that China's point of view of its civilization, appears in many respects as a Civilization of Commerce, this has always been. This can cause service wealth creation for the world. Kuznets cycles in their life cycles coincide with transport and also of the construction. These cross-cutting sectors in their internal functioning (the construction industry is a kind of working draft form: each project is a new company he must first sell – service logic – and simultaneously manage industrially in view the issue of timeliness, quality, and cost control, transportation is thanks to new forms of energy a new form to the source of the revival of each long cycle, but it also implies hidden costs at the service of economic organization) contain some features of global regulation economy and society.

<u>- Transport:</u> in the early history of the modern economy, the elasticity of the market was calculated from the distances thus the costs of transportation, even the interest rate reckoned by bankers (Chilosi and Volckart 2009) at the nerve centre of Europe (Rhine, Swiss German, Czechoslovakia) were based on the travel distance to reach potential customers, both physically and through information channels, and now ... at the age of information and Communication revolution: the transport issue arises increasingly in two directions at once physical and intangible (information transmission) with its resultant probably on the movement of money to manage its portfolio with the latest components of financial technology. And also raises issues of transportation of people ... All this mixed invites us to compare the transport to the nervous system of the economy. Provided it is highly important constituent of the fixed costs of organizations, and because of their importance they must be properly insured!

<u>- Construction:</u> basic element of the distinction between ecology (home management) and economy (trade management in both professional and private areas). So it would

appear like the first element of heritage that underpins much of the overall security of the economic system. Therefore we compare it to "the backbone of the economy".

The discovery of the Kuznets cycles and their importance clearly confirms the importance of these two sectors for the health of the economy. As the French proverb: "Quand le bâtiment va tout va!!" *"When the building goes all goes!"* Of course this is not inconsistent with the fact that China can be an engine of wealth creation around the world through trade. Factors guiding the creation of wealth in his direction may come from structuring sectors such as construction, transportation, logistics, advertising, marketing, trade. Maybe we can dare to assume today that the Kuznets cycle, around 1940, was not very noticeable in China, which fought to get out of underdevelopment, but today, when the world changed, as the word you ready to Aristotle: "the truth is in the middle". There would exist Kuznets cycles: as well perhaps in China.

2. Methodology. Epistemology.

A) Yesterday

For Kuznets, inequality knows an inversed U-curve. At the beginning of its development they grow quickly, because entrepreneurs playing a pioneering role are likely to get rich much faster than the rest of the population. In the second period, when society became more structured ... and thus better managed, it may well pay more in wages and benefits associated with social protection mechanisms that must accompany them. Income gaps between all the agents are reduced through the increase of the mass of employees properly treated. Today, many countries emerging out of the third world show more dynamism – both economic and demographic – that industrialized countries that prevailed to retain their technological edge. We can then say the problem should now be addressed at the global level. Is there an inversed U-shaped inequality [⌒] acting globally? If yes, how is it possible to manage it? Unfortunately, the theory of Kuznets shows the path of hope – the increase of wealth – and then hope that these resources would be better administered, managed, shared … but how this can be accomplished in the very long term or in the part of the World System – has become very complex and currently subject to a series of serious crises – financial, food, energy, geopolitics, social...

A second approach developed by Kuznets seems rather give us a direction that will take the phenomenon of wealth creation. But this does not seem to offer technical alternatives to achieve avert risks that could accompany that phenomenon of wealth creation. Unlike Keynes (1969: 112-152), Kuznets assumes that when a country's GNP increases, the propensity to consume of agents increases as long as earnings, even more. This leads us to believe that the increase in the propensity to consume more than income growth, accompanied by a growth, too, of the size of inequalities that are likely to occur with the phase of growth of income when in times of crisis the centre of the world economy (debt, crises closer to the centre of the global system as the financial crisis in 2007 and the subprime crisis the following year) is hurt … that would not find solutions without a new administration system. But given the unconscious factors at play (if the lower turning point can be easily calculated, assuming that one has even been able to determine if it was a problem of price or quantity of the image ...from "Loch Ness Monster of economic theory" proposed by C. Diebolt (Diebolt 2002: 11–16) sometimes appear to be sound!), is not it necessary to focus attention on finding the causes of the phenomenon at the start?

B) Today

It's about time, calendar that the actions of different agents can coordinate. The only Nobel Prize in Economics has spent his entire academic career in France, Maurice Allais, was once severely criticized for its introduction of the mathematics in economic models in France – as in many countries, France, the teaching of economics was a teaching in method and critical thinking by getting in touch with moral philosophy, law and economics itself: the whole was so low mathematized… after all even the "General Theory of Employment, Interest and Money" (Keynes 1969) does not include a single equation. It seems that building the economy system of equations as complex as astrophysics would not pose difficulties only if two criteria are at the same time also met in the strictest possible way, that means:

- ***The question of timing:***

Tell us where is. Usually, the hegemony is the master calendar. This gives it a monetary authority to fix the price of money, control the actions of most agents and how they are coordinated. The rejection of economics or its mathematization is often the rejection of the possible consequences of: the supervision – or likely to be – by the "Big Brother" imagined by George Orwell (1994). Time is likely to be a conditioning parameter for understanding the course of the series, and at the same time for those who reject the power that goes with it: it is possible to consider it – even if it is a parameter and not a single variable – as a discrete parameter, which goes without saying and that we would not limit the need to clarify the existence in the equations. The economy then runs the risk of slipping into a no computational or even informal way of being. Sometimes by circumstances of extremely cold conjuncture, awareness of the mystical dimension of time is emerging, and it will relate more readily to the "time of the universe" – that of nature, or, ... cosmological equations – that "World Time" – the one constructed actions and society started by humans – then we might be led to reject the international currency and all that goes with it.

- ***The issue of self-beliefs:***

Hence our second criterion: it must also be able to apply the equations in a space that is institutionally recognized in the economic system, a space where self-belief in human beings are respected. That these beliefs are of religious or humanistic essence, or even atheists (and) sometimes as long as they relate to a provable moral paradigm: beliefs, with preferences (these often come second in contradiction with these materials) and with the strategies (they are the result, the "precipitation" within the meaning of chemical interactions between the first two, even by including the strategies themselves, which constantly form, reform and change over collective action or individual actions), if we must consider autonomy the person in relation to the individual games ... There would be no problems using the "astrophysical equations", provided you know "where we are" therefore also the origin, the repository, from when we begin to measure and study the series of events ... trying to clarify the "nature" of the events described.

3. "Econométrie *(The French word)*". Econometrics (English translation for French words "économétrie" and "économétrique". The second word is more risk-tolerant than the former usual one).

A) Yesterday

Since the 1970s, "économétrie" has developed. It was used to compare schools of thought in economics and political economy in particular, in order to look at the consequences if we were to apply their recommendations. Thus, the natural movement of the model was to go with the consequences ... if we apply the model to make policies to follow. It carries out this test. We will always model the consequences and the consequences to the model, as if the underlying cause was necessarily known and controlled. The idea is to add the scientific tools of statistics to clear discussions about representations of reality and its possible actions. The objective there is clearly to avoid falling into pure ideology.

A second major reason for the massive development of the use of statistics – except the development of technological tools themselves, with the creation of econometric software – was the growing complexity of an economy that has become multi-function and multi-purpose... Many companies have tended to create their own "system", whether of information or decisions. Multinational firms, as to them: tend to make the macro economy contradict the opinion of the creators (Galbraith 1989) of macroeconomic theory. The macro-open economy that has begun to develop over the last decade of the twentieth century: does force to develop analytical tools of econometrics: ever pushed in order to mathematically predict the consequences of combined effects of several factors acting simultaneously.

However, logically, when one looks to the future consequences of possible decisions still to be taken, it may often happen that one approaches – or has the impression of being closer to – the discovery and understanding of the causes. Sometimes one may have the impression of discovering a fundamental underlying cause, or, the ultimate cause, analyzing reality. Therefore it was pedagogically useful to distinguish "économétrie" (the French most usual word) and "économétrique" ("econometrics" captures both French vocables). The first expressive vocable acts as a general theory of statistics applied to study of a phenomenon which can easily measure profitability – financial –; productivity – physical –; and link it to a representation of political economy which is known to remaining fairly well expected in terms of moral philosophy attached. In the second, on the contrary, given the intertwined phenomena, the approach is to screen the risk of calculating profitability. However we hope to go further in understanding the ultimate cause, thanks to improved visibility of reality that gives us the proven ability to filter the risk and overcome the obstacle that is usually in the good knowledge of reality. This must of course allow, in theory: to go further.

B) Today

Wolff pointed out already that any statistical theory begins with taking conscience (Wolff 1993: 233–246) of the more or less affirmed existence of a series. This decision of consciousness can be achieved in a "search for frequency" style of research: looking after a landmark event which always recurs with some frequency. It can also be realized as "residuals-diversion" style of approach of an existing series: so, if one understands that a determined series of events is a factor of the system, i.e. a root of mathematical explanation, one can then hope to reconstruct the inferential function and constitute a "residue", even if sometimes the latter appears a not desired (at first) consequence. Then, by mathematically integrating the residue: one can hope to reconstruct a vision, then a solid, consistent with reality, scientific theory.

But today, spectral analysis was developed with some risks of confusion that may result. Spectral analysis acts on about the frequency, assuming that the phenomena which are presumably found ought to correspond to the existence of "memory" of the entire system. However, if found through this phenomenon actually comparable by form, but possibly of different nature, structure…not only could we not be sure to undoubtedly reach to meet through it more "memory" – mobilizable to serve the entire system –... but the more we run the risk of going to misinterpretation! In biology, we know the example of plants that views from outside seem to belong to the same family, although they are genetically very different and cannot reproduce them. Unlike other plants look very different belong to the same family.

These new methods do not suffice to explain a phenomenon like that of Kondratieff cycles, which manages to create a "structure" by subsequently serve as a statistically robustly grounded “filter” for scientific interpretations. By construction through four different types of parameters, this theory resurrects an old mathematical mystery at the very foundation of the building of some famous Civilization – the "four elements" of the Greeks ... base that become five in Chinese. For Chinese in addition to air, water, earth, fire, he would immediately add the wood that allows even more to act on living nature of things and the ability to act generally paid humans. The transition from four to five in the spirit of fostering actions may be however, conceptually not easy to reach – beyond any description but switching to a dynamic, inclusive (and himself included) paradigm for social sciences. In the other direction, to find a semblance of unity, drop from four to three could be little easier: how to achieve that "squaring the circle"? The practical solution adopted by many researchers could be "calculating a direction" for the "creation of wealth" ... and reasoning based on three elements of "direction"… in failing to not concern the "meaning" to be intended to be the systematic ”fourth" element. This fourth element acting as a horizon of research would be devised to give meaning to the first three. It could well stabilize the whole system. In fact, when the system with four elements would change from a passive state to an active state, it could discern the existence of a fifth vector. He will represent this action whose effect produced by a single vector system will be to expand the size of the entire system. The reverse operation is possible: the fourth parameter can be "inhibited" by the entire system, including in some cases due to a simple "statistical noise" made about him by the overall system because emerged such a faulty understanding of its members of its ins and outs of her. In this case only three vectors will be clearly visible... as the carrier of meaning would be hidden. It probably why Kuznets has collected more successful in the West than Kondratieff did: it calculates directions of profit, regardless of whom when it comes to pay the tax dimension of things – including globally.

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Thus, for our part, we conducted an Economic History of Europe showing the progressive income collected by *homo monetarius* since 1800, a Monetary History of Europe (Jourdon 2009a) showing concerns Central Bank to secure and diversify the income of the *homo monetarius* within the World System since 1800. Our theory: it should partially rewrite history of money to reflect this new agent, *homo monetarius*. The following: 1) makes decisions allowing it to diversify and take financial risks, 2) increases its sense that its political philosophy ought to be designed so as to strengthen the quality of his insurance, 3) performs information transfer with the environment, with a monetary character of this information so as to better ensure the world around him. The next monetary Long Cycle (1992-2090) is the cycle of the new key currency, the Euro (Jourdon 2011a, 2011b). It follows the respective cycles Pound Sterling (1848-1945) and the Dollar (1917-2015). There will the € specify his reservations vis-à-vis the system embodied by $ (1980-2020), becoming the first reserve currency in the world while load-bearing debt of the World System (2015-2055), then weaken, decline and pass the baton to another key currency (2050-2090). The € brings with it a new social project: the balance between private property, social property and self-property, thus following the projects $ (balance between private ownership and social ownership) and £ (liberalism in defense of private property). The works of Kuznets same as that of Kondratieff remarkably fit into our perspective. They enrich it. If the Kuznets cycles show wealth creation, and those of Kondratieff the best way that is humanly imaginable to administer them, how to think the borders between sectors, countries ... to enhance coordination. An approach of semi-monetary long cycles (Jourdon 2008: 95–122; 2009b: 13–26) or monetary rethink could help both series of cycles – Kuznets and Kondratieff – find new paths of convergence.

Clearly, the long cycle of $ does not take completely into account through the lasting justice. On the contrary, the miracle last fifteen years is that good, from a more technical point of view, the holders of economic interest managed accuracy of their investments and investments while moving them towards new growth-areas, so their endeavour to reduce taxation income, and in the same time on the more general point of view there is a consensus on a relative stabilization, perhaps the calm before the storm. In the political discourse we observed the effects in Europe: so-called "the single thought" in whose name the right and the left defend the same values for tightly managing the euro, seems remarkably empty in positions asserted to a separate original social project. Europe is becoming a huge area of relative stability, which must then convince her split horizon, beyond its borders. More than ever, Kuznets and Kondratieff become inseparable in their paradise of Great Economists: the world today to 50% democratized (Jourdon 2010b) and 50% monetized, requires prudent management, but also a way in order to bridge unpredictable differences.

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[1]Tortajada (1991) provides a good discussion of the contribution of the School of Salamanca for the creation of economic science in Western Europe. It, as any human science initially draws its arguments in law, and also in the culture of the very religious late Middle Ages Era. Thus, "Les Deux Sources de la morale et de la religion" Bergson (1932) exposed at the time of the triumph of sociology in France and of the separation of powers between state power and that of the Church, we can cast her gaze back centuries and see the source of religious thought in the creation of Economics (School of Salamanca) and the pulsed source to moral philosophy came from Scotland, a century or two after the first source of inspiration. What might appear to us today as a rather boring presentation is very important to clarify the beginnings of the capitalist system. It is perhaps in order to no longer remember, or even deliberately conceal the source of religious thought that economists could have undertaken to relate their mathematical discipline to science nature. Around the time of the Theory of General Equilibrium of Walras (1874), they – maybe for such a reason – tried and rejected as much as possible its historically constituted discipline and logical links with the humanities ... But still, sources of religious thought related to the education system in the Middle Ages called Scholastic (Roover (de) 1971, Sierra-Bravo 1975), can afford to analyze or decompose the mechanism supported by contracts (those involving more trade and commerce than manufacture organization and large industries)… Studying contracts on one hand, and treaties on the other hand, the School of Salamanca (Azpilcueta (de), see Gazier (1978); Baeck 1987; Grice-Hutchinson 1952) leads to a perfectly coherent currency effects: they first reduced to commercial contracts and opening to finance (Mercado (de) 1569), will be extended with the discovery of America to monetary aspects (Hambleton 1955). On the other hand, weighing pieces that also evokes the concept of action at the same time that weight was considered at that time as related to the "weighing of souls" at the time of colonization by Spain and Portugal in South America and Central America, this would be regarded with horror today ... The interest rate was generated in response to scholarly considerations of time, society, wandering merchants, the convolutions browsers, calculations owners, tribulations diplomats working for States or for the Church of Rome. The interest rate, though, was "created "as opposed to the practice hitherto current rate of wear. And this major legal innovation was probably marked by the desire after weighing "souls", to save maximum on earth through finance, and therefore the Western economy and

capitalist economy! The influence of the School of Salamanca in the law prior to the birth and development of the capitalist system – first historically being a merchant capitalism -, was a coherently packaged legal corpus. It led the way in the doctrine of "fair price" (Gomez-Camacho 1985, Lapidus 1982, Molina (de) (1981)), in relation to well before the utilitarianisms with the concept of "balance of pleasures and pains". But such a concept understood in the sense that Spanish had in mind, came early to add a link with the law. It is notable that this allowed time for the Church in this country to think in depth the question of social poverty (Vitoria (de) 1928, 1934, 1933-5). And centuries later, Schumpeter, and Keynes, both felt surprised intellectually because it appeared necessary to them to return to the path of wear, and the way of interest, to think the relationship between law and economics before laying the foundations of macroeconomics (Keynes 1932, Schumpeter 1954; see also Dempsey 1948, Lapidus 1987, Melitz 1971). After that XXst century macroeconomics was calculated more in destination of the industrial sector, than in destination of the merchants or of the grossers…

[2]French and English thinkers of the same period were less influenced than their Spanish economist's colleagues in their thought by religion and by the Catholic Church. They impulsed their own original paths and initiated separate calculations. They had in common, however, research on the sources of social consensus. They mainly apprehended (in England and Scotland), or quite exclusively accepted (in France), such a social consensus: outside the religion. It consisted initially in the definition of a line between the moral sciences, and religion, on the basis of reflection concerning human understanding (Locke 1979). Here we may detect the laid historical foundations for solid economic theory vocation of the currency… the Spanish still more attached at that time to the concept of « fair price »! The theory of money announced and announced money has been driving macroeconomic theory. Macroeconomic theory has less to do with definition of currency or even of what is macroeconomics, it is much more concerned with actual recommendations in terms of monetary policy (that also means that macroeconomic theory generally speaking can barely solve its perspective issues in links with other science, with linguistic and sometimes with national conventions put it in legal terms). It appeared after golden period of a full series of « Treaty of Political Economy », the Theory of General Equilibrium by Walras (1874). It will address macroeconomics that caters to the official sphere of monetary circuits, not forgetting in the same time the semi-official ones, we mean credit effects and in a large conception the rather implicit study (legal preconditions often taken for granted) of all dynamic phenomena taking place in the economy, provided we study in the same time their consequences (for macroeconomics is based on taking account of the anticipations of economic agents). That last remark is able to open a wider research steering again than the only macroeconomics, which includes use of restrictive preconditions – once « reasonable » equations have been accepted about anticipations. In particular the number of channels in question will rather be limited in practice for reasons of policy effectiveness because of its quantitative nature – channel of money is official, channel of credit may appear to rely more on national conventions – although this is evolving extremely fast due to fast track pressure of globalization. The English and Scottish during sixteenth and seventeenth centuries claimed that the price of the currency was bound to result from a convention… All other prices would result more out of contracts

than of a convention: we mean basically commercial contracts. According to Locke (see Berthoud 1988, and Dang 1997), the average interest human beings use currency as a marker of their ownership links in an open game between morality, money and property, which is expected to deliver a « natural price » for the money. For French economist Turgot, money would instead be more opportunity for each individual in society to express their feelings related to social life (Giacometti 1984, Turgot 1970). Such an acceptation appears semantically being more located in an economy of the sign. While the English would deliberately prefer to locate it in a more structural sense, or at least more structured or, last but not least… more structured!

[3]After the National Revolutions took place in 1848, the birth of a number of National Central Banks, writing of a range of « Treaty of Political Economy », and last but not least industrialization, economic theory had to be completely redesigned. Longuet (1999) argues that Lachmann had thought about the major economic fluctuations that are often linked to major historical shocks, whereas the thought of Keynes would have been more suitable for relatively average magnitude of fluctuations in the context of an already stabilized financial institution…

[4]In fact we could present our arguments here otherwise, or even offer other choices on the comparative importance of schools in national economy compared with respect to a common heritage of Europe economists who prefigured some approaches used by the institutions currently at the power in the UE28. Just for the French approach and regarding thinking about money, for example the approach of S. Sismondi (1819) in this regard deserves to be mentioned, which anticipated a century thought on the circuit for future macro-economists. And we can also legitimately continue to question the relative importance of different sources of inspiration which have nourished the thought of Keynes. The role of early institutionalisms including Veblen (1970) might have been underestimated; the theory of the leisure class was not necessarily the teacup think for someone who wanted full employment – even in view of an increase in overall household consumption. Nevertheless, after 1945, it was long slightest concern sources such as Macroeconomics. Macroeconomics has grown tremendously and « the American », that is to say, that it adopted « factual, inferential, quantitative » methods. Such global methods included a way up to the « rational expectations » where no information can be put away if we really want to solve a recurring problem whatsoever. Rothschild (1999) notes yet after 1945 – and still more today – the existence of some method, a style, a different way of approaching problems that would distinguish a common way of being a European economist. It would be slightly easy to recognize it even beyond their respective national schools in economics - whose importance is very high. And it would also distinguish them from their peers located in America… Even earlier – before the European and World catastrophe of 1914-1945 – it seems to us that in England A. Marshall in an extremely accurate sense of analysis (1906) identified almost every problem – particularly between economics, law, and organization… And almost brought not any answer because every time, to bring a practicable theoretical solution it would have been necessary to make the « jump » between microeconomics then reached to a peak in the design, and at this time, still not existing quantity-based approach - called macro economy – and that was developed a few decades after that time. The four giants of thought during the interwar period, most probably Keynes,

Kondratieff, Coase (1937), Schumpeter, powerfully helped to define the style. We ought to add Kuznets to these four major funding fathers of further methods and controversies, but still should mention in the same time that Kuznets accomplished most of his results after war, in the NBER in Chicago – also Coase expatriated from England to the United States of America. Then, from the intersection traced by these authors and in particular Keynes and Kondratieff, roads diverged. But we believe: this is not the way of thinking as such that is most concerned. The latter in a still strongly growing discipline, can leave some guessing by the very style itself. Actually, this is the tool that seems more relevant here, that is to say: econometrics. That narrow path of the European economists often more worried about sources, seems to be slightly different from the approach of Americans who in their mainstream dominant way face a technically applied mathematical and justified by an extreme quantitative approach and having triumphed in most branches of science. It may have triumphed not only in macroeconomics – which would be the natural way *per se* – but even also in microeconomics and because of the triumph of finance in a globalized economy. In this field the art of mathematics is able to identify gaps or contradictions within models, and they will propose solutions to improve these models, while maintaining them core quantitative. Such a kern quantitative approach – and neo-liberal could not until the 1980s confront easily with economists from Russia and Eastern Europe who kept undeniable advance in the theory of cycles including long cycles – they had fashioned the initial approach. Solutions for the near future may reside – in trying to let us be inspired by Rothschild-type reasoned concern -: either in a suitable mathematical characterization of the roots of development, in the spirit and continuation of De Munck, Grinin, Korotayev, and Turchin (2006), Korotayev (2006), Korotayev, and Tsirel (2010); either in a more applied approach to go back to find the roots of the economic cycle throughout the last fifty years, or even longer political cycles the last century. The goal will be to detect how the divorce could occur; this could be the condition to avoid still being trapped in the current Civilization Crisis. Says so trivial, econometrics is the less “objective” discipline: which comprises continuously to choose, by placing data sets from each other.

HOW TO RECONCILE KUZNETS AND KONDRATIEFF?

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Abstract:

The debate between Kuznets and Kondratieff in the 1930s rests a new way, which gives some moral justification to the positions of both authors regarding long cycles' theory, with in scope an economic analysis framework fitted both to administration of growth and prediction of crises in 21st century. That attitude of Kuznets, very cautious against the theories of half a century long cycles, could be justified by uncertainties in respect of terminology, making it difficult at that time, construction of a matrix of progress, acceptable to all. Depending on whether one thinks in price or quantities, we will benchmark against the reversal of the low point of cycle or high point on the contrary, depending on how we link the demographic and monetary phenomena, and, third, the impact of the energy on the motor of economic production, it may be in the indeterminacy of how getting out of crisis on time… The scenarios will differ, depending on how we track either the time of cycle turns, or the time of the outclassing of the former model in economic governance. Two tracks seem likely promote reconciliation of Kuznets and Kondratieff. A "contingent" – "monetary" or "semi-monetary" – approach in the spirit of French scientist Simiand (and followers) to the economy in the international crisis, can meet this challenge of ensuring delicate transition period, provided the second condition is met for the establishment of robust regional framework giving way to building of relevant super fitted systems of qualifications.

The approach of the long cycles is one way of taking into account the long term in economics and a framework agreed by some historians. However, analyzing works referring to this approach reveals that many epistemological, methodological, theoretical and statistical problems epistemological remain unsolved. On the flattening of these problems and responses that researchers in economic history will be able to make will depend the future of this theoretical corpus.

As Kuznets wrote in 1930 about the theory of long cycles made by Kondratieff : "To establish the existence of cycles ... it must first demonstration that we hold there fluctuations which occur for a period approximately regular, simultaneously with the movements representing various important aspects of economic life ... and secondly, there should be some indication of what factors external or peculiarities of the economic system would be sufficient to make account of these recurrent fluctuations. As the old framework of analysis economic life will be in effect, the concept of this type of cycle could be accepted without question the cohesion of the economic life in general ... If the second condition, theoretically, is not met, no may establish a link between the findings on observations empirical evidence on a certain type of cycles

... with the wider context broad knowledge already established. Neither of these conditions has never been completed satisfactorily in terms of cycles Kondratieff ... The prevalence of cycles of fifty years has been established or in the volumes of production or employment, or more in one, physical, trade, ... no satisfactory theory has been provided for explain these swings of 50 who are supposed to return ... [1]

1. **The graphical representation of the cycle: a technical problem revealing a failure in theory.**

All authors wishing to deliver a theoretical construction of the long cycle were compared to identify each of the two phases. N. D. Kondratieff uses the terms "ascending waves" and "waves down." J. Schumpeter identified four phases: prosperity, recession, depression and recovery; other authors stick to the two terms "prosperity" and "depression". With its "phase A" and "phase B", Simiand invented a terminology which is disconnected from the word "growth" using a rather imprecise and shifting semantic content[2]. The terms "long periods of rapid growth" and "slow growth" are legion in Louis Fontvieille's.

This long cycle sequencing is based on volume indicators (production of various goods in different countries, or taken globally) and / or prices (of goods, labor, financial assets ...). The sharp increase in these variables is in the first period of a long cycle qualified of "growth", their stagnant states or to decline is to be found in the second so-called "depression". However, the use of these indicators alone as criteria for determining economic conditions raises questions. Are they sufficient to characterize the phases of economic movements? Higher prices and production they still occur in the first phase of the cycle, their decline in the second?

Theories incorporating only the mechanisms of growth based on technology often appear artificial in historians' eyes. The survey made by Kristine Ryland and Keith Smith - that focused on the analyses of N. D. Kondratieff and J. Schumpeter -, perfectly illustrates the reticence that these theoretical constructions often inspire [3]. Indeed, recourse to the theory of long cycles can by no means be solely mechanistic. In our view, there is no "technological determinism". The dialectic between innovation and technical training / qualification, lying in the center of our theoretical construction: opens the field of possibilities and grants to an important political decision. The transformations do not take place only because of the level reached by certain economic aggregates (savings, wages, consumption ...) but also by an appropriate government intervention.

The controversies over the nature of the period from 1815 to the Second Empire are indicative of the ambiguity of the indicators. Among economists, if Louis Fontvieille equates this period to a phase of stagnation, because of the decline in value of material production[4], Renato Di Ruzza would deny its existence instead[5]. Based on the rate of growth of GDP, some historians consider these years as a depression phase ending in 1835 [6] while others reject their character of a crisis[7].

Finally, the continued rise in prices in the 1970-1980 contradicts commonly accepted determinism between falling prices and conditions. These physical indicators and price introduce other ways: the interest focuses on the look upwards or downwards. However, the graphical representation of statistical series is simplistic thinking, impoverishes and even betrays impoverishes theory building: it opposes the rich meaningfulness along the movement, even ignores the nature of social relations.

The shape of the curves gives premium on determinants of higher and lower positions, and rollovers. The primacy of the graphical representation encourages a standard reading in the beginning cycle, that fact regardless of the timing and theoretical explanations accepted by the so-called period of growth which is followed by a period characterized by economic difficulties. It is a pure convention since any economic cycle has neither beginning nor end but only hyphenation deliberately chosen by the researcher. This convention implicitly generates a hierarchy: the premium growth phase on the phase of depression.

It may be that it should include the famous Kuznets sentence describing the theory of Kondratieff. Probably could be established about Kondratieff and Kuznets the idea that it is fertile enough symmetrical systems of thought: one vis-à-vis the other in their efforts towards truth. The thought of Kondratieff is interesting to delve into the mind of the system, taken on the overall, one thing that out of the great thinkers of the economy, only perhaps Karl Marx and Alfred Marshall, had risked undertaking it... But Western economic thought of the 20th century which has chosen to link Kuznets, refused to get there. It deals willingly System, seeking to control it. But can you control an object - in this very complex case - if one ignores its nature and does above all not want to know it? Among Russian thinkers, has developed this idea ... the System; in the West's ability to guess and formalize the "spark" that should force the system – when it reached its mature phase, even phase of turbulence – to change, reform institutionally ... Between the two schools, Viennese thinkers in the Austrian School of cycles – so including Schumpeter and Steindl ... - show that finance creates itself not only of wealth - a fact generally admitted - but also its own circuits which may be far more institutionalized than they are. But the position of Kuznets, advocating the creation of wealth, is to adopt an attitude of caution about the Kondratieff theory, perhaps because you can learn easily - at the time: in 1930 – the institutional implications.

The long cycle is represented by all authors as the succession of a phase of prosperity and another phase, called depression. Opinions differ about the origin of these phases in their essence, but this sequencing prosperity / depression seems obvious, be natural. However, in our view it is mere convention. This sequence appears to us to be sometimes a problem.

Far from describing an objective reality, the long cycle is, in our view, a metaphorical representation of a theoretical process[8]. The interest pushes the researcher to design this metaphorical representation so that it is the best tool for interpreting reality. Also, since the long cycle theorizes a transformation of the system and not merely copied on a wider basis, it can only be an open cycle, in our view it would look rather like a spiral. This feature of the economic cycle is implicit in most work on long cycles. But as it is usually organized on the metaphor of the phases "Prosperity", and "Depression: it will tend to give credence to the idea of a possible reversal of a pivot point, a balance, yet it is not so: the process is dynamic!

From a logical point of view, there cannot be a rapid growth phase of production, and expansion of the industrial capitalist mode of production, without a number of preconditions has been implemented, and also without an initial impulse has arisen. Insofar as we consider the phase of slow growth as a privileged time for processing system, we propose a sequence that gives the second row to the period of "Prosperity", which phase we prefer to call "Extension." But our position is not limited to this phase inversion. By organizing the mode of alternating Up/Down, the usual representation of the long cycle emphasizes the quantitative aspects in economic realities; it refers to the rate of GDP growth, the movement of prices or the profit rate.

We propose the following sequence: Phase One: “Transformation” Phase Two "Extension". During the Extension, the system tends to "good" function, meaning that production and distribution operations are generating high profits, and major productions do not encounter major problems in markets. The Extension also refers to the system's ability to expand into space, integrating new territories, new populations in the sphere of production and consumption. Gradually, this expansionist dynamic runs out because of a declining productivity of capital, which stands itself as a result of less effort concentrated on an innovative movement. So should open a new cycle, where the capital-labor ratio will change as a result of innovations, new training needs will develop throughout the phase transformation.

In describing this theoretical *cycle of economic transformation,* we focus on the role of processing, as opposed to short “classic” cycles that definitely appear as operating cycles of the economy.

The transition from the theoretical to the historical analysis is obviously very sensitive, and likely to raise many controversies.

However, the opening phases of institutional "depression" can be addressed in dealing jointly with the theory of the Kuznets and Kondratieff, if we integrate the current knowledge. “Spiraling transformation”, "institutional transformation" (Dupriez), "genetic sequencing" (Modelski), are phenomena easier to understand if we talk about the state of the core of the global system that owns a hegemonic power. This was done for the case of the United States by B. Berry. For him the U.S. economy is characterized by Kondratieff cycles that each includes three Kuznets which correspond to the types of institutional changes, that is to say, different modes of regulation.

Berry provides a novel approach, but still restricted to the United States of America. We choose to report it because the country, caught between a purely national approach and a global approach, it expresses in a different matter for the same periods, was always plenty to argue about global regulation. And any attempt at explanation which would cover the regulation brought by the European Union, must consider the experience brought about coming from the other side of the Atlantic. The approach of B. Berry is located halfway between economics and politics. In other words, there are monetary long cycles kindly noticeable[9].

The Kondratieff cycle is described as comprising three Kuznets cycles Kuznets, which mark three different ways to manage money. Thus since 1800 the United States, it would be passed through five cycles of Kondratieff, consisting of fourteen Kuznets cycles. Each time, the process repeats. It starts with a policy directed by conservatives, where the economy is riding a technological revolution, but where inequalities are widening. Monetarily, the policy is clearly deflationary. America finds her traditional values and does not mind too much from the outside, where "popular titles" to describe these “happy” years: the "era of good feelings" or the famous “Roaring Twenties”. Because of inequality, we go to another Kuznets cycle where the economy has a little less important aspect, but where politics is back in the saddle in its ability to manage a balance always challenged by progress. So it is the era of a more intense political competition. It is also a time when the United States must confront the challenging wars on their margins: Mexican War (1836-1848), Spanish-American War (1884-1896), Second World War and Korean War. This also is the era of political reform: President Washington with the Bill of Rights, “Jacksonian Democracy”, New Deal, President Clinton. Monetary policy stands normal. Finally, the third Kuznets is a period of growth but also inflation. If politics in the second Kuznets was centrist, the time now is up for an American version of the “revolutionary” mind: the "messianism." Indeed, the economic and social spheres appear restructured in a redefinition of the role of the nation's borders and frontier: whether physical or symbolic, they come extended. This was the case in Jefferson (Louisiana acquired) at the time of "manifest destiny" (continental expansion), that of "white man's burden" (the U.S. is taking an increasing role in the affairs of world). And yet, the post sixty-eight years of "global search", and "Great Society", and the speeches of G. W. Bush: all contribute to that insistence of American dream. But often, individuals also would "tend to confuse the growth and inflation". Out of the three “serial” Kuznets, this is the most likely of the three to be inflationary, inside or even outside the United States through the mechanism of the "world currency". But it is also the era of wars that call into question the more the United States in their hearts: War of 1812, Civil War, and First World War when the United States feel obliged to participate because they want to defend Europe because of a common model of civilization. This war affects them "more than their margins", unlike the ancient feud between English culture and Hispanic one. Finally: the Cold War. In the latter Kuznets, it is no longer the technological / economic direction, or the institutional / policy dimension, which is put forward… rather is it the ambiguous power of money. The latter, by its excessive expansion, said the Americans that it is going too far, he must return to traditional values. We will return to Keynesianism or monetarism. In the first Kuznets, one is a bit oriented towards the past, in the second to the present, in the third to the future. Money, as a link between present and future, is increasingly produced, and this sign of expectations that go too far and forced to return to the start. The monetary dimension, ambiguous and underlying the history of development: is pervasive in the presentation of B. Berry all through long. The building of monetary forms that are more complete: passes on to her by a cyclical and reflexive recovery.

We can see here that we would fall relatively somewhere in the middle between one mode of monetary operating, and policy regime of operation of the difficulty ... It happens when we are coming rapidly from the concept of wealth creation to go in the search for balance. Kuznets’s GNP is first and foremost a tool for measuring the production, thus the created wealth; this in itself does not presuppose the balance of institutional factors in particular: we believe they belong to another order.

2. The matrix phase of long-cycle: from concept to historical identification.

The problem of the genesis of the cyclical process requires deepening conceptual meaning and providing factual proofs. Why would there exist from the start of the cycles of equivalent periods, duration, amplitude? In the present state of our thinking, we would be looking instead for a phase of genesis and construction of the cyclic process. In the same vein, the cyclic process is not from eternity, so we raise the question of its outcome beyond it. Genesis phase might be to Great Britain from 1740 to 1780 and for France from 1770 to 1815.

The adjustment of Kondratieff cycles in three Kuznets cycles may be postulated easier for a dominant economy - indeed, hegemonic - such as that of the United States ... in which their power can provide the means for greater flexibility in their movements ... perhaps the semi periphery as Europe and Latin America will not have that much ease of movement, and should rather be content with two phases, a growth advantage to track the movement of the United States, the other of uncertainties requiring multiple back to reflexively reconstitute its forces.

For many authors, the long cycles of industrial capitalism begin in the 1780s. Van Gendered was from 1913, the first author to provide a periodization of long cycles in the light of the work of Stanley Jevons relative to price movements (1884): his first cycle began in 1790.According to the preferred series, the first cycle of N. D.

Kondratieff would have started either in the late 1780s or early in the following decade[10]. For his part, Joseph Schumpeter holds a single date: 1787. This periodization is confirmed by several economists in the 1970s[11]: 1785-90 by Gerhard Mensch, 1782 at Van Duijn, late eighteenth century to Ernest Mandel and the early 1790s by Louis Fontvieille[12]. Conversely, in the periodization of the long movements, several authors integrate the circumstances of each country. For American economists defenders of the social structures of accumulation, within the United States would begin the long movements in the 1840s[13]. Finally, for taking the neo Schumpeterian, Alfred Kleinknecht, the long-term movements begin at different dates in different countries[14].

Table 1
Periodization of long cycles long as most authors

	Prosperity Phase	Depression Phase
First Long Cycle	1770-1790 to 1810-1817	1810-1817 to 1844-1851
2nd Long Cycle	1844-1851 to 1890-1896	1890-1896 to 1890-1896
3rd Long Cycle	1890-1896 to 1914-1920	1914-1920 to 1945
4th Long Cycle	1945 to 1968-1970	1968-1970 to?

In our view, far from win every political space and as an intangible economy, the movement along the industry capitalism is historically constructed by inserting into the capitalist sphere areas previously dominated by precapitalist agro-craft typed social relations. With Immanuel Wallerstein, we see in the constant widening of the geographical base of capitalism a response to the process of proletarianization and urbanization, forcing to increase wage levels, and reduce the rate of profit[15]. We developed how this spatial dimension has been integrated by different theoretical schools referring to an analysis in terms of cycles or waves, and especially how the long term movement is inherently related to the spatial differentiation of the economy[16]. This is also related to that is often at regional level that can be constructed skills ... But the qualification system will often be – *ceteris paribus* - the system integrator of choice as a last resort.

Spurred on components of a political, economic and social type, the industrial revolution shook Britain to 1740 and, therefore, its economy recorded before the other, the first long fluctuations. This anticipation is reflected statistically. Out of four extracted series, which N. D. Kondratieff was used for dating his long cycle, three concern the country[17]. French industrialization, later and less massive, is presented more in the form of poles of industrialization rather than as a true industrial revolution. Before 1815, the capitalist mode of production has not reached a stage in France ripe for the emergence of long-term movements.

Therefore, it seems essential to improve the explanation underlying long-cycle by bringing an analysis of the construction process supporting long-term movements. However, this construction does not simultaneously happen in all economic areas: for is linked to the level of development of productive forces. We call those years in which this process is implemented "Matrix Phase". Characterized by the lifting of many "locks", "blocks", of some kinds - institutional, political, technological -, the matrix phase is the depletion of pre-capitalist forms and the emergence of industrial capitalist production relations, holders of long-term economic movements in this sense. It is both transition and rebuilding phase. The sequencing of the internal matrix phase, constitutively cycles and not just prior to them, does not yet correspond to that of a long cycle. The structural role of this phase would require theoretical and methodological developments[18]. Specifically, during the second half of the eighteenth century, certain trends in the capital – i.e. physical, financial, human and institutional -, have created the conditions for transition to industrial capitalism.

Historically, the first cycle of economic transformation could take place, for Great Britain, the country where the Industrial Revolution was born, roughly between 1750-1815: with a first Phase Transformation from 1750 to 1780, and a Phase Extension from 1780 to 1815. In contrast, the period 1770-1815 appears to France, as a period of genesis of industrial capitalism. The first cycle of economic transformation, therefore, would embrace the period from 1815 to 1870. A third round would cover the period 1870-1914, a fourth we estimate from 1918 to 1973. Since 1973, opened a new phase of transformation that has not yet generated, it seems, an expansion phase in Europe, but probably in the Asian region.

Table 2
Test for determining the processing cycles for Great Britain

Matrix Phase	1720-1750	
	Transformation Phase	Extension Phase
1st Cycle of transformation	1750 to 1780 (?)	1780 to 1815 (?)
2nd cycle of transformation	1815 to 1848	1849 to 1873
3rd cycle of transformation	1874 to 1896	1897 to 1919
4th round of transformation	1920 to 1947	1948 to 1973
5th processing cycle	1974 to 2000 (?)	2000 to?

The problem of dating the turning point in the fourth cycle of processing remains. In the 1980s, many authors, anticipating a little too mechanistic a period of 25 to 30 years in the phase of slow growth, predicted to a reversal in the 1990s. Then, this deadline for turnaround was extended: in 1998, Luigi Scandella sees it "around the turn of the century"[19]. More recently, Eric Bosserelle has attempted to provide a survey of different doctrines in this area[20]. This indecision on the identification of both constituent phases of long-cycle returns, in our view: a problematic. That problematic is the changing nature of the long standing trend of the capitalist economy.

Table 3
Test for determining the processing cycles for France

Matrix Phase	1780-1815	
	Phase transformation	Phase extension
First transformation cycle	1815 to 1848	1849 to 1873
2nd cycle of transformation	1874 to 1896	1897 to 1919
3nd cycle of transformation	1920 to 1947	1948 to 1973
4th cycle of transformation	1974 to 2000 (?)	2000 to?

This implies that these difficulties of phasing, and therefore designing and developing (technical, human, financial) qualification systems ... make it difficult to create the proper integration framework. Even in Western Europe where the existence of Kondratieff cycles seems confirmed by many authors since 1770 (by the authors against some doubts that such cycles still exist in our time), this problem leads the authors to Kuznets happy to adopt a simple principle of caution about the open question of the existence of Kondratieff cycles. Often, the economy was struggling to feed the people. GNP measurement seemed very well suited to this objective. All other measures than the growth of physical production, as a measure of the accumulation of an ideal good (happiness) or between the physical measurement and the extent ideational extent which would be swaying concepts that can accurately say an economy based on "the manipulation of symbols": such attempts would require deep theoretical overhaul investigating our "transformation phase" and setting assumptions for the relationship between Kuznets and Kondratieff cycles.

3. Flipping or exceeded?

In order to estimate that the current phase of the long cycle is drastically different from what as were previous depressive phases, we would have to deepen some questions about the nature of reversibility, the need of flip and its predictability, or the character of the transformations preparing the reversal of longstanding trend.

How can we formalize relationship between turning and overtaking? Is there any antinomy or complementarity between these two interpretations and the long-term dynamics of both economic and social forecasted events? When L. Fontvieille evokes exceeding the depressive phases, it means it does not see just back from a Phase A - type Kondratieff -, i. e. the emergence of a phase of growth after a long-term phase of slow growth[21]. This term means that it sees a pattern deep again, breaking the laws of economic transformation that he had previously highlighted. This raises the problem of tools for analysis, and scope for methodology.

If we think in terms of price or wage, or even quantity, we will not address the same issue from the point of turning down. Can this be the occasion of the release of a phase of depression towards resumption of growth, and thanks to a power relay the initial impetus from a new leading country - or sector -, for all its business partners? And what about the link between the economy and the impact that demographics will have on it? Point reversal would be well, perhaps an idea that could have been ... assessed whether there or not long after the fact! In terms of social relations, which would seem to be a price for an officer would be experienced as a quantity by another officer ... Hence the time required to formalize this phenomenon strongly glimpse!

Does the theory of Kuznets avoid it having to ask such questions? Two main theories of Kuznets put us on the trail of a process in two stages, which nowadays have an important new, given the current highly complex crisis!

For Kuznets, inequalities know a U curve. In the early developmental period, they grow quickly ... because entrepreneurs playing a pioneering role are likely to get rich much faster than the rest of the population. In the second period, when society became more structured ... and thus better managed, it may well pay more in wages and benefits associated with social protection mechanisms that must accompany them. Income gaps between all the agents are reduced through the increase of the mass of employees. Today, many countries emerging out of the third world, show more dynamism - both economic and demographic - than industrialized countries that prevailed to retain their technological edge. We can then say the problem should now be addressed at a global level, worldwide. Is there U-shaped graph showing inequality acting on the world scale? If yes, how is it possible to manage it? Kuznets theory shows the path of hope - the increase of wealth -… and then we can hope that these resources would be better administered, managed, shared ... but how this can be accomplished in the very long term or in the global system - become very complex and currently subject to a series of serious crises - financial, food, energy, geopolitics, social ... Does it not seem to be the sole creation of infrastructure regardless of social, political and institutional envelops which will preside? We can precise economic justice, sustainable development in order to prevent conflicts, and of course role of institutions so as to be able to smoothly manage them.

A second approach developed by Kuznets seems rather give us a direction that will take the phenomenon of wealth creation. But this does not seem to offer all but technical alternatives so as to achieve avert risks that could accompany the phenomenon. Unlike Keynes, Kuznets assumes that when a country's GNP increases, the propensity to consume of workers will increase as long as earnings, and even more! This leads us to believe that the increase in the propensity to consume more than income growth, accompanied by a growth, and too inequalities that are likely to occur with the phase of growth of income… would in times of crisis in the center of the world economy (debt, closer crises both to the center of the global system, and in time… as since 2007), cannot find solutions without offering a new administration system. But, given the unconscious factors at play (if the lower turning point can be easily calculated, assuming that one has even been able to determine if it was a problem of price or quantity ... the Like "Loch Ness Monster of economic theory" proposed by C. Diebolt [22] appear to be sound sometimes!), is not it necessary to focus attention on finding the causes of the phenomenon at the start?

The reference to passing integrates structural transformations, implies that the future can only be intrinsically different from the present, that the next phase of growth is not comparable to previous long cycles. Finally, the analysis in terms of authorized up-passing does not imply a resumption of sustainable growth rate.

The concept of reversal is itself ambiguous. Is it the restoration of growth rates comparable to those of years 1950-1970? Is it the restoration of a set of equilibrium affecting the labor market, financial markets, or the efficiency of the production system? How can we interpret the past returns to long phases of prosperity as reversals or as overruns?

Our recent studies lead us to affirm that the phase of the interwar period is already carrying the fundamental transformation that will express them clearly in the long cycle of post-Second World War. During the years 1920-1940, the overt forms of human development (increased efforts in education, social protection) coexist with forms of "dispossession" (lower qualifications under the influence of Taylorism)[23]. The nature of the next phase of prosperity is deeply marked by the dialectical relationship and its outcome.

*   *
*

We are aware of the rather exploratory reflections. The hypothesis of a generator matrix phase of a long standing trend analysis in the framework of the theory of long cycles remains to substantiate. It should initially be identified for each industrialized country this matrix phase. In a second step would be the articulation of the various movements for national long cycles form the international long cycle. Beyond this research program in itself significant because it refers to problems of statistical sources, methodology and econometric treatment, there is the prospective nature of this theoretical approach. What gives us tools of historical analysis to build the theoretical tools for tomorrow?

If the Kuznets cycles show the creation of wealth, and those Kondratieff the best way humanly imaginable to administer them, how to think the borders between sectors, countries ... to strengthen coordination[24]. An approach to semi-monetary long cycles [25] or money could afford to rethink the two series of cycles – Kuznets and Kondratieff – not necessarily always oppose them.

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A dynamic monetary multi-sectoral model of production

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Though Keynes entitled his magnum opus *The general theory of employment, interest **and money*** (Keynes 1936), he acknowledged that money did not feature heavily in his technical analysis, and that he saw a substantial continuity between monetary analysis and the Marshallian model of supply and demand:

> whilst it is found that money enters into the economic scheme in an essential and peculiar manner, *technical monetary detail falls into the background*. A monetary economy, we shall find, is essentially one in which changing views about the future are capable of influencing the quantity of employment and not merely its direction. But *our method* of analyzing the economic behavior of the present under the influence of changing ideas about the future is one which *depends on the interaction of supply and demand, and is in this way linked up with our fundamental theory of value*. We are thus led to a more general theory, which includes the classical theory with which we are familiar, as a special case. (Keynes 1936, p. xxii)

After Keynes, macroeconomics fragmented around the importance of both uncertainty—implicit in the statement above that “changing views about the future are capable of influencing the quantity of employment”, but strongly explicit elsewhere (Keynes 1936; Keynes 1937)—and money. Both concepts disappeared from mainstream macroeconomic analysis, to be replaced initially by IS-LM analysis—in which an exogenously determined money played a minor role, but uncertainty disappeared (Hicks 1937; Minsky 1975; Hicks 1981)—and ultimately by Real Business Cycle modeling (Kydland and Prescott 1982), in which “rational expectations” neutered uncertainty completely (Lucas 1972), and money was entirely absent.

On the periphery of the profession, a rump of self-described “Post Keynesians” clung to the position that both money and uncertainty were essential aspects of macroeconomics. Going far further than Keynes himself, this rump incorporated Schumpeter's arguments on the essential role of *endogenously created* money in financing growth (Schumpeter 1927; Schumpeter 1934; Moore 1979) and Fisher's debt-deflation perspective (Fisher 1933) to develop the “Financial Instability Hypothesis” (Minsky 1975; Minsky 1977; Minsky 1982; Minsky 1993), while it also rejected Marshallian analysis—following on this issue Sraffa (Sraffa 1926; Robertson, Sraffa et al. 1930) rather than Keynes. Others added insights from theoretical developments like complexity theory, which post-dated Keynes, to argue that the macro-economy was inherently cyclical (Goodwin 1967; Goodwin 1986; Goodwin 1990).

This rump was ignored by the mainstream, which over time expunged not only uncertainty and money but even Keynes himself from macroeconomics (despite the fact that the dominant segment of the mainstream described its work as “New Keynesian”). Mainstream macroeconomics became applied neoclassical microeconomics, as Oliver Blanchard, founding editor of the journal *AER: Macro*, outlined in his survey of macroeconomics in 2009.

> The most visible outcomes of this new approach are the dynamic stochastic general equilibrium (DSGE) models. They are models derived from micro foundations—that is, utility maximization by consumers-workers; value maximization by firms; rational expectations; and a full specification of imperfections, from nominal rigidities to some of the imperfections discussed above—and typically estimated by Bayesian methods. (Blanchard 2009, p. 223)

As the end of the first decade of the 21st century approached, the mainstream was triumphal. At the policy level, it took the credit for the decline in economic volatility since the early 1980s:

> As it turned out, the low-inflation era of the past two decades has seen not only significant improvements in economic growth and productivity but also a marked reduction in economic volatility, both in the United States and abroad, a phenomenon that has been dubbed "the Great Moderation." Recessions have become less frequent and milder, and quarter-to-quarter volatility in output and employment has declined significantly as well. The sources of the Great Moderation remain somewhat controversial, but as I have argued elsewhere, *there is evidence for the view that improved control of inflation has contributed in important measure to this welcome change in the economy*. (Bernanke 2004; emphasis added)

At the level of pure theory, a similar contentment prevailed. Though he acknowledged one notable dissenter (Solow 2008), Blanchard's survey was unequivocal:

> The state of macro is good. (Blanchard 2009, p. 210)

Few more poorly timed statements have ever been made by prominent economists. This paper was first published online as a working paper in August 2008 (Blanchard 2008)—one year after the event that is now regarded as the beginning of the financial crisis (New York Times 2007) and 8 months after the NBER's date for the commencement of the Great Recession (NBER 2011). Its publication as a journal paper in May 2009 preceded the NBER's date for the end of this recession by one month (a decision that I expect will prove premature).

Blanchard was forced into recanting his optimism less than a year later (Blanchard, Dell'Ariccia et al. 2010). But while he criticized macroeconomic policy prior to the crisis, he remained a believer in neoclassical theory itself:

> Identifying the flaws of existing policy is (relatively) easy. Defining a new macroeconomic policy framework is much harder... *It is important to start by stating the obvious, namely, that the baby should not be thrown out with the bathwater. Most of the elements of the pre-crisis consensus, including the major conclusions from macroeconomic theory, still hold*. Among them, the ultimate targets remain output and inflation stability. The natural rate hypothesis holds, at least to a good enough approximation, and policymakers should not design policy on the assumption that there is a long-term trade-off between inflation and unemployment. Stable inflation must remain one of the major goals of monetary policy. Fiscal sustainability is of the essence, not only for the long term but also in

> affecting expectations in the short term. (Blanchard, Dell'Ariccia et al. 2010, p. 207; emphasis added)

Blanchard's unwillingness to countenance the possibility that the Great Recession may be a Kuhnian critical anomaly for neoclassical macroeconomics (Bezemer 2011) is representative of this school of thought:

> Indeed, the extreme severity of this great recession makes it tempting to argue that new theories are required to fully explain it... But … *it would be premature to abandon more familiar models just yet*. (Ireland 2011, p. 1; emphasis added)

As a representative of the Post Keynesian and complexity theory rump, and one of the handful of economists to foresee the Great Recession (Keen 1995; Keen 2000; Keen 2006; Keen 2007; Keen 2007; Bezemer 2009; Bezemer 2011), I could not disagree more with Blanchard and his colleagues. Though neoclassical economists believe they are being methodologically sound in applying microeconomic concepts to model the macro-economy, deep research long ago established that this is a fallacy. The Sonnenschein-Mantel-Debreu conditions alone establish that *even the microeconomics of demand in a single market cannot be derived by extrapolation from the behavior of a single utility-maximizing agent*, let alone the macroeconomics of the whole economy. As Solow himself noted in the paper cited in Blanchard (2009, p. 210):

> Suppose you wanted to defend the use of the Ramsey model as the basis for a descriptive macroeconomics. What could you say? ...
>
> You could claim that … there is no other tractable way to meet the claims of economic theory. *I think this claim is a delusion.* We know from the Sonnenschein-Mantel-Debreu theorems that the only universal empirical aggregative implications of general equilibrium theory are that excess demand functions should be continuous and homogeneous of degree zero in prices, and should satisfy Walras' Law. Anyone is free to impose further restrictions on a macro model, but they have to be justified for their own sweet sake, not as being required by the principles of economic theory. Many varieties of macro models can be constructed that satisfy those basic requirements without imposing anything as extreme and prejudicial as a representative agent in a favorable environment. (Solow 2008, p. 244; emphasis added; see also Solow 2001 and 2003)

I cover the myriad flaws in neoclassical macroeconomics in much more detail in Keen 2011b; suffice it to say here that, far from it being unwise to “throw the baby out with the bathwater”, *neoclassical macroeconomics should never have been conceived in the first place*. The Great Recession will hopefully prove to be the Biblical economic flood needed to finally sink this superficially appealing but fundamentally flawed vision of how the macro-economy functions.

How do I fault thee? Let me count the ways

The flaws of neoclassical macroeconomics are almost too numerous to enumerate, but the key weaknesses are:

1. Treating a complex monetary market economy as a barter system;
2. Assuming that the macro-economy is either in equilibrium (partial or general, perfect or imperfect), or that it will return to equilibrium rapidly if disturbed;
3. Modeling the entire economy using "applied microeconomics" and ignoring social class, when the Sonnenschein-Mantel-Debreu conditions (Sonnenschein 1972; Sonnenschein 1973; Kirman 1989; Shafer and Sonnenschein 1993) establish that, as Kirman put it:

   > "we may well be forced to theorise in terms of groups who have collectively coherent behaviour. Thus demand and expenditure functions if they are to be set against reality must be defined at some reasonably high level of aggregation. The idea that we should start at the level of the isolated individual is one which we may well have to abandon" (Kirman 1992, p. 138);

4. Obliterating uncertainty from macroeconomic theory with the absurd proposition that a rational individual is someone who can accurately foresee the future—which is what "rational expectations" really means;[1]
5. Persisting with a simplistic "money multiplier" model of money creation when the empirical evidence against this model is overwhelming (Holmes 1969; Moore 1979; Moore 1988; Kydland and Prescott 1990); and
6. Ignoring the pivotal roles of credit and debt in the macro-economy.

All these flaws are absent from the non-neoclassical rump—especially in the work of Minsky. But what the rump lacks, in comparison to the neoclassical mainstream, is a coherent mathematical expression of its model that is widely accepted within that school. In this paper I contribute to the development of such a model (though I appreciate that my model is a long way from being accepted by my peers) using a modeling framework—which I call *Monetary Circuit Theory* (*MCT*)—that, in contrast to the neoclassical litany of sins above:

1. Treats the economy as inherently monetary;
2. Makes no assumptions about the nature of equilibrium and models the economy dynamically;
3. Models behavior at the level of social classes rather than isolated agents;
4. Presumes rational but not prophetic behavior: people in social classes act in what they perceive as their best interests given information available, but do not attempt to forecast the future state of the economy (and they cannot do so in any case, because of the well-known features of complex systems);
5. Models the endogenous creation of money by the banking sector in a pure credit economy (later extensions will incorporate fiat money creation by governments); and

1 Lucas stated as much in the paper in which he introduced "rational expectations" into macroeconomics, by stating that rational expectations was identical to assuming that future expectations were correct: "the hypothesis of adaptive expectations was rejected as a component of the natural rate hypothesis on the grounds that, under some policy [the gap between expected future inflation and actual future inflation] is non-zero. As the impossibility of a non-zero value for Expression 6 is taken as an essential feature of the natural rate theory, *one is led simply to adding the assumption that Expression 6 is zero as an additional axiom*, or to assume that expectations are rational in the sense of Muth." (Lucas 1972, p. 54)

6. Gives credit and debt the pivotal roles in economic theory that the Great Recession has shown they have in the real world.

A framework for monetary macroeconomics

At one level, *MCT* is deceptively simple: all demand in the macroeconomy is treated as originating in bank accounts, where, in accordance with the empirical literature (Holmes 1969; Moore 1979, 1988; Kydland and Prescott 1990), the banking system has the capacity to endogenously create new credit-based money. The development of the framework is described elsewhere (see Keen 2006b, 2008, 2009); here I will simply illustrate *MCT* with the financial flows used in the model of the 19th century "free banking" system in Keen (2010).[2] The core of *MCT* is a tabular layout of the financial relations between the economic entities in the model, where each column represents an aggregate bank account, and each row represents operations on and between those accounts.[3]

Table 1: Sample Financial Flows Godley Table

	Assets		***Liabilities***		***Equity***
Account Name	*Bank Vault*	*Firm Loan*	*Firm Deposit*	*Worker Deposit*	*Bank Equity*
Symbol	$B_V(t)$	$F_L(t)$	$F_D(t)$	$W_D(t)$	$B_E(t)$
Initial conditions	100	0	0	0	0
Lend Money	-A		A		
Record Loan		A			
Compound Debt		B			
Service Debt			-B		B
Record Payment		-B			
Deposit Interest			C		-C
Wages			-D	D	
Deposit Interest				E	-E
Consume			F+G	-F	-G
Repay Loan	H		-H		

[2] The table differs slightly from that in the paper, since the columns have been re-ordered and renamed in accordance with standard accounting practice..

[3] The table is similar to the Social Accounting Matrix approach of Wynne Godley (see Godley & Lavoie 2007a and 2007b), but has several differences that are explained in Keen 2009, pp. 162-167—notably that row operations do not have to sum to zero, and the economy is modeled in continuous rather than discrete time.

Record Repayment		-H			

Using a symbolic algebra program,[4] the placeholders A to H are then replaced by suitable functions:[5]

$$\begin{aligned}
A &\rightarrow \frac{B_V(t)}{\tau_V(t)} \\
B &\rightarrow r_L(t)\cdot F_L(t) \\
C &\rightarrow r_D(t)\cdot F_D(t) \\
D &\rightarrow \frac{F_D(t)}{\tau_F(t)} \\
E &\rightarrow r_D(t)\cdot W_D(t) \\
F &\rightarrow \frac{W_D(t)}{\tau_W(t)} \\
G &\rightarrow \frac{B_E(t)}{\tau_B(t)} \\
H &\rightarrow \frac{F_L(t)}{\tau_L(t)}
\end{aligned} \tag{1.1}$$

The program then automatically derives a set of differential equations for this system, which can be analyzed symbolically or simulated numerically:

$$\begin{aligned}
\frac{d}{dt}B_V(t) &= \frac{F_L(t)}{\tau_L(t)} - \frac{B_V(t)}{\tau_V(t)} \\
\frac{d}{dt}F_L(t) &= \frac{B_V(t)}{\tau_V(t)} - \frac{F_L(t)}{\tau_L(t)} \\
\frac{d}{dt}F_D(t) &= \frac{B_V(t)}{\tau_V(t)} - r_L(t)\cdot F_L(t) + r_D(t)\cdot F_D(t) - \frac{F_D(t)}{\tau_F(t)} + \frac{W_D(t)}{\tau_W(t)} + \frac{B_E(t)}{\tau_B(t)} - \frac{F_L(t)}{\tau_L(t)} \\
\frac{d}{dt}W_D(t) &= \frac{F_D(t)}{\tau_F(t)} + r_D(t)\cdot W_D(t) - \frac{W_D(t)}{\tau_W(t)} \\
\frac{d}{dt}B_E(t) &= r_L(t)\cdot F_L(t) - r_D(t)\cdot F_D(t) - r_D(t)\cdot W_D(t) - \frac{B_E(t)}{\tau_B(t)}
\end{aligned} \tag{1.2}$$

[4] This system has been implemented in the commercial programs Mathcad (www.ptc.com/mathcad), Mathematica (www.wolfram.com) and Matlab (www.matlab.com), and a prototype of a standalone monetary simulation system called QED—for "Quesnay Economic Dynamics"—is freely downloadable from my website http://www.debtdeflation.com/blogs/qed/.

[5] I explain the functions used in the exposition of the multisectoral model below.

This covers the financial side of the economy. The real economy is coupled to this via a price mechanism (and links between the wages flow—which determines employment—and investment, which is not shown in the simple model in Table 1, but which determines the capital stock in a larger model).

The price mechanism is derived analytically in Keen 2010 (pp. 17-18), and corresponds to the extensive empirical literature into how firms actually set prices—which has nothing to do with marginal cost and marginal revenue (see Lee 1998, Blinder et al. 1998, and Keen & Standish 2006 and 2010) but instead represents a markup on the wage costs of production

$$\frac{d}{dt}P = -\frac{1}{\tau_P}\cdot\left(P - \frac{1}{(1-\sigma)}\cdot\frac{W}{a}\right) \qquad (1.3)$$[6]

The real economy itself is modeled using Goodwin's growth cycle (Goodwin 1967; see also Blatt 1983, pp. 204-216), but expressed in absolute values (Employment, Wages, etc.) rather than ratios (rate of employment, wages share of output) as in Goodwin's original model.

Applying the framework: a "corn economy" with a financial crisis

The sample Godley Table shown in Table 1 has to be extended to allow for investment, which as Schumpeter argued is the sound basis on which the credit system endogenously creates new debt-based money (Schumpeter 1934, pp. 95-101).

Table 2: Godley Table for Corn Economy Model

	Assets		***Liabilities***		***Equity***
Account Name	*Bank Vault*	*Firm Loan*	*Firm Deposit*	*Worker Deposit*	*Bank Equity*
Symbol	$B_V(t)$	$F_L(t)$	$F_D(t)$	$W_D(t)$	$B_E(t)$
Lend from Vault	-A		A		
Record Loan		A			
Compound Debt		B			
Service Debt			-C		C
Record Payment		-C			
Debt-financed Investment			D		
Record Investment Loan		D			
Wages			-E	E	
Deposit Interest			F	G	-(F+G)

[6] τ_P is the time constant in price setting, σ is the share of income going to capitalists, and *a* is labor productivity.

Consumption			H+I	-H	-I
Repay Loan	J		-J		
Record Repayment		-J			

This Godley Table results in the following generic system of financial flows:

$$\begin{aligned}
&\frac{d}{dt}B_V(t) = J - A \\
&\frac{d}{dt}F_L(t) = A - J + B - C + D \\
&\frac{d}{dt}F_D(t) = A - J - C + D - E + F + H + I \\
&\frac{d}{dt}W_D(t) = E + G - H \\
&\frac{d}{dt}B_E(t) = C - F - G - I
\end{aligned} \tag{1.4}$$

The substitutions for this table are show in Equation (1.5); the rates of lending, investment and loan repayment (respectively A, D and J in Table 2) are now functions of the rate of profit, and wage payments (E) are now wages times employment.

$$\begin{aligned}
&A \rightarrow \frac{B_V(t)}{\tau_V(\pi_r(t))} \\
&B \rightarrow r_L(t)\cdot F_L(t) \\
&C \rightarrow r_L(t)\cdot F_L(t) \\
&D \rightarrow Inv(\pi_r(t))\cdot P(t)\cdot Y_R(t) \quad \text{Investment function times Output} \\
&E \rightarrow W(t)\cdot\frac{Y_R(t)}{a(t)} \quad \text{Wage rate times employment} \\
&F \rightarrow r_D(t)\cdot F_D(t) \\
&G \rightarrow r_D(t)\cdot W_D(t) \\
&H \rightarrow \frac{W_D(t)}{\tau_W} \\
&I \rightarrow \frac{B_E(t)}{\tau_B} \\
&J \rightarrow \frac{F_L(t)}{\tau_L(\pi_r(t))}
\end{aligned} \tag{1.5}$$

The basic causal cycle in the Goodwin model (to which the financial flows above are attached) is quite simple. Causation flows from left to right in equations (1.6) to (1.14):

- The level of the physical capital stock K_R determines the level of physical output Y_R per year:

$$\frac{K_R(t)}{v} = Y_R(t) \tag{1.6}$$

- Output per year determines employment L:

$$\frac{Y_R(t)}{a(t)} = L(t) \tag{1.7}$$

- The rate of employment $L/N = \lambda$ determines the rate of change of the money wage—thus linking the physical sector to the monetary sector; in keeping with Phillips's original intentions (and in contrast to most macroeconomic models), the wage change function includes a reaction to the rate of change of employment and the level of inflation, as well as a nonlinear reaction to the level of employment:

$$W(t)\cdot\left(P_h\left(\lambda(t)\right)\right) + \omega\cdot\frac{1}{\lambda(t)}\cdot\frac{d}{dt}\lambda(t) + \frac{1}{P(t)}\cdot\frac{d}{dt}P(t) \Rightarrow \frac{d}{dt}W(t) \tag{1.8}$$

- The money wage determines the rate of change of the price level P:

$$-\frac{1}{\tau_P}\cdot\left(P(t) - \frac{1}{(1-\sigma)}\cdot\frac{W(t)}{a(t)}\right) \Rightarrow \frac{d}{dt}P(t) \tag{1.9}$$

- The monetary value of output $P\cdot Y_R$ minus wages $W\cdot L$ determines profit:

$$P(t)\cdot Y_R(t) - W(t)\cdot L(t) - \left(r_L\cdot F_L(t) - r_D F_D(t)\right) = \Pi(t) \tag{1.10}$$

- The rate of profit $\frac{\Pi(t)}{P(t)\cdot K_R(t)} = \pi_r$ determines investment (and hence the amount of new credit money needed should desired investment exceed profit) and investment minus depreciation δ determines the rate of economic growth g:

$$\frac{Inv\left(\pi_r(t)\right)}{v} - \delta = g(t) \tag{1.11}$$

- The integral of investment determines the capital stock:

$$g(t)\cdot K_R(t) \Rightarrow \frac{d}{dt}K_R(t) \tag{1.12}$$

- The rate of change of the employment rate is the rate of growth minus the rates of growth of labor productivity and population:

$$\lambda(t)\cdot\left(g(t) - (\alpha+\beta)\right) \Rightarrow \frac{d}{dt}\lambda(t) \tag{1.13}$$

- Equations for growth in labor productivity and population complete the model:

$$\begin{aligned}\alpha \cdot a(t) &\Rightarrow \frac{d}{dt}a(t)\\ \beta \cdot N(t) &\Rightarrow \frac{d}{dt}N(t)\end{aligned} \tag{1.14}$$

The rates of lending (A), debt-financed investment (D) and loan repayment (J) are modeled as nonlinear functions of the rate of profit, while the Phillips Curve is also a nonlinear function of the level of employment. The basic function used in all cases is a generalized exponential function where the arguments to the function are an (x_c,y_c) coordinate pair, the function's slope at that point *s*, and its minimum *m*:

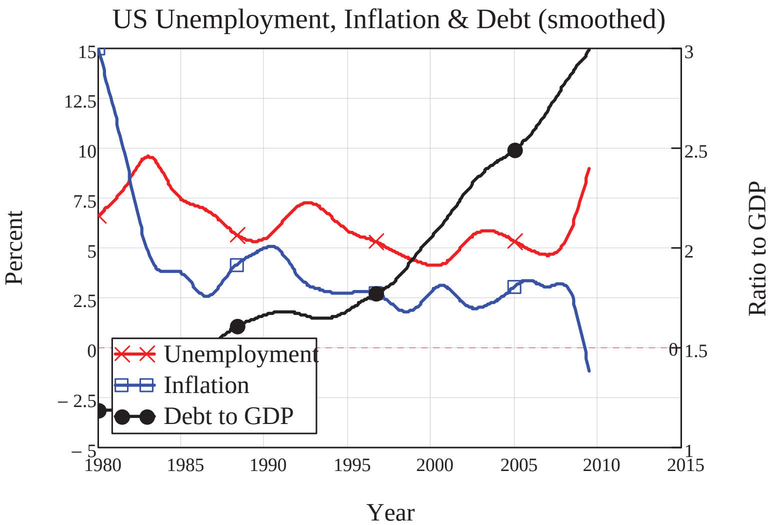
$$genexp(x, x_c, y_c, s, m) = (y_c - m) \cdot e^{\frac{s}{y_c - m}\cdot(x - x_c)} + m \tag{1.15}$$

The complete model is described by a set of ten differential equations:

$$\begin{aligned}
&\frac{d}{dt}B_V(t) = \frac{F_L(t)}{\tau_L(\pi_r(t))} - \frac{B_V(t)}{\tau_V(\pi_r(t))}\\
&\frac{d}{dt}F_L(t) = \frac{B_V(t)}{\tau_V(\pi_r(t))} - \frac{F_L(t)}{\tau_L(\pi_r(t))} + Inv(\pi_r(t)) \cdot P(t) \cdot Y_R(t)\\
&\frac{d}{dt}F_D(t) = \frac{B_V(t)}{\tau_V(\pi_r(t))} - \frac{F_L(t)}{\tau_L(\pi_r(t))} - r_L(t) \cdot F_L(t) + Inv(\pi_r(t)) \cdot P(t) \cdot Y_R(t) - W(t) \cdot \frac{Y_R(t)}{a(t)} + r_D(t) \cdot F_D(t) + \frac{W_D(t)}{\tau_W} + \frac{B_E(t)}{\tau_B}\\
&\frac{d}{dt}W_D(t) = W(t) \cdot \frac{Y_R(t)}{a(t)} + r_D(t) \cdot W_D(t) - \frac{W_D(t)}{\tau_W}\\
&\frac{d}{dt}B_E(t) = r_L(t) \cdot F_L(t) - r_D(t) \cdot F_D(t) - r_D(t) \cdot W_D(t) - \frac{B_E(t)}{\tau_B}\\
&\frac{d}{dt}K_R(t) = g(t) \cdot K_R(t)\\
&\frac{d}{dt}\lambda(t) = \lambda(t) \cdot (g(t) - (\alpha + \beta))\\
&\frac{d}{dt}W(t) = W(t) \cdot (P_h(\lambda(t))) + \omega \cdot \frac{1}{\lambda(t)} \cdot \frac{d}{dt}\lambda(t) + \frac{1}{P(t)} \cdot \frac{d}{dt}P(t)\\
&\frac{d}{dt}a(t) = \alpha \cdot a(t)\\
&\frac{d}{dt}N(t) = \beta \cdot N(t)
\end{aligned} \tag{1.16}$$

Given suitable initial conditions and parameter values, this highly nonlinear monetary model can generate the stylized facts of the last 20 years of macroeconomic data: an apparent "Great Moderation" in employment and inflation—which was actually driven by an exponential growth in private debt—followed by a "Great Recession" in which unemployment explodes, inflation turns to deflation, and the debt level—absent of bankruptcy and government intervention—goes purely exponential as unpaid interest is compounded.

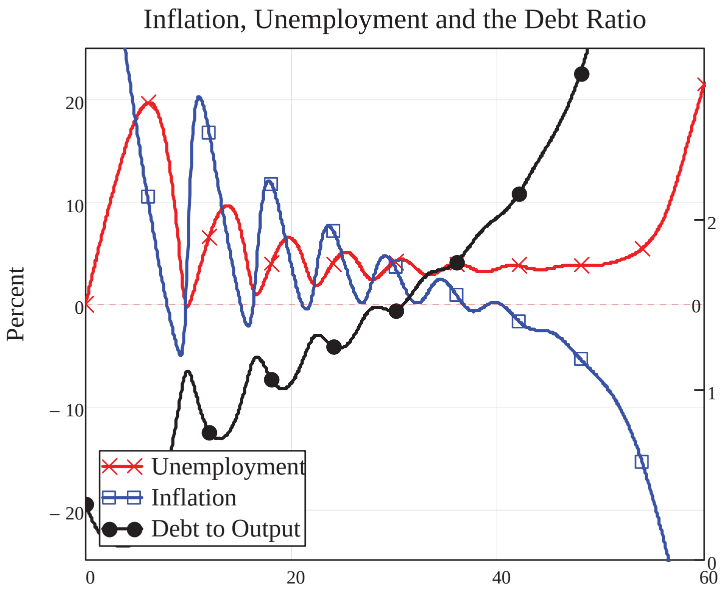
Figure 1: US Data 1980-2008



As a complex systems model, the behavior of this system depends upon its initial conditions as well as upon its inherent dynamics. In Keen 2011 I used a set of initial conditions that resulted in both a Great Moderation and a Great Recession—with no change to the underlying parameters of the system—to indicate that this model fits Minsky's criteria for a successful model of capitalism:

> Can "It"—a Great Depression—happen again? And if "It" can happen, why didn't "It" occur in the years since World War II? These are questions that naturally follow from both the historical record and the comparative success of the past thirty-five years. *To answer these questions it is necessary to have an economic theory which makes great depressions one of the possible states in which our type of capitalist economy can find itself.*(Minsky 1982 , p. 5; emphasis added)

Figure 2: Simulation Results with uncalibrated constant parameter values



This model captures the macroeconomic experience of the last 2 decades far more effectively than any neoclassical model. However, the Holy Grail of economics has always been to model the complex dynamic process by which commodities are produced using other commodities and labor. In the next section I show that a structured extension of this corn economy model—with financial flows determining demand, and production modeled using Goodwin's growth cycle—can generate a coherent dynamic monetary multisectoral model of production.

A dynamic monetary multisectoral model of production

First a strong caveat: this model is very tentative, and many refinements need to be made. However even in its tentative state, it shows that a monetary, dynamic multisectoral model of production can be constructed.

The model reproduces the structure of the preceding corn economy model, extended to multiple commodities in both production (with each sector needing to purchase inputs from other sectors proportional to its desired output level), and consumption. I also address one of the weaknesses of input-output analysis—that purchases within a sector are not explicitly shown—by the simple expedient of splitting each sector in two. There are 4 sectors in this simple "proof of concept" model (notionally Capital Goods, Consumer Goods, Agriculture and Energy).

The Godley Table for this system has 19 system states— Bank Reserve, Bank Equity and Worker Deposit accounts as in the single sectoral model, plus two Deposit and two Loan accounts per sector—and 16 financial operations—debt compounding, debt repaying, money relending and wages payment as in the single sectoral model, plus one intersectoral purchase for production and one for consumption per sector. A stylized representation of these flows is given in Table 3 (the intersectoral flows are only partially indicated).

Table 3: Stylized representation of multiectoral Godley Table

	Assets				**Liabilities**		**Equity**
Account	Bank Reserve	Sector 1 Loan	Sector 2 Loan	Sector 1 Deposit	Sector 2 Deposit	Worker Deposit	Bank Equity
Symbol	$B_R(t)$	$F_{L1}(t)$	$F_{L1}(t)$	$F_{D1}(t)$	$F_{D2}(t)$	$W_D(t)$	$B_E(t)$
Compound Debt		A_1	A_2				
Deposit Interest				B_1	B_2		
Wages				$-C_1$	$-C_2$	C_1+C_2	
Worker Interest						-D	-D
Investment K				E	-E		
Intersectoral C				-F	F		
Intersectoral A				-G	G		
Intersectoral E				-H	H		
Consumption K				I	-I		
Consumption C				-J	J		
Consumption A				-K	K		
Consumption E				-L	L		
Pay Interest				-M			M
Repay Loans	N			-N			
Recycle Reserves	-O	O		O			
New Money		P		P			

An extract from the actual Godley Table for this system (as implemented in Mathcad) is shown in Figure 3.

Figure 3: 7 of the 19 columns in the multisectoral Godley Table

$T_{.1} \rightarrow$

"Type"	0	1	−1	−1	−1	−1	0
"Name"	"BR"	"LK1"	"DK1"	"DK2"	"DC1"	"WD"	"BE"
"Symbol"	$B_{.R}(t)$	$F_{.LK1}(t)$	$F_{.DK1}(t)$	$F_{.DK2}(t)$	$F_{.DC1}(t)$	$W_{.D}(t)$	$B_{.E}(t)$
"Compound Interest"	0	A1	0	0	0	0	0
"Deposit Interest"	0	0	B1	B2	B3	0	−B1 − B2 − B3 − B4 − B5 − B6 − B7 − B8
"Wages"	0	0	−C1	−C2	−C3	C1 + C2 + C3 + C4 + C5 + C6 + C7 + C8	0
"Worker Interest"	0	0	0	0	0	D1	−D1
"Investment Demand for K"	0	0	E2 − E1 + E3 + E5 + E7	E1 − E2 + E4 + E6 + E8	−E3	0	0
"Intersectoral Demand for C"	0	0	−F1	−F2	F1 − F3 + F4 + F5 + F7	0	0
"Intersectoral Demand for A"	0	0	−G1	−G2	−G3	0	0
"Intersectoral Demand for E"	0	0	−H1	−H2	−H3	0	0
"Consumption K"	0	0	$I2 - I1 + I3 + I5 + I7 + \frac{I9}{2} + \frac{I10}{2}$	$I1 - I2 + I4 + I6 + I8 + \frac{I9}{2} + \frac{I10}{2}$	−I3	−I9	−I10
"Consumption C"	0	0	−J1	−J2	$J1 - J3 + J4 + J5 + J7 + \frac{J9}{2} + \frac{J10}{2}$	−J9	−J10
"Consumption A"	0	0	−K1	−K2	−K3	−K9	−K10
"Consumption E"	0	0	−L1	−L2	−L3	−L9	−L10
"Pay Interest"	0	−M1	−M1	−M2	−M3	0	M1 + M2 + M3 + M4 + M5 + M6 + M7 + M8
"Repay Loans"	N1 + N2 + N3 + N4 + N5 + N6 + N7 + N8	−N1	−N1	−N2	−N3	0	0
"Recycle Reserves"	−O1 − O2 − O3 − O4 − O5 − O6 − O7 − O8	O1	O1	O2	O3	0	0
"New Money"	0	P1	P1	P2	P3	0	0

The rate of profit is now net of intersectoral purchases for each sector, and of course there is a different rate of profit in each sector. Intersectoral purchases of inputs differ for each sector, and are proportional to the labor input needed to produce the required output in each sector—signified by $\sigma_{1,2}$ where the first subscript represents the sector purchasing the inputs and the second the sector from which the inputs are purchased. Equation (1.17) shows the rate of profit formulae for the capital goods and consumer good sectors:

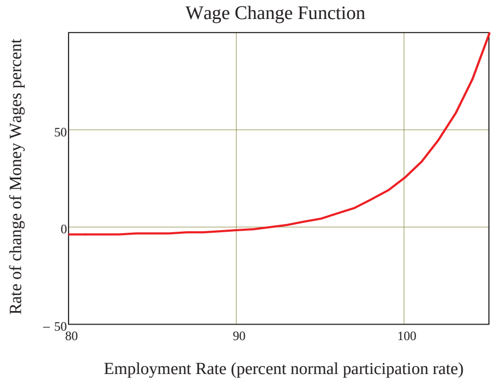
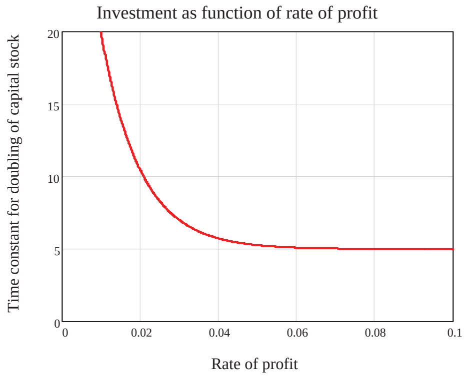
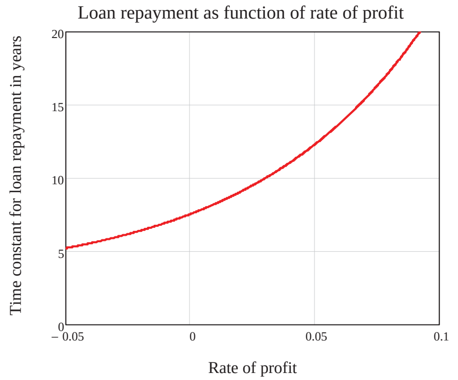
$$\begin{aligned}
&\frac{\Pi_K(t) = P_K(t)\cdot Q_K(t) - W(t)\cdot L_K(t) - \sum_{S\neq K}^{n}\left(\sigma_{KS}\cdot W(t)\cdot L_S(t)\right) - \left(r_L\cdot K_L(t) - r_D K_D(t)\right)}{P_K(t)\cdot K_K(t)}\\
&\frac{\Pi_C(t) = P_C(t)\cdot Q_C(t) - W(t)\cdot L_C(t) - \sum_{S\neq C}^{n}\left(\sigma_{CS}\cdot W(t)\cdot L_S(t)\right) - \left(r_L\cdot C_L(t) - r_D C_D(t)\right)}{P_K(t)\cdot K_C(t)}
\end{aligned} \tag{1.17}$$

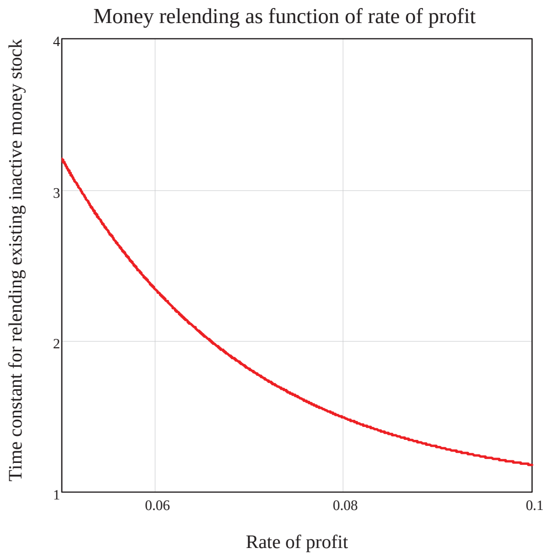
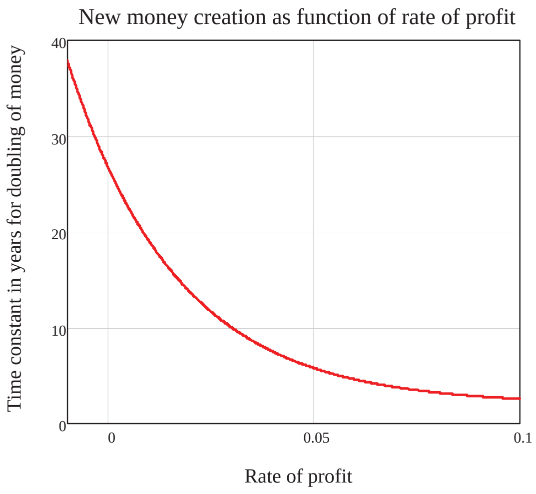
As with the single sectoral model, behavior in five crucial areas is modeled as a nonlinear response to a relevant variable:[7]

- The rate of change of money wages as a function of the rate of employment;
- The time constant in investment decisions τ_{PR} as a function of the rate of profit;
- The time constant in loan repayment as a function of the rate of profit;
- The time constant in money relending as a function of the rate of profit;
- The time constant in new money creation as a function of the rate of profit;

[7] I have had this described to me as "an assumption of irrational behavior" by neoclassical economists who are accustomed to the assumption of rational expectations. I find this accusation bizarre, since nothing could be more irrational than to assume that agents in a complex system can accurately predict its future course—and yet this is precisely what "rational expectations" entails. In my models, people react rationally to the information they believe is relevant and that they have at hand, but they cannot and do not predict the future course of the economy—or if they try to, their predictions will be wrong. Assuming fallibility is not the same as assuming irrationality!

Table 4: Parameters for Behavioral Functions

	$genexp(x, 92\%, 0\%, 1, -4\%)$
	$genexp(x, 3\%, 7\,years, -200, 5\,years)$
	$genexp(x, 3\%, 10\,years, 100, 3\,years)$

	$genexp(x, 3\%, 7\,years, -300, 1\,year)$
	$genexp(x, 3\%, 10\,years, -300, 2\,years)$

With the purchases of intermediate inputs taken care of in the monetary demand component of the model, production in each sector is modeled as lagged response to installed capital, and employment is a lagged response to output. The functions for the consumer goods sector, which are representative of those for the other sectors, are shown in Equation (1.18):

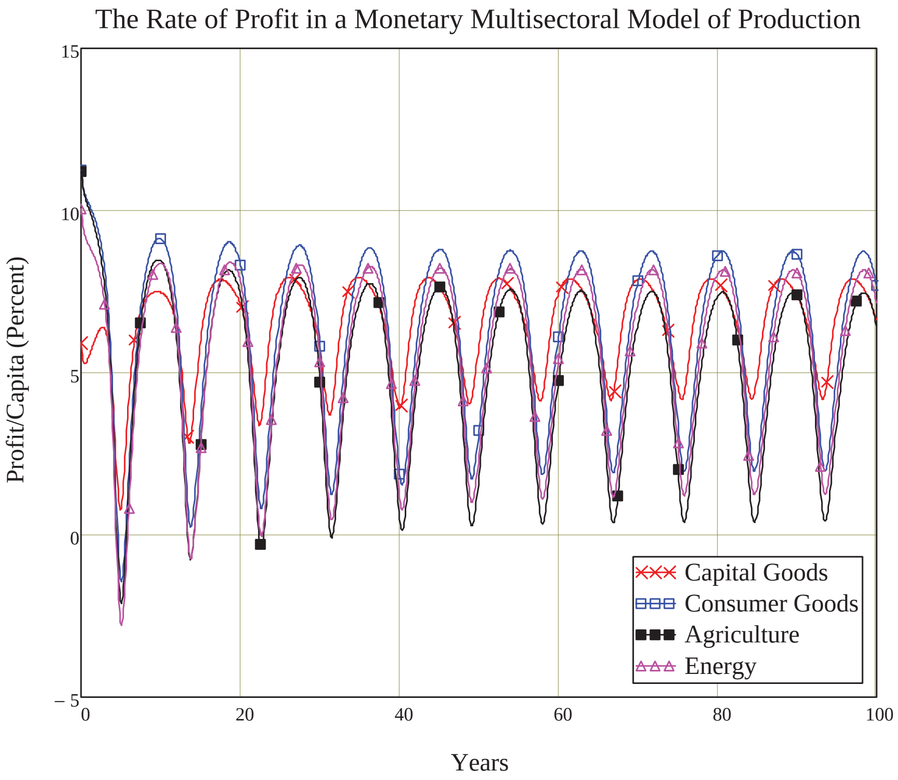
$$\begin{aligned}
&\frac{d}{dt}K_C(t)=\frac{F_{DK}(t)}{\tau_{PR}(\pi_C(t))\cdot P_K(t)}-\gamma K_C(t) && \text{Capital Stock}\\
&\frac{d}{dt}Q_C(t)=-\frac{1}{\tau_{QC}}\cdot\left(Q_C(t)-\frac{1}{v_C}\cdot K_C(t)\right) && \text{Output}\\
&\frac{d}{dt}L_C(t)=-\frac{1}{\tau_{LC}}\cdot\left(L_C(t)-\frac{1}{a_C(t)}\cdot Q_C(t)\right) && \text{Labor}\\
&\frac{d}{dt}P_C(t)=-\frac{1}{\tau_{PC}}\cdot\left(P_C(t)-\frac{W(t)}{a_C(t)\cdot(1-s_C)}\right) && \text{Price Level}\\
&\frac{d}{dt}a_C(t)=\alpha\cdot a_C(t) && \text{Labor Productivity}
\end{aligned} \tag{1.18}$$

The full model is a system of $(2\cdot 2\cdot n+3)+5\cdot n+1$ differential equations, where n is the number of sectors, and the first set of terms specifies the equations in the financial sectsor, the second the equations in production, and the final equation is for population growth. In this sample 4-sector model, this results in a system of 40 nonlinear ODEs.

Results

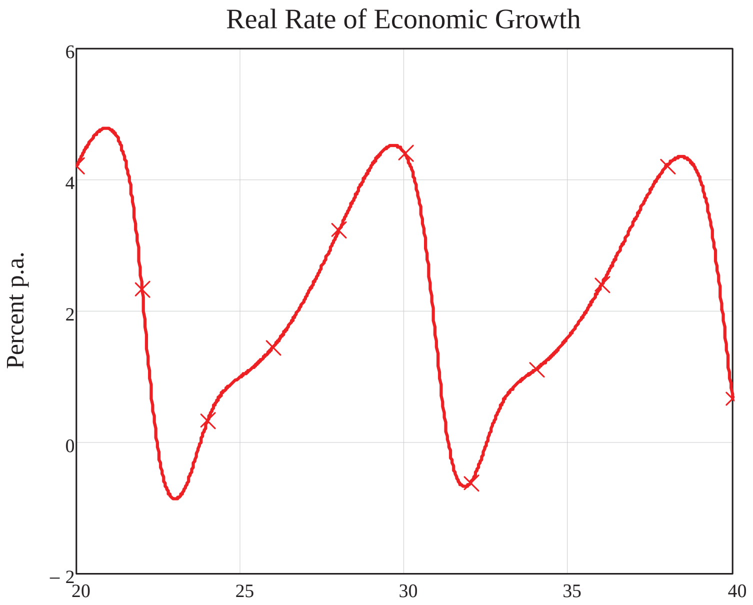
The rate of profit varied between sectors, and, once the system had settled into its limit cycle, ranged from 0.4% p.a. and 8.7%.

Figure 4



The aggregate real rate of economic growth varied between minus 1 and plus 5 percent p.a., and growth followed a sawtooth pattern:

Figure 5

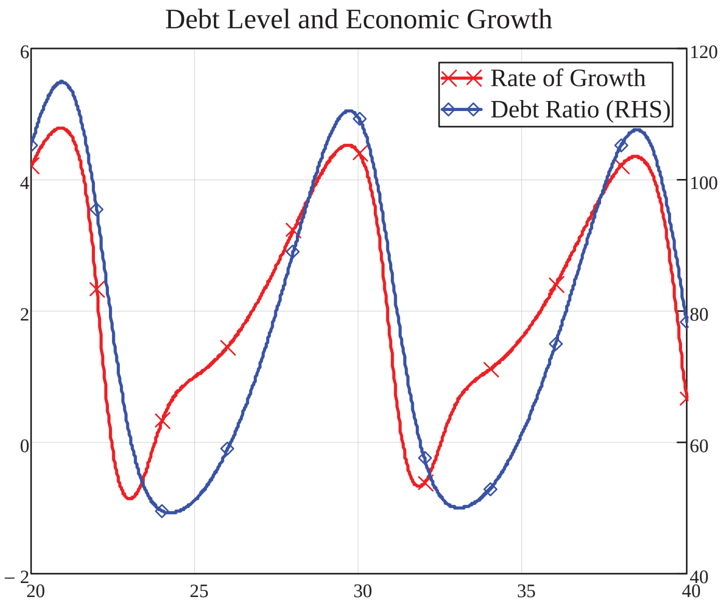


This shape corresponds with the stylized nature of the business cycle, as Blatt observed:

> In the real world, upswings are slow; downswings go with an almighty rush. In the words of Galbraith:
>
> "The usual image of the business cycle was of a wavelike movement, and the waves of the sea were the accepted metaphor… The reality in the nineteenth and early twentieth centuries was, in fact, much closer to the teeth of a ripsaw which go up on a gradual plane on one side and drop precipitately on the other…" (Blatt 1983, pp. 203-204, citing Galbraith 1975, p. 104)

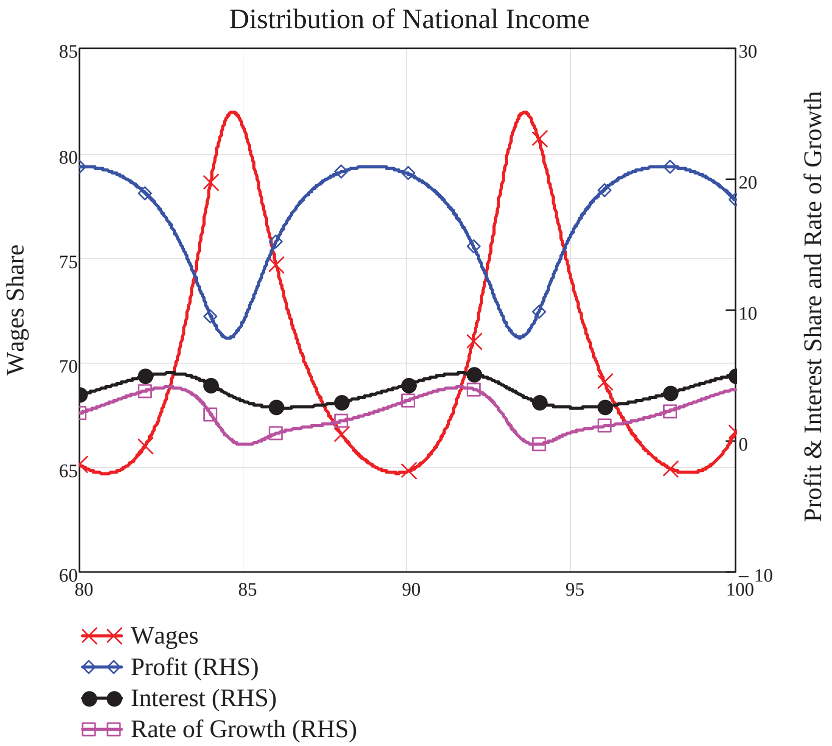
The growth rate and the debt to output level moved together, and the debt ratio cycled between 50 and 110 percent of GDP.

Figure 6



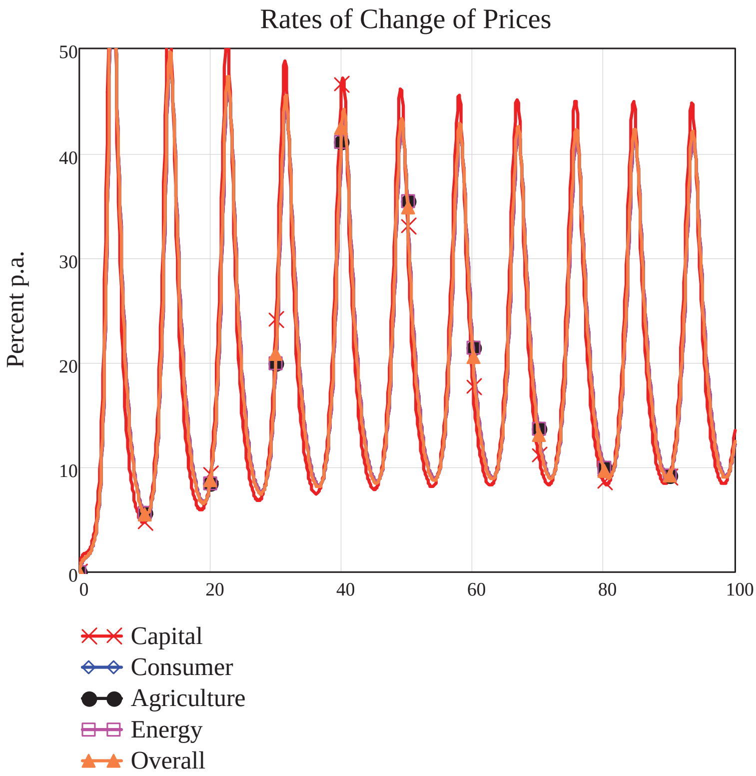
The distribution of income was realistic, though the dynamics were rather more volatile than in actual data:

Figure 7



The rate of inflation was unrealistic, with a minimum of 8 percent p.a. and a maximum of 45 percent.

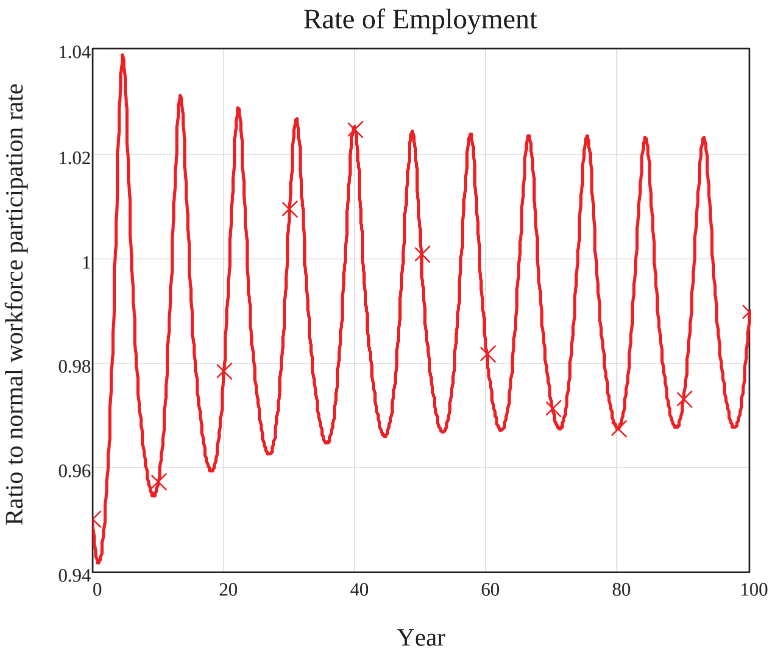
Figure 8



These last two empirical weaknesses probably reflect the specification for the Phillips curve,[8] and the tendency of the model to operate in over-full employment (defined as a ratio of 1 in this simple model) given the parameters used for capitalist and banker behavior.

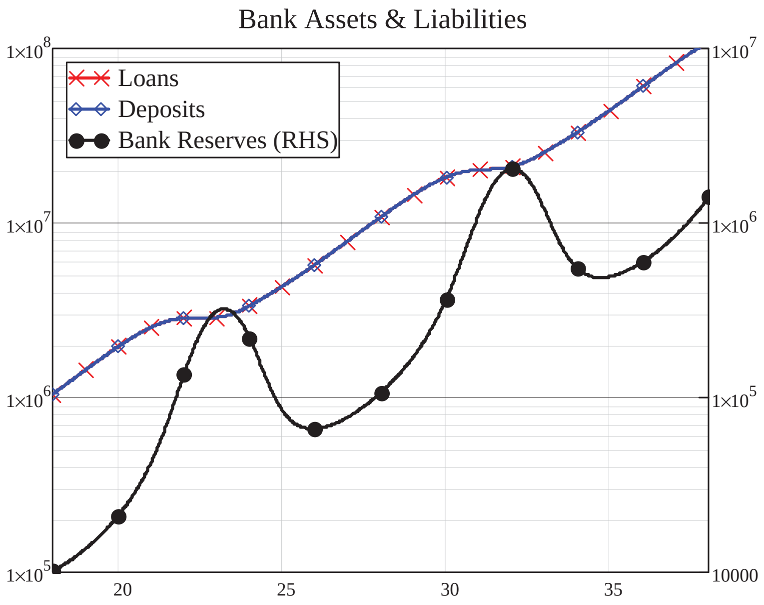
8 In this simple model, the population level effectively meant the available workforce—rather than the total population, with a large proportion of that being not of working age. I also used a simple single factor Phillips Curve, rather than the more realistic 3-factor function used in the preceding single sector model.

Figure 9



Finally, financial dynamics were an essential part of this model: money is far from neutral in this model (and in the real world). Periods of falling economic growth coincided with an increase in bank reserves, and a decline in the level of loans.

Figure 10



Conclusion

Though this preliminary model has many shortcomings, the fact that it works at all shows that it is possible to model the dynamic process by which prices and outputs are set in a multisectoral economy. The failure of the neoclassical school to achieve this objective—which it has had since the time of Walras—may relate to the abstractions it made with the intention of making this process easier to model. These devices—everything from Walras's tatonnement, to ignoring the role of money—may in fact be why they failed. The real world is complex and the real economy is monetary, and complex monetary models are needed to do it justice.

Given the complexity of this model and the sensitivity of complex systems to initial conditions, it is rather remarkable that an obvious limit cycle developed out of an arbitrary set of parameter values and initial conditions—with most (but by no means all) variables in the system keeping within realistic bounds. A conjecture is that this limit cycle is a manifestation of the well-known instability of an input-output matrix (Jorgenson 1960; Jorgenson 1960; Jorgenson 1961; Jorgenson 1961; Hahn 1963; Blatt 1983; Fleissner 1990; Heesterman 1990; Johnson 1993), combined with nonlinear relations that reverse the instability properties of the system as it diverges from its equilibrium. This conjecture was first made by Blatt in a discussion of both the historical evidence of the business cycle and the dual instability of the equilibrium growth path:

> At this stage of the argument, we feel free to offer a *conjecture*: The repeated development of an unstable state of the economy is associated with, and indeed is

> an unavoidable consequence of, the local instability of the state of balanced growth. (Blatt 1983, p. 161)

The presence of monetary buffers—in the guise of deposit accounts—surely also plays a role in the system's capacity, despite its instability, to stay within realistic bounds, in contrast to most (if not all) other dynamic multisectoral models.

I doubt that Kuznets would have been surprised by the failure of equilibrium-oriented attempts to build dynamic multisectoral models of economic growth, since he argued long ago that dynamics had to be different to statics, and in particular that the fetish with equilibrium had to be abandoned:

> According to the economists of the past and to most of their modern followers, static economics is a direct stepping stone to the dynamic system, and may be converted into the latter by the introduction of the general element of change… According to other economists, the body of economic theory must be cardinally rebuilt, if dynamic problems are to be discussed efficiently…
>
> … as long as static economics will remain a strictly unified system based upon the concept of equilibrium, and continue to reduce the social phenomenon to units of rigidly defined individual behavior, its analytic part will remain of little use to any system of dynamic economics… *the static scheme in its entirety, in the essence of its approach, is neither a basis, nor a stepping stone towards a proper discussion of dynamic problems*. Kuznets, S. (1930, pp. 422-428, 435-436; emphasis added)

Yet the static approach—masquerading as dynamics via word games such as using the moniker "Dynamic Stochastic General Equilibrium" to describe bastardized Ramsay-Solow equilibrium growth models—still dominate economics, even after the continuing disaster of the crisis of 2007. Part of the reason for this persistence, I believe, is the seductive simplicity of the "Marshallian Cross" that forms the basis of education in economics: it conforms to Henry Menchen's aphorism that "Explanations exist; they have existed for all time; there is always a well-known solution to every human problem—neat, plausible, and wrong".[9] For economics to escape the trap of static equilibrium thinking, we need an alternative foundation methodology that is neat, plausible, and—at least to a first approximation—right.

I offer this model and the tools used to construct it as a first step towards such a neat, plausible and generally correct approach to macroeconomics. A colleague has implemented the Godley Table method for building a dynamic model of financial flows in a prototype dynamic modeling program QED, which is freely downloadable from my blog.[10] A Mathematica implementation is being developed as part of a project with the CSIRO,[11] and it will also be freely available from my blog when it is completed. The ultimate objective is to develop a standalone dynamic monetary macroeconomic modeling tool that is more suited to financial flows than existing systems dynamics programs like Simulink (http://www.mathworks.com/products/simulink/), Vensim (http://www.vensim.com/) and Vissim (http://www.vissim.com/).

[9] See http://en.wikiquote.org/wiki/H._L._Mencken.

[10] Go to http://www.debtdeflation.com/blogs/qed/; QED stands for "Quesnay Economic Dynamics".

[11] The Commonwealth Scientific and Industrial Research Organisation is Australia's public research institution.

The global economy was blindly led into our current financial crisis by an economics profession that had deluded itself into the belief that such phenomena cannot occur. Hopefully, during this crisis, monetary macroeconomic dynamics will finally supplant the static method against which Kuznets inveighed so eloquently at the start of capitalism's previous great financial crisis.

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Acknowledgements

This work results from a collaborative research effort between the United Nations Environment Program (UNEP) and CSIRO Sustainable Ecosystems to establish a regional report on Resource Efficiency: Economics and Outlook for Asia-Pacific. I thank Peter Humphreys of the UWS School of Accounting (and previously Manager in the Group Accounting Research and Policy Section of the Commonwealth Bank of Australia) for advice on banking practice.

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